



Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test II – October 2025 Semester V

Class : III UG
Branch : Mathematics

Time : 2 Hours
Max. Marks : 60

23BMAC10 – Advanced Algebra

Course Outcomes:

CO1: understand the basic concepts of group actions and their applications.

CO2 : recognize and use the Sylow theorems to characterize certain finite groups.

CO3 : know the fundamental concepts in ring theory such as the concepts of ideals, quotient rings, integral domains and fields.

CO4 : learn in detail about polynomial rings.

CO5 : grasp the fundamental properties of finite field extensions and classification of finite fields.

PART A

6 x 1 = 6

Choose the Correct Answer

- In the ring of integers, the ideal $n\mathbb{Z}$ is prime if and only if ____ CO3K2
a. n is integer b. n is prime c. n is composite d. $n < 1$
- In a Euclidean ring R , if $b \neq 0$ is not a unit in R , then CO3K2
a. $d(a) < d(ab)$ b. $d(a) = d(ab)$ c. $d(a) \leq d(ab)$ d. $d(a) = d(b)$
- If $f(x), g(x)$ are non-zero elements in $F[x]$ then $\deg(fg)$ CO4K1
a. $= \deg(f) + \deg(g)$ b. $< \deg(f) + \deg(g)$ c. $\leq \deg(f) + \deg(g)$ d. $= \deg(g)$
- The content of the polynomial $f(x) = a_0 + a_1x + \dots + a_nx^n$ where a_i 's are integers is CO4K1
a. $\text{LCM}(a_0, a_1, \dots, a_n)$ b. $\text{GCD}(a_0, a_1, \dots, a_n)$ c. 1 d. a_0
- A field K is said to be an extension of F if CO5K1
a. $F \subseteq K$ b. $K \subseteq F$ c. $F \subset K$ d. $K \subseteq F$
- If L is a finite extension of F and K is a subfield of L which contains F , then CO5K1
a. $[K:L]$ b. $[L:K]$ c. $[K:F] / [L:F]$ d. $[L:F] / [K:F]$

Part B

3 x 6 = 18

Answer ALL questions

- a. Show that R/A is a field if and only if A is a maximal ideal. CO3K3
(or)
- b. State and prove Fermat's theorem. CO3K3
- a. State and prove the division algorithm. CO4K4
(or)
- b. If $f(x)$ and $g(x)$ are primitive polynomials, then prove that $f(x)g(x)$ is a primitive polynomial. CO4K4
- a. If L is an algebraic extension of K and if K is an algebraic extension of F , then show that L is an algebraic extension of F . CO5K3
(or)
- b. Prove that the elements in K which are algebraic over F form a field a subfield of K . CO5K3

Part C
Answer ALL questions

3 x 12 = 36

10. a. State and prove first isomorphism theorem for rings.

CO3K4

(or)

10. b. State and prove unique factorization theorem

CO3K4

11. a. State and prove the Eienstein Criterion.

CO4K3

(or)

11. b. Show that if R is a unique factorization domain and if $p(x)$ is a primitive polynomial in $R[x]$, then it can be factored in a unique way as the product of irreducible elements in $R[x]$.

CO4K4

12. a. Show that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F .

CO5K3

(or)

12. b. If L is an finite extension of K and if K is an finite extension of F , then show that L is an finite extension of F .

CO5K4

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