3

γ Generalized Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

3.1 Introduction

Mappings are important tools for studying the properties of spaces and for constructing the new spaces from the existing spaces. Balachandran, Sundaram, and Maki (1991) have introduced generalized continuous maps in topological spaces. Warren (1978) gave the characterizations of fuzzy continuous functions characterized by the closure of fuzzy sets, a sub basis of a fuzzy topology and fuzzy neighbourhoods. Jun, Kang and Song (2005) have introduced intuitionistic fuzzy irresolute and continuous mappings. Here in this chapter we have introduced intuitionistic fuzzy γ generalized continuous mappings, intuitionistic fuzzy contra γ generalized continuous mappings, intuitionistic fuzzy almost γ generalized continuous mappings, intuitionistic fuzzy almost contra γ generalized continuous mappings. Also we have provided the relations between these continuous mappings. Moreover intuitionistic fuzzy γ generalized irresolute mappings, intuitionistic fuzzy contra γ generalized irresolute mappings and intuitionistic fuzzy completely γ generalized continuous mappings are introduced and studied. Furthermore we have provided some properties of those continuous mappings and discussed some fascinating propositions.

3.2 Intuitionistic fuzzy γ generalized continuous mappings

In this section we have introduced intuitionistic fuzzy γ generalized continuous mappings and investigated some of their properties.

Definition 3.2.1: A mapping $f: (X, \tau) \to (Y, \sigma)$ is called an ***intuitionistic fuzzy γ generalized*** (IFγG) ***continuous mapping*** if $f^{-1}(V)$ is an IFγGCS in (X, τ) for every IFCS V of (Y, σ).

Example 3.2.2: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ where $\mu_{G_1}(a) = 0.5_a$, $\mu_{G_1}(b) = 0.6_b$, $\nu_{G_1}(a) = 0.5_a$, $\nu_{G_1}(b) = 0.4_b$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$

where $\mu_{G_2}(a) = 0.4_a$, $\mu_{G_2}(b) = 0.3_b$, $\nu_{G_2}(a) = 0.6_a$, $\nu_{G_2}(b) = 0.7_b$, $G_3 = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v)\rangle$ where $\mu_{G_3}(u) = 0.7_u$, $\mu_{G_3}(v) = 0.8_v$, $\nu_{G_3}(u) = 0.3_u$, $\nu_{G_3}(v) = 0.2_v$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here $G_3^c = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$ is an IFCS in Y and $f^{-1}(G_3^c) = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$ is an IFγGCS in (X, τ). Therefore f is an IFγG continuous mapping.

Proposition 3.2.3: Every IF continuous mapping is an IFγG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. Since every IFCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.2.4: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG continuous mapping but not an IF continuous mapping, since $G_3^c = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFCS in X, as $cl(f^{-1}(G_3^c)) = G_1^c \neq f^{-1}(G_3^c)$.

Proposition 3.2.5: Every IF semi continuous mapping is an IFγG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF semi continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.2.6: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG continuous mapping but not an IF semi continuous mapping, since $G_3^c = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3^c))) = int(G_1^c) = G_2 \nsubseteq f^{-1}(G_3^c)$.

Proposition 3.2.7: Every IF pre continuous mapping is an IFγG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF pre continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.2.8: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG continuous mapping but not an IF pre continuous mapping, since $G_3^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFPCS in X, as $cl(int(f^{-1}(G_3^c))) = cl(G_2) = G_1^c \nsubseteq f^{-1}(G_3^c)$.

Proposition 3.2.9: Every IFα continuous mapping is an IFγG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFα continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X. Since every IFαCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.2.10: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim , G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG continuous mapping but not an IFα continuous mapping, since $G_3^c = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFαCS in X, as $cl(int(cl(f^{-1}(G_3^c)))) = cl(int(G_1^c)) = cl(G_2) = G_1^c \nsubseteq f^{-1}(G_3^c)$.

Proposition 3.2.11: Every IFγ continuous mapping is an IFγG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFγ continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFγCS in X. Since every IFγCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

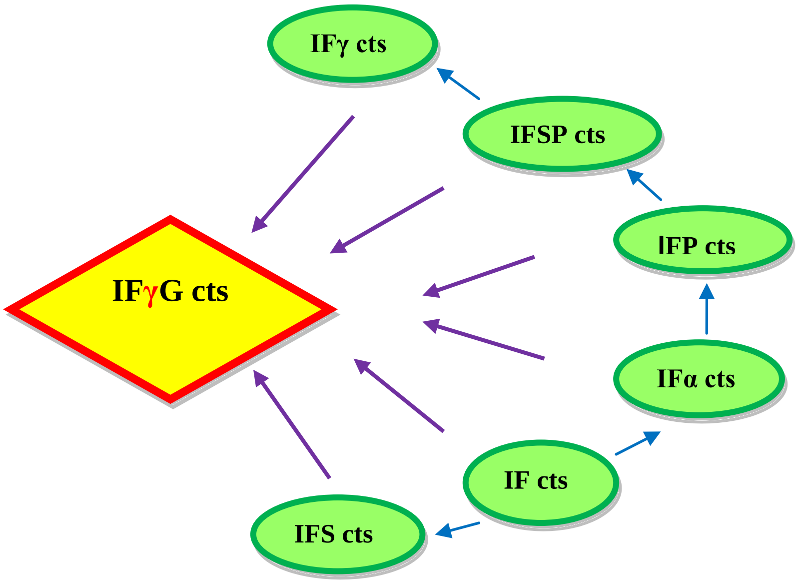
Example 3.2.12: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG continuous mapping but not an IFγ continuous mapping, since $G_3^c = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFγCS in X as $int(cl(f^{-1}(G_3^c))) \cap cl(int(f^{-1}(G_3^c))) = 1_\sim \cap 1_\sim = 1_\sim \not\subseteq f^{-1}(G_3^c)$.

Proposition 3.2.13: Every IF semipre continuous mapping is an IFγG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF semipre continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSPCS in X. Since every IFSPCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.2.14: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG continuous mapping but not an IF semipre continuous mapping since $f^{-1}(G_3^c)$ is not an IFSPCS in X, as there exists no IFPCS B in X such that $int(B) \subseteq f^{-1}(G_3^c) \subseteq B$ in X.

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram 'cts' means continuous. The reverse implications are not true in general in the below diagram.



Proposition 3.2.15: A mapping f: (X, τ) → (Y, σ) is an IFγG continuous mapping if and only if the inverse image of each IFOS in Y is an IFγGOS in X.

Proof: Necessity: Let A be an IFOS in Y. This implies A^c is an IFCS in Y. Then $f^{-1}(A^c)$ is an IFγGCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFγGOS in X.

Sufficiency: Let A be an IFCS in Y. Then A^c is an IFOS in Y. By hypothesis $f^{-1}(A^c)$ is IFγGOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an IFγGOS in X. Therefore $f^{-1}(A)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Proposition 3.2.16.: If f: (X, τ) → (Y, σ) is an IFγG continuous mapping then for each IFP $p_{(\alpha,\beta)}$ of X and each $A \in \sigma$ such that $f(p_{(\alpha,\beta)}) \in A$, there exists an IFγGOS B of X such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$.

Proof: Let $p_{(\alpha,\beta)}$ be an IFP of X and $A \in \sigma$ such that $f(p_{(\alpha,\beta)}) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis, B is an IFγGOS in X such that $p_{(\alpha,\beta)} \in B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Proposition 3.2.17: If f: (X, τ) → (Y, σ) is an IFγG continuous mapping then for each IFP $p_{(\alpha,\beta)}$ of X and each A ∈ σ such that $f(p_{(\alpha,\beta)})\ _q$ A, there exists an IFγGOS B of X such that $p_{(\alpha,\beta)\ q}$ B and f(B) ⊆ A.

Proof: Let $p_{(\alpha,\beta)}$ be an IFP of X and A ∈ σ such that $f(p_{(\alpha,\beta)})\ _q$ A. Put B = f^{-1}(A). Then by hypothesis, B is an IFγGOS in X such that $p_{(\alpha,\beta)\ q}$ B and f(B) = f (f^{-1} (A)) ⊆ A.

Proposition 3.2.18: Let f: (X, τ) → (Y, σ) be an IFγG continuous mapping, then f is an IFγ continuous mapping if X is an $IF\gamma_\gamma T_{1/2}$ space.

Proof: Let V be an IFCS in Y. Then f^{-1} (V) is an IFγGCS in X, by hypothesis. Since X is an $IF\gamma_\gamma T_{1/2}$ space, f^{-1} (V) is an IFγCS in X. Hence f is an IFγ continuous mapping.

Proposition 3.2.19: Let f: (X, τ) → (Y, σ) be an IFγG continuous mapping, then f is an IF continuous mapping if X is an $IF\gamma_c T_{1/2}$ space.

Proof: Let V be an IFCS in Y. Then f^{-1} (V) is an IFγGCS in X, by hypothesis. Since X is an $IF\gamma_c T_{1/2}$ space, f^{-1} (V) is an IFCS in X. Hence f is an IF continuous mapping.

Proposition 3.2.20: Let f: (X, τ) → (Y, σ) be an IFγG continuous mapping and let g: (Y, σ) → (Z, δ) be an IF continuous mapping then g o f : (X, τ) → (Z, δ) is an IFγG continuous mapping.

Proof: Let V be an IFCS in Z. Then g^{-1}(V) is an IFCS in Y, by hypothesis. Since f is an IFγG continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFγGCS in X. Hence g o f is an IFγG continuous mapping.

Proposition 3.2.21: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IF\gamma_\gamma T_{1/2}$spaces:

(i) f is an IFγG continuous mapping,

(ii) f^{-1}(B) is an IFγGOS in X for each IFOS B in Y,

(iii) for each IFP $p_{(\alpha,\beta)}$ in X and for every IFOS B in Y such that $f(p_{(\alpha,\beta)})$ ∈ B, there exists an IFγGOS A in X such that $p_{(\alpha,\beta)}$ ∈ A and f(A) ⊆ B.

Proof: (i) ⇒ (ii) is obvious from the Proposition 3.2.15.

(ii) ⇒ (iii) Let B be any IFOS in Y and let $p_{(\alpha,\beta)} \in$ X. Given $f(p_{(\alpha,\beta)}) \in$ B. By hypothesis $f^{-1}(B)$ is an IFγGOS in X. Take A = $f^{-1}(B)$. Then $p_{(\alpha,\beta)} \in f^{-1}(B)$ = A. This implies $p_{(\alpha,\beta)} \in$ A and $f(A) = f(f^{-1}(B)) \subseteq B$.

(iii) ⇒ (i) Let A be an IFCS in Y. Then its complement, say B is an IFOS in Y. Let $p_{(\alpha,\beta)} \in$ X and $f(p_{(\alpha,\beta)}) \in$ B. Then there exists an IFγGOS, say C in X such that $p_{(\alpha,\beta)} \in$ C and f(C) $\subseteq$ B. Therefore $p_{(\alpha,\beta)} \in C \subseteq f^{-1}(f(C) \subseteq f^{-1}(B)$ and hence $f^{-1}(B)$ is an IFγGOS in X by Proposition 2.4.13. That is $f^{-1}(A^c)$ is an IFγGOS in X and hence $f^{-1}(A)$ is an IFγGCS in X. Thus f is an IFγG continuous mapping.

Proposition 3.2.22: Let f: (X, τ) → (Y, σ) be a mapping that satisfies $f^{-1}(int(B)) \subseteq (int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))))$ for every IFS B in Y. Then f is an IFγG continuous mapping.

Proof: Let B be an IFOS in Y. Then int(B) = B, and by hypothesis $f^{-1}(B) \subseteq (int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))))$. This implies $f^{-1}(B)$ is an IFγOS in X. Therefore it is an IFγGOS in X and hence f is an IFγG continuous mapping.

Proposition 3.2.23: Let f: (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if X is an $IF\gamma_{\gamma}T_{1/2}$ space:

(i) f is an IFγG continuous mapping,

(ii) If B is an IFOS in Y then $f^{-1}(B)$ is an IFγGOS in X,

(iii) $f^{-1}(int(B)) \subseteq (int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))))$ for every IFS B in Y.

Proof: (i) ⇒ (ii) is obviously true by Proposition 3.2.15.

(ii) ⇒ (iii) Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IFγGOS in X. Since X is an $IF\gamma_{\gamma}T_{1/2}$ space, $f^{-1}(int(B))$ is an IFOS in X and hence an

IFγOS in X. Therefore $f^{-1}(int(B)) \subseteq (int(cl(f^{-1}(intB))) \cap cl(int(f^{-1}(intB)))) \subseteq (int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))))$.

(iii) ⇒ (i) Let B be an IFCS in Y. Then its complement, say A is an IFOS in Y, then int(A) = A. Now by hypothesis $f^{-1}(int(A)) \subseteq (int(cl(f^{-1}(A))) \cap cl(int(f^{-1}(A))))$. This implies $f^{-1}(A) \subseteq (int(cl(f^{-1}(A))) \cap cl(int(f^{-1}(A))))$. Hence $f^{-1}(A)$ is an IFγOS in X. Since every IFγOS is an IFγGOS, $f^{-1}(A)$ is an IFγGOS in X. Thus $f^{-1}(B)$ is an IFγGCS in X, since $f^{-1}(A) = f^{-1}(B^c)$. Hence f is an IFγG continuous mapping.

Proposition 3.2.24: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IF\gamma_\gamma T_{1/2}$ spaces:

(i) f is an IFγG continuous mapping,

(ii) $(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B)))) \subseteq f^{-1}(\gamma cl(B))$ for each IFCS B in Y,

(iii) $f^{-1}(\gamma int(B)) \subseteq (int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B))))$ for each IFOS B of Y,

(iv) $f(int(cl(A)) \cap cl(int(A))) \subseteq cl(f(A))$ for each IFS A of X.

Proof: (i) ⇒ (ii) Let B be an IFCS in Y. Then $f^{-1}(B)$ is an IFγGCS in X, by hypothesis. Since X is an $IF\gamma_\gamma T_{1/2}$ space, $f^{-1}(B)$ is an IFγCS in X. Therefore $(int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B)))) \subseteq f^{-1}(B) = f^{-1}(\gamma cl(B))$.

(ii) ⇒ (iii) can be easily proved by taking complement in (ii).

(iii) ⇒ (iv) Let A ⊆ X. Then B = f(A) in Y and therefore $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Here int(f(A)) = int(B) is an IFOS in Y. Then (iii) implies that $f^{-1}(\gamma int(int(B))) \subseteq (int(cl(f^{-1}(int(B)))) \cup cl(int(f^{-1}(int(B))))) \subseteq (int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B))))$. Now $(int(cl(A^c)) \cup cl(int(A^c)))^c \subseteq (int(cl(f^{-1}(B)^c)) \cup cl(int(f^{-1}(B)^c)))^c \subseteq (f^{-1}(\gamma int(int(B^c))))^c$. Therefore $int(cl(A)) \cap cl(int(A)) \subseteq f^{-1}(\gamma cl(cl(B)))$. Now $f(int(cl(A)) \cap cl(int(A))) \subseteq f(f^{-1}(\gamma cl(cl(B)))) \subseteq cl(B) = cl(f(A))$.

(iv) ⇒ (i) Let B be any IFCS in Y, then $f^{-1}(B)$ is an IFS in X. By hypothesis $f(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B)))) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B) = B$. Now $(int(cl(f^{-1}(B))) \cap$

cl(int(f^{-1}(B)))) ⊆ f^{-1} (f((int(cl(f^{-1}(B))) ∩ cl(int(f^{-1}(B))))) ⊆ f^{-1} (B). This implies f^{-1} (B) is an IFγCS and hence it is an IFγGCS in X. Thus f is an IFγG continuous mapping.

Proposition 3.2.25: A mapping f: (X, τ) → (Y, σ) is an IFγG continuous mapping if cl(int(cl(f^{-1}(A)))) ⊆ f^{-1}(cl(A)) for every IFS A in Y.

Proof: Let A be an IFOS in Y then A^c is an IFCS in Y. By hypothesis, cl(int(cl(f^{-1}(A^c)))) ⊆ f^{-1}(cl(A^c)) = f^{-1}(A^c). Now (int(cl(int(f^{-1}(A)))))^c = cl(int(cl(f^{-1}(A^c)))) ⊆ f^{-1}(A^c) = (f^{-1}(A))^c. This implies f^{-1}(A) ⊆ int(cl(int(f^{-1}(A)))). Hence f^{-1}(A) is an IFαOS and hence it is an IFγGOS. Therefore f is an IFγG continuous mapping.

Proposition 3.2.26: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an IFγ$_\gamma$ $T_{1/2}$ space:

(i) f is an IFγG continuous mapping,

(ii) f^{-1} (B) is an IFγGCS in X for every IFCS B in Y,

(iii) (int(cl(f^{-1} (A))) ∩ cl(int(f^{-1} (A)))) ⊆ f^{-1} (cl(A)) for every IFS A in Y.

Proof: (i) ⇒ (ii) is obvious from Definition 3.2.1.

(ii) ⇒ (iii) Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, f^{-1} (cl(A)) is an IFγGCS in X. Since X is an IFγ$_\gamma$ $T_{1/2}$ space, f^{-1} (cl(A)) is an IFγCS. Therefore (int(cl(f^{-1} (cl(A)))) ∩ cl(int(f^{-1} (cl(A))))) ⊆ f^{-1} (cl(A)). Now (int(cl(f^{-1} (A))) ∩ cl(int(f^{-1} (A)))) ⊆ (int(cl(f^{-1} (cl(A)))) ∩ cl(int(f^{-1} (cl(A))))) ⊆ f^{-1} (cl(A)).

(iii) ⇒ (i) Let A be an IFCS in Y. By hypothesis (int(cl(f^{-1} (A))) ∩ cl(int(f^{-1} (A)))) ⊆ f^{-1} (cl(A)) = f^{-1} (A). This implies f^{-1} (A) is an IFγCS in X and hence it is an IFγGCS. Thus f is an IFγG continuous mapping.

Proposition 3.2.27: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y that satisfies f^{-1}(int(B)) = int(cl(f^{-1}(B))) for every IFS B in Y. Then f is an IFγG continuous mapping.

Proof: Let B be an IFOS in Y, then int(B) = B. By hypothesis $f^{-1}(B) = int(cl(f^{-1}(B)))$. This implies $f^{-1}(B)$ is an IFROS in X. Therefore it is an IFγGOS in X. Hence f is an IFγG continuous mapping.

Proposition 3.2.28: Let f: (X, τ) → (Y, σ) be an IFγG continuous mapping, then f is an IFP continuous mapping if X is an $IF\gamma_pT_{1/2}$ space.

Proof: Since every IFγGCS is an IFPCS in an $IF\gamma_pT_{1/2}$ space, the proof is obvious.

Proposition 3.2.29: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y that satisfies $f^{-1}(int(B)) \subseteq cl(int(f^{-1}(B)))$ for every IFS B in Y. Then f is an IFγG continuous mapping.

Proof: Let B be an IFOS in Y. Then int(B) = B, by hypothesis $f^{-1}(B) = f^{-1}(int(B)) \subseteq cl(int(f^{-1}(B)))$. This implies $f^{-1}(B)$ is an IFSOS in X. Therefore it is an IFγGOS in X. Hence f is an IFγG continuous mapping.

Proposition 3.2.30: If f: (X, τ) → (Y, σ) is an IFγG continuous mapping and g: (Y, σ) → (Z, δ) is an IFG continuous mapping and Y is an $IFT_{1/2}$ space, then $g \circ f$: (X, τ) → (Z, δ) is an IFγG continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFGCS in Y, by hypothesis. Since Y is an $IFT_{1/2}$ space, $g^{-1}(V)$ is an IFCS in Y. Therefore $f^{-1}(g^{-1}(V))$ is an IFγGCS in X, by hypothesis. Hence $g \circ f$ is an IFγG continuous mapping.

Proposition 3.2.31: Let f : (X, τ) → (Y, σ) be an IFγG continuous mapping, then

(i) f is an IF continuous mapping if X is an $IF\gamma_c T_{1/2}$ space,

(ii) f is an IFP continuous mapping if X is an $IF\gamma_p T_{1/2}$ space.

Proof: (i) Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFγGCS in X, by hypothesis. Since X is an $IF\gamma_c T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous mapping.

(ii) Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFγGCS in X, by hypothesis. Since X is an $IF\gamma_pT_{1/2}$ space, $f^{-1}(A)$ is an IFPCS in X. Hence f is an IFP continuous mapping.

Proposition 3.2.32: Let f : (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if X is an IFγ_c $T_{1/2}$ space:

(i) f is an IFγG continuous mapping

(ii) for each IFP $p_{(\alpha,\beta)}$ in X and for every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFγGOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$

(iii) for each IFP $p_{(\alpha,\beta)}$ in X and for every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFγGOS B in X such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$

Proof: (i) ⇒ (ii) Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha,\beta)}) \in C \subseteq A$. Since f is an IFγG continuous mapping, $f^{-1}(C) = B$ (say), is an IFγGOS in X and $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \in f^{-1}(C) \subseteq f^{-1}(A)$. Therefore $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.

(ii) ⇒ (iii) Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFγGOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$, by hypothesis. Therefore $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(iii) ⇒ (i) Let B be an IFOS in Y and let $p_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(p_{(\alpha,\beta)}) \in B$. Therefore B is an IFN of $f(p_{(\alpha,\beta)})$. Then, by hypothesis there exists an IFγGOS A in X such that $p_{(\alpha,\beta)} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Therefore $f^{-1}(B)$ is an IFγGOS in X. Hence f is an IFγG continuous mapping.

Definition 3.2.33: Let (X, τ) be an IFTS. Then the ***γ generalized closure*** (γgcl(A)) for any IFS A is defined as,

$$\gamma gcl(A) = \cap \{K \,/\, K \text{ is an IF}\gamma\text{GCS in X and } A \subseteq K\}.$$

Remark 3.2.34: If A is an IFγGCS, then γgcl(A) = A but the converse is not true as the intersection of two non - trivial IFγGCSs need not be an IFγGCS.

Proposition 3.2.35: Let f: (X, τ) → (Y, σ) be an IFγG continuous mapping. Then the following conditions hold:

(i) $f(\gamma gcl(A)) \subseteq cl(f(A))$, for every IFS A in X

(ii) $\gamma gcl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, for every IFS B in Y

Proof: (i) Let A be any IFS in X. Then f(A) is an IFS in Y and cl(f(A)) is an IFCS in Y. Since f is an IFγG continuous mapping, f^{-1}(cl(f(A))) is an IFγGCS in X. That is γgcl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A))). Therefore f(γgcl(A)) ⊆ f(γgcl(f^{-1}(f(A)))) ⊆ f(γgcl(f^{-1}(cl(f(A)))) ⊆ f(f^{-1}(cl(f(A)))⊆ cl(f(A)).

(ii) Replacing A by f^{-1} (B) in (i), we get f(γgcl(f^{-1}(B))) ⊆ cl(f(f^{-1} (B))) ⊆ cl(B). Hence γgcl(f^{-1}(B)) ⊆ f^{-1} (f(γgcl(f^{-1}(B))) ⊆ f^{-1} (cl(B)), for every IFS B in Y.

3.3 Intuitionistic fuzzy contra γ generalized continuous mappings

In this section we have introduced intuitionistic fuzzy contra γ generalized continuous mappings and investigated some of their properties.

Definition 3.3.1: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy contra γ generalized*** (IF contra γG) ***continuous mapping*** if f^{-1}(V) is an IFγGCS in (X, τ) for every IFOS V of (Y, σ).

Example 3.3.2: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩, G_2 = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ and G_3 = ⟨y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG continuous mapping.

Proposition 3.3.3: Every IF contra continuous mapping is an IF contra γG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF contra continuous mapping. Let V be an IFOS in Y. Then f^{-1}(V) is an IFCS in X. Since every IFCS is an IFγGCS, f^{-1}(V) is an IFγGCS in X. Hence f is an IF contra γG continuous mapping.

Example 3.3.4: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩, G_2 = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ and G_3 = ⟨y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG continuous mapping but not an IF contra continuous mapping, since $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$ is an IFOS in Y but $f^{-1}(G_3)$ is not an IFCS in X, as $cl(f^{-1}(G_3)) = G_1^c \neq f^{-1}(G_3)$

Proposition 3.3.5: Every IF contra semi continuous mapping is an IF contra γG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF contra semi continuous mapping. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IF contra γG continuous mapping.

Example 3.3.6: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG continuous mapping but not an IF contra semi continuous mapping, since $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$ is an IFOS in Y but $f^{-1}(G_3)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3))) = int(G_1^c) = G_2 \not\subseteq f^{-1}(G_3)$.

Proposition 3.3.7: Every IF contra pre continuous mapping is an IF contra γG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF contra pre continuous mapping. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IF contra γG continuous mapping.

Example 3.3.8: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG continuous mapping but not an IF contra pre continuous mapping, since $G_3 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFOS in Y but $f^{-1}(G_3)$ is not an IFPCS in X, as $cl(int(f^{-1}(G_3))) = cl(G_2) = G_1^c \not\subseteq f^{-1}(G_3)$.

Proposition 3.3.9: Every IF contra α continuous mapping is an IF contra γG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) $\to$ (Y, σ) be an IF contra α continuous mapping. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFαCS in X. Since every IFαCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IF contra γG continuous mapping.

Example 3.3.10: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG continuous mapping but not an IF contra α continuous mapping, since $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$ is an IFOS in Y but $f^{-1}(G_3)$ is not an IFαCS in X, as $cl(int(cl(f^{-1}(G_3)))) = cl(int(G_1^c)) = cl(G_2) = G_1^c \not\subseteq f^{-1}(G_3)$.

Proposition 3.3.11: Every IF contra γ continuous mapping is an IF contra γG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) $\to$ (Y, σ) be an IF contra γ continuous mapping. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFγCS in X. Since every IFγCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IF contra γG continuous mapping.

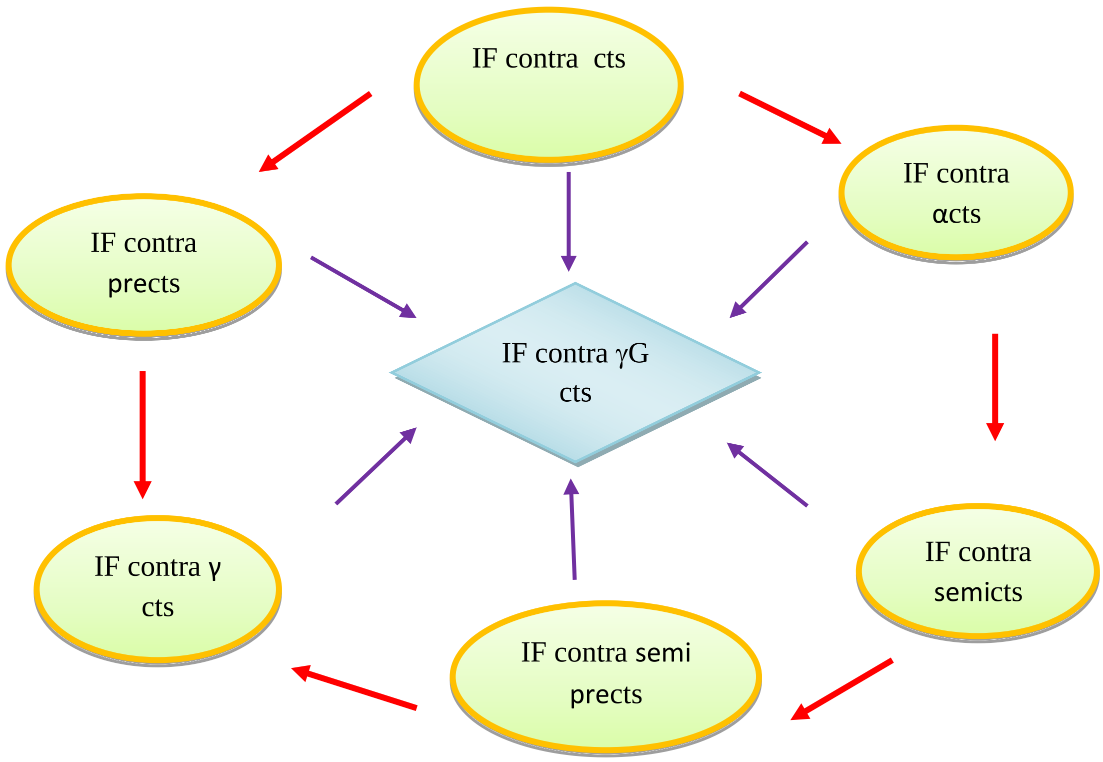
Example 3.3.12: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG continuous mapping but not an IF contra γ continuous mapping, since $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$ is an IFOS in Y but $f^{-1}(G_3)$ is not an IFγCS in X, as $int(cl(f^{-1}(G_3))) \cap cl(int(f^{-1}(G_3))) = 1_\sim \cap 1_\sim = 1_\sim \not\subseteq f^{-1}(G_3)$.

Proposition 3.3.13: Every IF contra semipre continuous mapping is an IF contra γG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF contra semipre continuous mapping. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFSPCS in X. Since every IFSPCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IF contra γG continuous mapping.

Example 3.3.14: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.2_v), (0.5_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim , G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim , G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG continuous mapping but not an IF contra semipre continuous mapping, since G_3 is an IFOS in Y whereas $f^{-1}(G_3)$ is not an IFSPCS in X as there exists no IFPCS B in X such that int(B) ⊆ $f^{-1}(G_3)$ ⊆ B in X.

The relation between various types of intuitionistic fuzzy contra continuous mappings and IF contra γG continuous mapping is given in the following diagram. In this diagram 'cts.' means continuous mapping.



The reverse implications are not true in general in the above diagram.

Proposition 3.3.15: A mapping f: (X, τ) → (Y, σ) is an IF contra γG continuous mapping if and only if the inverse image of each IFCS in Y is an IFγGOS in X.

Proof: Necessity: Let A be an IFCS in Y. This implies A^c is an IFOS in Y. Then $f^{-1}(A^c)$ is an IFγGCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFγGOS in X.

Sufficiency: Let A be an IFOS in Y. Then A^c is an IFCS in Y. By hypothesis $f^{-1}(A^c)$ is IFγGOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an IFγGOS in X. Therefore $f^{-1}(A)$ is an IFγGCS in X. Hence f is an IF contra γG continuous mapping.

Proposition 3.3.16: Let f: (X, τ) → (Y, σ) be a bijective mapping, suppose that one of the following properties hold:

(i) $f^{-1}(cl(B)) \subseteq int(\gamma cl(f^{-1}(B)))$ for each IFS B in Y,

(ii) $cl(\gamma int(f^{-1}(B))) \subseteq f^{-1}(int(B))$ for each IFS B in Y,

(iii) $f(cl(\gamma int(A))) \subseteq int(f(A))$ for each IFS A in X,

(iv) $f(cl(A)) \subseteq int(f(A))$ for each IFγOS A in X.

Then f is an IF contra γG continuous mapping.

Proof: (i) ⇒ (ii) is obvious by taking complement of (i).

(ii) ⇒ (iii) Let A ⊆ X. Put B = f(A) in Y. This implies $A = f^{-1}(f(A)) = f^{-1}(B)$ in X. Now $cl(\gamma int(A)) = cl(\gamma int(f^{-1}(B))) \subseteq f^{-1}(int(B))$ by (ii). Therefore $f(cl(\gamma int(A))) \subseteq f(f^{-1}(int(B))) = int(B) = int(f(A))$.

(iii) ⇒ (iv) Let A ⊆ X be an IFγOS. Then γint(A) = A. By hypothesis, $f(cl\gamma(int(A))) \subseteq int(f(A))$. Therefore $f(cl(A)) = f(cl(\gamma int(A))) \subseteq int(f(A))$.

Suppose (iv) holds. Let A be an IFOS in X. Then f(A) is an IFS in Y and γint(f(A)) is an IFγOS in Y. Hence by hypothesis, $f^{-1}(cl(\gamma int(f(A)))) \subseteq int(f^{-1}(\gamma int(f(A)))) \subseteq int(f^{-1}(f(A))) = int(A) = A$. Therefore $cl(\gamma int(f(A))) = f(f^{-1}(cl(\gamma int(f(A))))) \subseteq f(A)$. Now cl(int(f(A))) ⊆ cl(γint(f(A))) ⊆ f(A). This implies f(A) is an IFPCS in Y and hence an IFγGCS in Y. Thus f is an IF contra γG continuous mapping.

Proposition 3.3.17: Let f: (X, τ) → (Y, σ) be a mapping. Suppose that one of the following properties hold:

(i) f(γcl(A)) ⊆ int(f(A)) for each IFS A in X,

(ii) γcl(f^{-1}(B)) ⊆ f^{-1}(int(B)) for each IFS B in Y,

(iii) f^{-1}(cl(B)) ⊆ γint(f^{-1}(B)) for each IFS B in Y.

Then f is an IF contra γG continuous mapping.

Proof: (i) ⇒ (ii) Let B ⊆ Y. Then f^{-1}(B) is an IFS in X. By hypothesis, f(γcl(f^{-1}(B))) ⊆ int(f(f^{-1}(B))) ⊆ int(B). Now γcl(f^{-1}(B)) ⊆ f^{-1}(f(γcl(f^{-1}(B)))) ⊆ f^{-1}(int(B)).

(ii) ⇒ (iii) is obvious by taking complement in (ii).

Suppose (iii) holds. Let A be an IFCS in Y. Then cl(A) = A and f^{-1}(A) is an IFS in X. Now f^{-1}(A) = f^{-1}(cl(A)) ⊆ γint(f^{-1}(A)) ⊆ f^{-1}(A), by hypothesis. This implies f^{-1}(A) is an IFγOS in X and hence an IFγGOS in X. Therefore f is an IF contra γG continuous mapping.

Proposition 3.3.18: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then f is an IF contra γG continuous mapping if cl(f(A)) ⊆ f(γint(A)) for every IFS A in X.

Proof: Let A be an IFCS in Y. Then cl(A) = A and f^{-1}(A) is an IFS in X. By hypothesis cl(f(f^{-1}(A))) ⊆ f(γint(f^{-1}(A))). Since f is bijective, f(f^{-1}(A)) = A. Therefore A = cl(A) = cl(f(f^{-1}(A))) ⊆ f(γint(f^{-1}(A))). Now f^{-1}(A) ⊆ f^{-1}(f(γint(f^{-1}(A)))) = γint(f^{-1}(A)) ⊆ f^{-1}(A). Hence f^{-1}(A) is an IFγOS in X and hence an IFγGOS in X. Thus f is an IF contra γG continuous mapping.

Proposition 3.3.19: If f: (X, τ) → (Y, σ) is a mapping where X is an IF$\gamma_\gamma T_{1/2}$ space, then the following are equivalent:

(i) f is an IF contra γG continuous mapping

(ii) For each IFP $p_{(\alpha,\beta)} \in X$ and for each IFCS B containing $f(p_{(\alpha,\beta)})$, there exists an IFγOS $A \subseteq X$ and $p_{(\alpha,\beta)} \in A$ such that $A \subseteq f^{-1}(B)$

(iii) For each IFP $p_{(\alpha,\beta)} \in X$ and for each IFCS B containing $f(p_{(\alpha,\beta)})$, there exists an IFγOS $A \subseteq X$ and $p_{(\alpha,\beta)} \in A$ such that $f(A) \subseteq B$

Proof: (i) ⇒ (ii) Let B be an IFCS in Y. Let $p_{(\alpha,\beta)}$ be an IFP in X such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \in f^{-1}(B)$. By hypothesis $f^{-1}(B)$ is an IFγGOS in X. Since X is an IF$\gamma_\gamma T_{1/2}$ space, $f^{-1}(B)$ is an IFγOS in X. Now let $A = \gamma int(f^{-1}(B)) \subseteq f^{-1}(B)$. Then $A \subseteq f^{-1}(B)$.

(ii) ⇒ (iii) Let B be an IFCS in Y. Let $p_{(\alpha,\beta)}$ be an IFP in X such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \in f^{-1}(B)$. By hypothesis $f^{-1}(B)$ is an IFγOS in X and $A \subseteq f^{-1}(B)$. This implies $f(A) \subseteq f(f^{-1}(B)) \subseteq B$.

(iii) ⇒ (i) Let B be an IFCS in Y and let $p_{(\alpha,\beta)} \in X$ and $f(p_{(\alpha,\beta)}) \in B$. By hypothesis there exists an IFγOS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$. This implies $p_{(\alpha,\beta)} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. That is $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since A is an IFγOS, $A = \gamma int(A) \subseteq \gamma int(f^{-1}(B))$. Therefore $p_{(\alpha,\beta)} \in \gamma int(f^{-1}(B))$. But $f^{-1}(B) = \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} \{p_{(\alpha,\beta)}\} \subseteq \gamma int(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence $f^{-1}(B)$ is an IFγOS in X and hence $f^{-1}(B)$ is an IFγGOS in X. Thus f is an IF contra γG continuous mapping.

Proposition 3.3.20: If f: (X, τ) → (Y, σ) is an IF contra γG continuous mapping, where X is an IF$\gamma_\gamma T_{1/2}$ space, then the following conditions hold:

(i) $\gamma cl(f^{-1}(B)) \subseteq f^{-1}(int(\gamma cl(B)))$ for every IFOS B in Y

(ii) $f^{-1}(cl(\gamma int(B))) \subseteq \gamma int(f^{-1}(B))$ for every IFCS B in Y

Proof: (i) Let $B \subseteq Y$ be an IFOS. By hypothesis $f^{-1}(B)$ is an IFγGCS in X. Since X is an $IF\gamma_{\gamma}T_{1/2}$ space, $f^{-1}(B)$ is an IFγCS in X. This implies $\gamma cl(f^{-1}(B)) = f^{-1}(B) = f^{-1}(int(B)) \subseteq f^{-1}(int(\gamma cl(B)))$.

(ii) can be proved easily by taking the complement of (i).

Proposition 3.3.21: If f: $(X, \tau) \rightarrow (Y, \sigma)$ is an IF contra γG continuous mapping and g: $(Y, \sigma) \rightarrow (Z, \delta)$ is an IF continuous mapping then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is an IF contra γG continuous mapping.

Proof: Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFOS in Y, since g is an IF continuous mapping. Since f is an IF contra γG continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFγGCS in X. Therefore g o f is an IF contra γG continuous mapping.

Proposition 3.3.22: If f: $(X, \tau) \rightarrow (Y, \sigma)$ is an IF contra γG continuous mapping and g: $(Y, \sigma) \rightarrow (Z, \delta)$ is an IF contra continuous mapping then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is an IFγG continuous mapping.

Proof: Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFCS in Y, since g is an IF contra continuous mapping. Since f is an IF contra γG continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFγGOS in X. Therefore g o f is an IFγG continuous mapping.

Proposition 3.3.23: For a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$, where X is an $IF\gamma_{\gamma}T_{1/2}$ space, the following are equivalent:

(i) f is an IF contra γG continuous mapping

(ii) For every IFCS A in Y and for every IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$ then $p_{(\alpha,\beta)}\ _q\ \gamma int(f^{-1}(A))$

(iii) For every IFCS in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$ then there exists an IFγGOS B such that $p_{(\alpha,\beta)}\ _q\ B$ and $f(B) \subseteq A$

Proof: (i) ⇒ (ii) Let f be an IF contra γG continuous mapping. Let $A \subseteq Y$ be an IFCS and let $p_{(\alpha,\beta)} \in X$. Also let $f(p_{(\alpha,\beta)})\ _q\ A$, then $p_{(\alpha,\beta)}\ _q\ f^{-1}(A)$. By hypothesis $f^{-1}(A)$ is an IFγGOS in X. Since X is an $IF\gamma_{\gamma}T_{1/2}$**,** $f^{-1}(A)$ is an IFγOS in X. Hence $\gamma int(f^{-1}(A)) = f^{-1}(A)$. This implies $p_{(\alpha,\beta)}\ _q\ \gamma int(f^{-1}(A))$.

(ii) ⇒ (i) Let $A \subseteq Y$ be an IFCS, then $f^{-1}(A)$ is an IFS in X. Let $p_{(\alpha,\gamma)} \in X$ and let $f(p_{(\alpha,\beta)})_q\ A$ then $p_{(\alpha,\beta)\,q}\ f^{-1}(A)$. By hypothesis, this implies $p_{(\alpha,\beta)\,q}\ \gamma int(f^{-1}(A))$. That is $f^{-1}(A) \subseteq \gamma int(f^{-1}(A))$. But $\gamma int(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $\gamma int(f^{-1}(A)) = f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFγOS in X and hence an IFγGOS in X**.** This implies f is an IF contra γG continuous mapping.

(ii) ⇒ (iii) Let $A \subseteq Y$ be an IFCS, then $f^{-1}(A)$ is an IFS in X. Let $p_{(\alpha,\beta)} \in X$. Also let $f(p_{(\alpha,\beta)})_q\ A$ then $p_{(\alpha,\beta)\,q}\ f^{-1}(A)$. By hypothesis, this implies $p_{(\alpha,\beta)\,q}\ \gamma int(f^{-1}(A))$. That is $f^{-1}(A) \subseteq \gamma int(f^{-1}(A))$. But $\gamma int(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $\gamma int(f^{-1}(A)) = f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFγOS in X and hence an IFγGOS in X**.** Let $f^{-1}(A) = B$. Therefore $p_{(\alpha,\beta)\,q}\ B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

(iii) ⇒ (ii) Let $A \subseteq Y$ be an IFCS, then $f^{-1}(A)$ is an IFS in X. Let $p_{(\alpha,\beta)} \in X$. Also let $f(p_{(\alpha,\beta)})_q\ A$ then $p_{(\alpha,\beta)\,q}\ f^{-1}(A)$. By hypothesis, there exists an IFγGOS B in X such that $p_{(\alpha,\beta)\,q}\ B$ and $f(B) \subseteq A$. Let $B = f^{-1}(A)$. Since X is an $IF\gamma_\gamma T_{1/2}$ space, $f^{-1}(A)$ is an IFγOS in X and $\gamma int(f^{-1}(A)) = f^{-1}(A)$. Therefore $p_{(\alpha,\beta)\,q}\ \gamma int(f^{-1}(A))$.

Proposition 3.3.24: A mapping $f: (X, \tau) \to (Y, \sigma)$ is an IF contra γG continuous mapping if and only if $f^{-1}(\gamma cl(B)) \subseteq \gamma int(f^{-1}(cl(B)))$ for every IFS B in Y, where X is an $IF\gamma_\gamma T_{1/2}$ space.

Proof: Necessity: Let $B \subseteq Y$ be an IFS. Then cl(B) is an IFCS in Y. By hypothesis, $f^{-1}(cl(B))$ is an IFγGOS in X. Since X is an $IF\gamma_\gamma T_{1/2}$ space, $f^{-1}(cl(B))$ is an IFγOS in X. Therefore $f^{-1}(\gamma cl(B)) \subseteq f^{-1}(cl(B)) = \gamma int(f^{-1}(cl(B)))$.

Sufficiency: Let $B \subseteq Y$ be an IFCS. Then $cl(B) = B$. By hypothesis, $f^{-1}(\gamma cl(B)) \subseteq \gamma int(f^{-1}(cl(B))) = \gamma int(f^{-1}(B))$. But $\gamma cl(B) = B$. Therefore $f^{-1}(B) = f^{-1}(\gamma cl(B)) \subseteq \gamma int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFγOS in X and hence an IFγGOS in X. Hence f is an IF contra γG continuous mapping.

Proposition 3.3.25: A mapping $f: (X, \tau) \to (Y, \sigma)$ is an IF contra γG continuous mapping if $f^{-1}(\gamma cl(B)) \subseteq int(f^{-1}(B))$ for every IFS B in Y.

Proof: Let $B \subseteq Y$ be an IFCS. Then $cl(B) = B$. Since every IFCS is an IFγCS, $\gamma cl(B) = B$. Now by hypothesis, $f^{-1}(B) = f^{-1}(\gamma cl(B)) \subseteq int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = int(f^{-1}(B))$. Therefore $f^{-1}(B)$ is an IFOS in X. Hence f is an IF contra continuous mapping. Then by Proposition 3.3.3, f is an IF contra γG continuous mapping.

Proposition 3.3.26: Let (X, τ) and (Y, σ) be any two IFTSs. Let $f: (X, \tau) \to (Y, \sigma)$ be any mapping. If the graph $g: X \to X \times Y$ of f is an IF contra γG continuous mapping then f is also an IF contra γG continuous mapping.

Proof: Let A be an IFOS in (Y, σ). By definition $f^{-1}(A) = 1_\sim \cap f^{-1}(A) = g^{-1}(1_\sim \times A)$. Since g is IF contra γG continuous mapping, $g^{-1}(1_\sim \times A)$ is an IFγGCS in (X, τ). Now $f^{-1}(A)$ is an IFγGCS in (X, τ). Thus f is an IF contra γG continuous mapping.

Proposition 3.3.27: Let $f: (X, \tau) \to (Y, \sigma)$ be an IF contra γG continuous mapping, then

(i) f is an IF contra continuous mapping if X is an IF$\gamma_c T_{1/2}$ space,
(ii) f is an IF contra γ continuous mapping if X is an IF$\gamma_\gamma T_{1/2}$ space,
(iii) f is an IF contra pre continuous mapping if X is an IF$\gamma_p T_{1/2}$ space.

Proof: (i) Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFγGCS in X, by hypothesis. Since X is an IF$\gamma_c T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF contra continuous mapping.

(ii) Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFγGCS in X, by hypothesis. Since X is an IF$\gamma_\gamma T_{1/2}$ space, $f^{-1}(A)$ is an IFγCS in X. Hence f is an IF contra γ continuous mapping.

(iii) Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFγGCS in X, by hypothesis. Since X is an IF$\gamma_p T_{1/2}$ space, $f^{-1}(A)$ is an IFPCS in X. Hence f is an IF contra pre continuous mapping.

Proposition 3.3.28: Let $f: (X, \tau) \to (Y, \sigma)$ and $g: (Y, \sigma) \to (Z, \delta)$ be mappings. Then the following conditions are equivalent if X is an IF$\gamma_c T_{1/2}$ space:

(i) $g \circ f : (X, \tau) \to (Z, \delta)$ is an IF contra γG continuous mapping
(ii) $cl(int(cl(g \circ f)^{-1}(B)) \subseteq (g \circ f)^{-1}(B)$for every IFOS in Z

Proof: (i) $\Longrightarrow$ (ii): Let B be any IFOS in Z. Then $(g \circ f)^{-1}(B)$ is an IFγGCS in X by hypothesis. Since X is an IF$\gamma_c T_{1/2}$ space, $(g \circ f)^{-1}(B)$ is an IFCS in X. Therefore

$cl((g \circ f)^{-1}(B)) = ((g \circ f)^{-1}(B))$. Now $cl(int(cl(g \circ f)^{-1}(B))) \subseteq cl(cl(g \circ f)^{-1}(B))) = cl((g \circ f)^{-1}(B)) = (g \circ f)^{-1}(B)$. This implies $cl(int(g \circ f)^{-1}(B)) \subseteq (g \circ f)^{-1}(B)$.

(ii) ⟹ (i): Let B be any IFCS in Z. Then its complement B^c is an IFOS in Z. By hypothesis, $cl(int(cl(g \circ f)^{-1}(B))) \subseteq (g \circ f)^{-1}(B)$. Hence $(g \circ f)^{-1}(B^c)$ is an IFαCS in X. Since every IFαCS is an IFγGCS, $(g \circ f)^{-1}(B^c)$ is an IFγGCS in X. Hence $g \circ f$ is an IF contra γG continuous mapping.

3.4 Intuitionistic fuzzy almost γ generalized continuous mappings

In this section we have introduced intuitionistic fuzzy almost γ generalized continuous mappings, intuitionistic fuzzy almost contra γ generalized continuous mappings and investigated some of their properties.

Definition 3.4.1: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy almost γ generalized*** (IF almost γG) ***continuous mapping*** if $f^{-1}(V)$ is an IFγGCS in (X, τ) for every IFRCS V of (Y, σ).

Example 3.4.2: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost γG continuous mapping in X.

Proposition 3.4.3: Every IF continuous mapping is an IF almost γG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. Since every IFCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IF almost γG continuous mapping.

Example 3.4.4: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Then

$\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF almost γG continuous mapping but not an IF continuous mapping, since $G_3^c = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFCS in X, as $cl(f^{-1}(G_3^c)) = 1_\sim \neq f^{-1}(G_3^c)$.

Proposition 3.4.5: Every IF semi continuous mapping is an IF almost γG continuous mapping but not conversely in general.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be an IF semi continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IF almost γG continuous mapping.

Example 3.4.6: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF almost γG continuous mapping but not an IF semi continuous mapping, since $G_3^c = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3))) = 1_\sim \not\subseteq f^{-1}(G_3^c)$.

Proposition 3.4.7: Every IF pre continuous mapping is an IF almost γG continuous mapping but not conversely in general.

Proof: Let f: $(X, \tau) \to (Y, \sigma)$ be an IF pre continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IF almost γG continuous mapping.

Example 3.4.8: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$, $G_4 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is an IF almost γG continuous mapping but not an IF pre continuous mapping, since $G_3^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFPCS in X, as $cl(int(f^{-1}(G_3^c))) = cl(G_2) = G_1^c \nsubseteq f^{-1}(G_3^c)$.

Proposition 3.4.9: Every IFα continuous mapping is an IF almost γG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFα continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X. Since every IFαCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IF almost γG continuous mapping.

Example 3.4.10: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is an IF almost γG continuous mapping but not an IFα continuous mapping, since $G_3^c = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFαCS in X, as $cl(int(cl(f^{-1}(G_3^c)))) = 1_\sim \nsubseteq f^{-1}(G_3^c)$.

Proposition 3.4.11: Every IFγ continuous mapping is an IF almost γG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFγ continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFγCS in X. Since every IFγCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IF almost γG continuous mapping.

Example 3.4.12: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost γG continuous mapping but not an IFγ continuous mapping, since $G_3^c = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFγCS in X, as $int(cl(f^{-1}(G_3^c))) \cap cl(int(f^{-1}(G_3^c))) = 1_\sim \cap 1_\sim = 1_\sim \nsubseteq f^{-1}(G_3^c)$.

Proposition 3.4.13: Every IF semipre continuous mapping is an IF almost γG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be an IF semipre continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFSPCS in X. Since every IFSPCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IF almost γG continuous mapping.

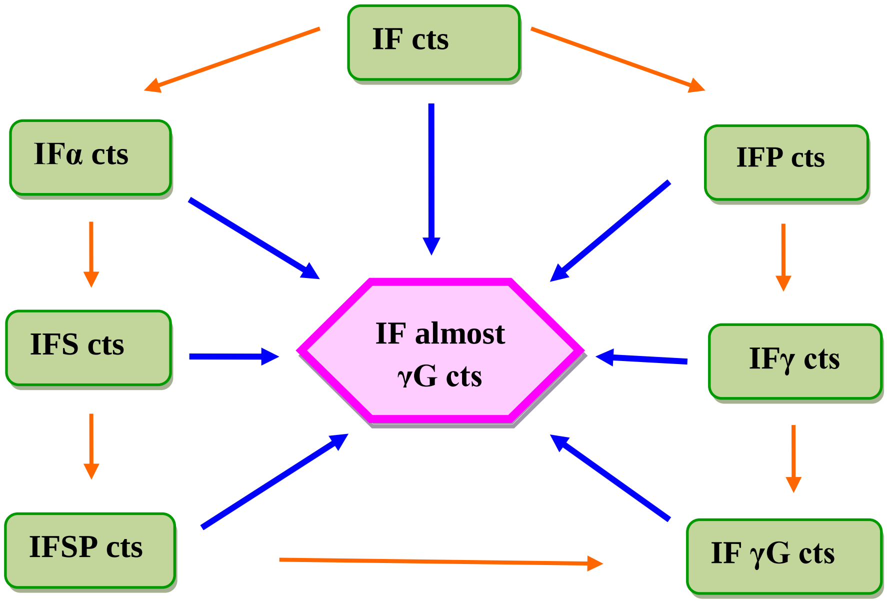
Example 3.4.14: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.2_v), (0.5_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost γG continuous mapping but not an IF semipre continuous mapping, since G_3^c is an IFCS in Y whereas $f^{-1}(G_3^c)$ is not an IFSPCS in X, as there exists no IFPCS B in X such that $int(B) \subseteq f^{-1}(G_3^c) \subseteq B$ in X.

Proposition 3.4.15: Every IFγG continuous mapping is an IF almost γG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) $\rightarrow$ (Y, σ) be an IFγG continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IF almost γG continuous mapping.

Example 3.4.16: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b)\rangle$, $G_2 = \langle y, (0.2_u, 0.2_v), (0.5_u, 0.8_v)\rangle$ and $G_3 = \langle y, (0.2_u, 0.2_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Here f is an IF almost γG continuous mapping but not an IFγG continuous mapping, since $G_2^c = \langle y, (0.5_u, 0.8_v), (0.2_u, 0.2_v)\rangle$ is an IFCS in Y but $f^{-1}(G_2^c)$ is not an IFγGCS in X, as $f^{-1}(G_2^c) \subseteq G_1$ where $\gamma cl(f^{-1}(G_2^c)))) = 1_\sim \nsubseteq G_1$.

The relation between various types of intuitionistic fuzzy continuous mappings with IF almost γG continuous mapping is given in the following diagram. In this diagram 'cts' means continuous mapping. The reverse implications are not true in general in the below diagram.



Proposition 3.4.17: A mapping f: (X, τ) → (Y, σ) is an IF almost γG continuous mapping if and only if the inverse image of each IFROS in Y is an IFγGOS in X.

Proof: Necessity: Let A be an IFROS in Y. This implies A^c is an IFRCS in Y. Since f is an IF almost γG continuous mapping, $f^{-1}(A^c)$ is an IFγGCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFγGOS in X.

Sufficiency: Let A be an IFRCS in Y. This implies A^c is an IFROS in Y. By hypothesis $f^{-1}(A^c)$ is an IFγGOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFγGCS in X. Hence f is an IF almost γG continuous mapping.

Proposition 3.4.18: Let $p_{(\alpha,\beta)}$ be an IFP in X. A mapping f: (X, τ) → (Y, σ) is an IF almost γG continuous mapping if for every IFOS A in Y with $p_{(\alpha,\beta)} \in A$, there exists an IFOS B in X with $p_{(\alpha,\beta)} \in B$ such that $f^{-1}(A)$ is intuitionistic fuzzy dense in B.

Proof: Let A be an IFROS in Y. Then A is an IFOS in Y. Let $p_{(\alpha,\beta)} \in$ A, then there exists an IFOS B in X such that $p_{(\alpha,\beta)} \in$ B and cl(f^{-1}(A)) = B. Therefore cl(f^{-1}(A)) is also an IFOS in X and int(cl(f^{-1}(A))) = cl(f^{-1}(A)). Now f^{-1}(A) ⊆ cl(f^{-1}(A)) = int(cl(f^{-1}(A))). This implies f^{-1}(A) is an IFγOS in X and hence an IFγGOS in X. Thus f is an IF almost γG continuous mapping.

Proposition 3.4.19: Let f: (X, τ) → (Y, σ) be a mapping. If f^{-1}(γint(B)) ⊆ γint(f^{-1}(B)) for every IFS B in Y, then f is an IF almost γG continuous mapping.

Proof: Let B ⊆ Y be an IFROS. By hypothesis, f^{-1}(γint(B)) ⊆ γint(f^{-1}(B)). Since B is an IFROS, it is an IFγOS in Y. Therefore γint(B) = B. Hence f^{-1}(B) = f^{-1}(γint(B)) ⊆ γint(f^{-1}(B)) ⊆ f^{-1}(B). This implies f^{-1}(B) is an IFγOS in X and hence f^{-1}(B) is an IFγGOS in X. Thus f is an IF almost γG continuous mapping.

Remark 3.4.20: The converse of the above Proposition is true if B ⊆ Y is an IFROS and X is an $IF\gamma_{\gamma}T_{1/2}$ space.

Proof: Let f be an IF almost γG continuous mapping. Let B be an IFROS in Y then by hypothesis f^{-1}(B) is an IFγGOS in X. Since X is an $IF\gamma_{\gamma}T_{1/2}$ space, f^{-1}(B) is an IFγOS in X. Therefore f^{-1}(γint(B)) ⊆ f^{-1}(B) = γint(f^{-1}(B)).

Proposition 3.4.21: Let f: (X, τ) → (Y, σ) be a mapping. If γcl(f^{-1}(B)) ⊆ f^{-1}(γcl(B))for every IFS B in Y, then f is an IF almost γG continuous mapping.

Proof: Let B ⊆ Y be an IFRCS. By hypothesis, γcl(f^{-1}(B)) ⊆ f^{-1}(γcl(B)). Since B is an IFRCS, it is an IFγCS in Y. Therefore γcl(B) = B. Hence f^{-1}(B) = f^{-1}(γcl(B)) ⊇ γcl(f^{-1}(B)) ⊇ f^{-1}(B). This implies f^{-1}(B) is an IFγCS in X and hence f^{-1}(B) is an IFγGCS in X. Thus f is an IF almost γG continuous mapping.

Remark 3.4.22: The converse of the above Proposition is true if B ⊆ Y is an IFRCS and X is an $IF\gamma_{\gamma}T_{1/2}$ space.

Proof: Let f be an IF almost γG continuous mapping. Let B be an IFRCS in Y then by hypothesis f^{-1}(B) is an IFγGCS in X. Since X is an $IF\gamma_{\gamma}T_{1/2}$ space, f^{-1}(B) is an IFγCS in X. Therefore γcl(f^{-1}(B)) = f^{-1}(B) ⊆ f^{-1} (γcl(B)).

Proposition 3.4.23: Let f: (X, τ) → (Y, σ) be a mapping where X is an IFγ_γ $T_{1/2}$ space. Then the following are equivalent:

(i) f is an IF almost γG continuous mapping

(ii) γcl(f^{-1}(A)) ⊆ f^{-1}(cl(A)) for every IFγOS A in Y

(iii) γcl(f^{-1}(A)) ⊆ f^{-1}(cl(A)) for every IFSOS A in Y

(iv) f^{-1}(A) ⊆ γint(f^{-1}(int(cl(A)))) for every IFPOS A in Y

Proof: (i) ⇒ (ii) Let A be an IFγOS in Y. Then cl(A) is an IFRCS in Y . By hypothesis f^{-1}(cl(A)) is an IFγGCS in X. Since X is an IFγ_γ $T_{1/2}$ space, f^{-1}(cl(A)) is an IFγCS in X. This implies γcl(f^{-1}(cl(A))) = f^{-1}(cl(A)). Now γcl(f^{-1}(A)) ⊆ γcl(f^{-1}(cl(A))) = f^{-1}(cl(A)). Thus γcl(f^{-1}(A)) ⊆ f^{-1}(cl(A)).

(ii) ⇒ (iii) Since every IFSOS is an IFγOS, proof is obvious.

(iii)⇒(i) Let A be an IFRCS in Y. Then A = cl(int(A)). Therefore A is an IFSOS in Y. By hypothesis, γcl(f^{-1}(A)) ⊆ f^{-1}(cl(A)) = f^{-1}(A) ⊆ γcl(f^{-1}(A)). Hence f^{-1}(A) is an IFγCS and hence is an IFγGCS in X. Thus f is an IF almost γG continuous mapping.

(i) ⇒ (iv) Let A be an IFPOS in Y. Then A ⊆ int(cl(A)). Since int(cl(A)) is an IFROS in Y, by hypothesis, f^{-1}(int(cl(A))) is an IFγGOS in X. Since X is an IF$\gamma_\gamma T_{1/2}$ space, f^{-1}(int(cl(A))) is an IFγOS in X. Therefore f^{-1}(A) ⊆ f^{-1}(int(cl(A))) ⊆ γint(f^{-1}(int(cl(A)))).

(iv) ⇒ (i) Let A be an IFROS in Y . Then A is an IFPOS in X. By hypothesis, f^{-1}(A) ⊆ γint(f^{-1}(int(cl(A)))) = γint(f^{-1}(A)) ⊆ f^{-1}(A). This implies f^{-1}(A) is an IFγOS in X and hence is an IFγGOS in X. Therefore f is an IF almost γG continuous mapping.

Definition 3.4.24: Let A be an IFS in an IFTS (X, τ). Then the ***γ generalized interior*** of A is defined as

γgint(A) = ∪ {G / G is an IFγGOS in X and G ⊆ A},

It is to be noted that for any IFS A in (X, τ), we have γgcl(A^c) = (γgint(A))^c and γgint(A^c) = (γgcl(A))^c.

Remark 3.4.25: If an IFS A in an IFTS (X, τ) is an IFγGOS in X, then γgint(A) = A. But the converse may not be true in general, since union of two non - trivial IFγGOSs need not be an IFγGOS in X.

Proposition 3.4.26: Let f: (X, τ) → (Y, σ) be a mapping. If f is an IF almost γG continuous mapping, then $\gamma gcl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every IFγOS A in Y.

Proof: Let A be an IFγOS in Y. Then cl(A) is an IFRCS in Y. By hypothesis $f^{-1}(cl(A))$ is an IFγGCS in X. Then $\gamma gcl(f^{-1}(cl(A))) = f^{-1}(cl(A))$ and $\gamma gcl(f^{-1}(A)) \subseteq \gamma gcl(f^{-1}(cl(A))) = f^{-1}(cl(A))$.

Proposition 3.4.27: Let f: (X, τ) → (Y, σ) be a mapping. If f is an IF almost γG continuous mapping, then $\gamma gcl(f^{-1}(cl(A))) = f^{-1}(cl(\gamma int(A)))$ for every IFγOS A in Y.

Proof: Let A be an IFγOS in Y. Then γint(A) = A and cl(A) is an IFRCS in Y. By hypothesis $f^{-1}(cl(A))$ is an IFγGCS in X. Then $\gamma gcl(f^{-1}(cl(A))) = f^{-1}(cl(A)) = f^{-1}(cl(\gamma int(A)))$.

Proposition 3.4.28: Let f: (X, τ) → (Y, σ) be a mapping where X is an $IF\gamma_\gamma T_{1/2}$ space. If f is an IF almost γG continuous mapping, then $int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))) \subseteq f^{-1}(\gamma cl(B))$ for every IFRCS B in Y.

Proof: Let B ⊆ Y be an IFRCS. By hypothesis, $f^{-1}(B)$ is an IFγGCS in X. Since X is an $IF\gamma_\gamma T_{1/2}$ space, $f^{-1}(B)$ is an IFγCS in X. Therefore $\gamma cl(f^{-1}(B)) = f^{-1}(B)$. Now $int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))) \subseteq f^{-1}(B) \cup int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))) \subseteq \gamma cl(f^{-1}(B)) = f^{-1}(B) = f^{-1}(\gamma cl(B))$. Hence $int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))) \subseteq f^{-1}(\gamma cl(B))$.

Proposition 3.4.29: Let f: (X, τ) → (Y, σ) be a mapping where X is an $IF\gamma_\gamma T_{1/2}$ space. If f is an IF almost γG continuous mapping, then $f^{-1}(\gamma int(B)) \subseteq int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B)))$ for every IFROS B in Y.

Proof: This Proposition can be easily proved by taking complement in Proposition 3.4.28.

Remark 3.4.30: The composition of two IF almost γG continuous mappings need not be an IF almost γG continuous mapping in general as seen in the following example.

Example 3.4.31: Let X = {a, b}, Y = {u, v}, Z = {p, q}, G_1 = ⟨x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)⟩, G_2 = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩, G_3 = ⟨y, (0.5_u, 0.5_v), (0.5_u, 0.5_v)⟩ and G_4 = ⟨z, (0.5_p, 0.4_q), (0.5_p, 0.6_q)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$}, σ = {$0_\sim$, G_3, $1_\sim$} and δ = {$0_\sim$, G_4, $1_\sim$} are IFTs on X, Y and Z respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v and g: (Y, σ) → (Z, δ) by g(u) = p and g(v) = q. Here f and g are IF almost γG continuous mapping but their composition g o f : (X, τ) → (Z, δ) defined by g(f(a)) = p and g(f(b)) = q is not an IF almost γG continuous mapping. Since G_4^c = ⟨z, (0.5_p, 0.6_q), (0.5_p, 0.4_q)⟩ is an IFRCS in Z, but $f^{-1}(g^{-1}(G_4^c)$ is not an IFγGCS in X as $f^{-1}(g^{-1}(G_4^c)) \subseteq G_1, G_2$ whereas $\gamma cl(f^{-1}(g^{-1}(G_4^c))) = 1_\sim \nsubseteq G_1, G_2$.

Proposition 3.4.32: Let f: (X, τ) → (Y, σ) be an IF almost γG continuous mapping and g: (Y, σ) → (Z, δ) is an IF continuous mapping then g∘ f : (X, τ) → (Z, δ) is an IF almost γG continuous mapping.

Proof: Let V be an IFRCS in Z. Since every IFRCS is an IFCS in Z. Then $g^{-1}(V)$ is an IFCS in Y. Since f is an IFγG continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFγGCS in X. Hence g∘f is an IF almost γG continuous mapping.

Proposition 3.4.33: The following are equivalent for a mapping f: (X, τ) → (Y, σ) where X is an IF$\gamma_\gamma T_{1/2}$ space:

(i) f is an IF almost γG continuous mapping

(ii) $\gamma cl(f^{-1}(A)) \subseteq f^{-1}(\alpha cl(A))$ for every IFγOS A in Y

(iii) $\gamma cl(f^{-1}(A)) \subseteq f^{-1}(\alpha cl(A))$ for every IFSOS A in Y

(iv) $f^{-1}(A) \subseteq \gamma int(f^{-1}(scl(A)))$ for every IFPOS A in Y

Proof: (i) ⇒ (ii) Let A be an IFγOS in Y. Then cl(A) is an IFRCS in Y . By hypothesis $f^{-1}(cl(A))$ is an IFγGCS in X and hence is an IFγCS in X, as X is an IF$\gamma_\gamma T_{1/2}$ space. This implies $\gamma cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$. Now $\gamma cl(f^{-1}(A)) \subseteq \gamma cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$. Since cl(A) is an IFRCS, cl(int(cl(A))) = cl(A). Now $\gamma cl(f^{-1}(A)) \subseteq f^{-1}(cl(A)) = f^{-1}(cl(int(cl(A)))) \subseteq f^{-1}(A \cup cl(int(cl(A)))) \subseteq f^{-1}(\alpha cl(A))$. Hence $\gamma cl(f^{-1}(A)) \subseteq f^{-1}(\alpha cl(A))$.

(ii) ⇒ (iii) Let A be an IFSOS in Y. Since every IFSOS is an IFγOS, the proof is obvious.

(iii) ⇒ (i) Let A be an IFRCS in Y. Then A = cl(int(A)). Therefore A is an IFSOS in Y. By hypothesis, $\gamma cl(f^{-1}(A)) \subseteq f^{-1}(\alpha cl(A)) \subseteq f^{-1}(cl(A)) = f^{-1}(A) \subseteq \gamma cl(f^{-1}(A))$. That is $\gamma cl(f^{-1}(A)) = f^{-1}(A)$. Hence $f^{-1}(A)$ is an IFγCS and hence is an IFγGCS in X. Thus f is an IF almost γG continuous mapping.

(i) ⇒ (iv) Let A be an IFPOS in Y. Then A ⊆ int(cl(A)). Since int(cl(A)) is an IFROS in Y, by hypothesis, $f^{-1}(int(cl(A)))$ is an IFγGOS in X. Since X is an $IF\gamma_\gamma T_{1/2}$ space, $f^{-1}(int(cl(A)))$ is an IFγOS in X. Therefore $f^{-1}(A) \subseteq f^{-1}(int(cl(A))) = \gamma int (f^{-1}(int(cl(A)))) \subseteq \gamma int(f^{-1}(A \cup int(cl(A)))) = \gamma int(f^{-1}(scl(A)))$. That is $f^{-1}(A) \subseteq \gamma int(f^{-1}(scl(A)))$.

(iv) ⇒ (i) Let A be an IFROS in Y. Then A is an IFPOS in Y. By hypothesis, $f^{-1}(A) \subseteq \gamma int(f^{-1}(scl(A)))$. This implies $f^{-1}(A) \subseteq \gamma int(f^{-1}(A \cup int(cl(A)))) = \gamma int(f^{-1}(A \cup A)) = \gamma int(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $f^{-1}(A)$ is an IFγOS in X and hence an IFγGOS in X. Thus f is an IF almost γG continuous mapping.

Proposition 3.4.34: A function f: (X, τ) → (Y, σ) is an IF almost γG continuous mapping where X is an $IF\gamma_\gamma T_{1/2}$ space if and only if $f^{-1}(A) \subseteq \gamma gint (f^{-1}(int(cl(A))))$ for an IFPOS A of Y .

Proof: Necessity: Let A be an IFPOS in Y. Then A ⊆ int(cl(A)) and int(cl(A)) is an IFROS in Y. Since f is an IF almost γG continuous mapping, $f^{-1}(int(cl(A)))$ is an IFγGOS in X and hence we obtain that $f^{-1}(A) \subseteq f^{-1}(int(cl(A))) = \gamma gint(f^{-1}(int(cl(A))))$.

Sufficiency: Let A be an IFROS in Y. Then A is an IFPOS in Y. By hypothesis, $f^{-1}(A) \subseteq \gamma gint(f^{-1}(int(cl(A))))) = \gamma gint(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $\gamma gint(f^{-1}(A)) = f^{-1}(A) = \gamma gint(f^{-1}(A))$, by hypothesis. Hence $f^{-1}(A)$ is an IFγGOS in X as X is an $IF\gamma_\gamma T_{1/2}$ space. Hence f is an IF almost γG continuous mapping.

Definition 3.4.35: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy almost contra γ generalized*** (IF almost contra γG) ***continuous mapping*** if $f^{-1}(V)$ is an IFγGCS in (X, τ) for every IFROS V of (Y, σ).

Example 3.4.36: Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF almost contra γG continuous mapping.

Proposition 3.4.37: A mapping f: $(X, \tau) \to (Y, \sigma)$ is an IF almost contra γG continuous mapping if and only if $f^{-1}(V)$ is an IFγGOS in (X, τ) for every IFRCS V of (Y, σ).

Proof: As $f^{-1}(A^c) = (f^{-1}(A))^c$, the proof is obvious.

Proposition 3.4.38: Let f: $(X, \tau) \to (Y, \sigma)$ be a mapping that satisfies $f^{-1}((int(cl(B))) \subseteq cl(int(f^{-1}(B)))$ for every IFS B in Y. Then f is an IF almost contra γG continuous mapping.

Proof: Let B be an IFRCS in Y. Then $cl(int(B)) = B$, by hypothesis $f^{-1}(B) = f^{-1}(cl(int(B)) \subseteq cl(int(f^{-1}(B)))$. This implies $f^{-1}(B)$ is an IFSOS in X. Therefore it is an IFγGOS in X. Hence f is an IF almost contra γG continuous mapping.

Proposition 3.4.39: A mapping f: $(X, \tau) \to (Y, \sigma)$ is an IF almost contra γG continuous mapping if $f^{-1}(\gamma cl(B)) \subseteq int(f^{-1}(B))$ for every IFS B in Y.

Proof: Let $B \subseteq Y$ be an IFRCS. Since every IFRCS is an IFγCS, $\gamma cl(B) = B$. Now by hypothesis, $f^{-1}(B) = f^{-1}(\gamma cl(B)) \subseteq int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = int(f^{-1}(B))$. Therefore $f^{-1}(B)$ is an IFOS in X and hence an IFγGOS in X. Then f is an IF almost contra γG continuous mapping.

Proposition 3.4.40: For a mapping f: $(X, \tau) \to (Y, \sigma)$, where X is an $IF\gamma_\gamma T_{1/2}$ space, the following are equivalent:

(i) f is an IF almost contra γG continuous mapping

(ii) For every IFRCS A in Y and for every IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$ then $p_{(\alpha,\beta)}\ _q\ \gamma int(f^{-1}(A))$

(iii) For every IFRCS in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$ then there exists an IFγGOS B such that $p_{(\alpha,\beta)}\ _q\ B$ and $f(B) \subseteq A$

Proof: (i) ⇒ (ii) Let f be an IF almost contra γG continuous mapping. Let A ⊆ Y be an IFRCS and let $p_{(\alpha,\beta)}$ ∈ X. Also let $f(p_{(\alpha,\beta)})\ _q$ A, then $p_{(\alpha,\beta)\ q}$ f^{-1}(A). By hypothesis f^{-1}(A) is an IFγGOS in X. Since X is an IF$\gamma_\gamma T_{1/2}$, f^{-1} (A) is an IFγOS in X. Hence γint(f^{-1}(A)) = f^{-1}(A). This implies $p_{(\alpha,\beta)\ q}$ γint(f^{-1}(A)).

(ii) ⇒ (i) Let A ⊆ Y be an IFRCS, then f^{-1}(A) is an IFS in X. Let $p_{(\alpha,\beta)}$ ∈ X and let $f(p_{(\alpha,\beta)})\ _q$ A then $p_{(\alpha,\beta)\ q}$ f^{-1}(A). By hypothesis, this implies $p_{(\alpha,\beta)\ q}$ γint(f^{-1}(A)). That is f^{-1}(A) ⊆ γint(f^{-1}(A)). But γint(f^{-1}(A)) ⊆ f^{-1}(A). Therefore γint(f^{-1}(A)) = f^{-1}(A) and f^{-1}(A) is an IFγOS in X. Hence f^{-1}(A) is an IFγGOS in X and hence f is an IF almost contra γG continuous mapping.

(ii) ⇒ (iii) Let A ⊆ Y be an IFRCS, then f^{-1}(A) is an IFS in X. Let $p_{(\alpha,\beta)}$ ∈ X. Also let $f(p_{(\alpha,\beta)})\ _q$ A then $p_{(\alpha,\beta)\ q}$ f^{-1}(A). By hypothesis this implies $p_{(\alpha,\beta)\ q}$ γint(f^{-1}(A)). That is f^{-1}(A) ⊆ γint(f^{-1}(A)). But γint(f^{-1}(A)) ⊆ f^{-1}(A). Therefore γint(f^{-1}(A)) = f^{-1}(A). Thus f^{-1}(A) is an IFγOS in X and hence an IFγGOS in X**.** Let f^{-1}(A) = B. Therefore $p_{(\alpha,\beta)\ q}$ B and f(B) = f(f^{-1}(A)) ⊆ A.

(iii) ⇒ (ii) Let A ⊆ Y be an IFRCS, then f^{-1}(A) is an IFS in X. Let $p_{(\alpha,\beta)}$ ∈ X. Also let $f(p_{(\alpha,\beta)})\ _q$ A then $p_{(\alpha,\beta)\ q}$ f^{-1}(A). By hypothesis there exists an IFγGOS B in X such that $p_{(\alpha,\gamma)\ q}$ B and f(B) ⊆ A. Let B = f^{-1}(A). Since X is an IF$\gamma_\gamma T_{1/2}$ space, f^{-1}(A) is an IFγOS in X and γint(f^{-1}(A)) = f^{-1}(A). Therefore $p_{(\alpha,\beta)\ q}$ γint(f^{-1}(A)).

3.5 Intuitionistic fuzzy γ generalized irresolute mappings

In this section we have introduced intuitionistic fuzzy γ generalized irresolute mappings, intuitionistic fuzzy contra γ generalized irresolute mappings and studied some of their properties.

Definition 3.5.1: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy γ generalized*** (IFγG) ***irresolute mapping*** if f^{-1}(V) is an IFγGCS in (X, τ) for every IFγGCS V of (Y, σ).

Example 3.5.2: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)⟩, G_2 = ⟨x, (0.5_a, 0.5_b), (0.4_a, 0.4_b)⟩ and G_3 = ⟨y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG irresolute mapping.

Proposition 3.5.3: Let f: (X, τ) → (Y, σ) be an IFγG irresolute mapping, then f is an IFγG continuous mapping but not conversely in general.

Proof: Let f be an IFγG irresolute mapping. Let V be any IFCS in Y. Then V is an IFγGCS in Y and by hypothesis f^{-1}(V) is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.5.4: Let X = {a, b}, Y = {u, v}, G_1 = ⟨x, (0.6_a, 0.8_b), (0.4_a, 0.2_b)⟩, G_2 = ⟨x, (0.5_a, 0.5_b), (0.4_a, 0.4_b)⟩ and G_3 = ⟨y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)⟩. Then τ = {$0_\sim$, G_1, G_2, $1_\sim$} and σ = {$0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG continuous mapping but not an IFγG irresolute mapping. Since the IFS A = ⟨y, (0.5_u, 0.5_v), (0.5_u, 0.4_v)⟩ is an IFγGCS in Y but f^{-1} (A) = ⟨x, (0.5_a, 0.5_b), (0.5_a, 0.4_b)⟩ is not an IFγGCS in X, as γcl(f^{-1} (A)) = $1_\sim \nsubseteq G_1, G_2$ where as $f^{-1}(A) \subseteq G_1, G_2$.

Proposition 3.5.5: A mapping f: (X, τ) → (Y, σ) is an IFγG irresolute mapping if and only if the inverse image of each IFγGOS in Y is an IFγGOS in X.

Proof: Necessity: Let A be an IFγGOS in Y. This implies A^c is an IFγGCS in Y. Then $f^{-1}(A^c)$ is an IFγGCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, f^{-1}(A) is an IFγGOS in X.

Sufficiency: Let A be an IFγGCS in Y. Then A^c is an IFγGOS in Y. By hypothesis $f^{-1}(A^c)$ is IFγGOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an IFγGOS in X. Therefore f^{-1}(A) is an IFγGCS in X. Hence f is an IFγG irresolute mapping.

Proposition 3.5.6: The composition of two IFγG irresolute mappings is an IFγG irresolute mapping.

Proof: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, δ) be IFγG irresolute mappings, let V be an IFγGCS in Z. Then $g^{-1}(V)$ is an IFγGCS in Y, by hypothesis. Since f is an IFγG irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFγGCS in X. Hence g o f is an IFγG irresolute mapping.

Proposition 3.5.7: Let f: (X, τ) → (Y, σ) be an IFγG irresolute mapping and g: (Y, σ) → (Z, δ) is an IFγG continuous mapping, then g o f : (X, τ) → (Z, δ) is an IFγG continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFγGCS in Y. Since f is an IFγG irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFγGCS in X. Hence g o f is an IFγG continuous mapping.

Proposition 3.5.8: If f: (X, τ) → (Y, σ) is an IFγG continuous mapping and g: (Y, σ) → (Z, δ) is an IFγG irresolute mapping where Y is an IF$\gamma_c T_{1/2}$ space, then g o f : (X, τ) → (Z, δ) is an IFγG irresolute mapping.

Proof: Let V be an IFγGCS in Z. Then $g^{-1}(V)$ is an IFγGCS in Y as g is an IFγG irresolute mapping. Since Y is an IF$\gamma_c T_{1/2}$space, $g^{-1}(V)$ is an IFCS in Y. Therefore $f^{-1}(g^{-1}(V))$ is an IFγGCS in X, by hypothesis. Hence g o f is an IFγG irresolute mapping.

Proposition 3.5.9: Let f: (X, τ) → (Y, σ) be an IFγG continuous mapping, then f is an IFγG irresolute mapping if (Y, σ) is an IF$\gamma_c T_{1/2}$space.

Proof: Let A be an IFγGCS in Y. Since Y is an IF$\gamma_c T_{1/2}$space, A is an IFCS in Y. By hypothesis $f^{-1}(A)$ is an IFγGCS in X. Therefore f is an IFγG irresolute mapping.

Proposition 3.5.10: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IF$\gamma_\gamma T_{1/2}$ spaces:

(i) f is an IFγG irresolute mapping,
(ii) $f^{-1}(B)$ is an IFγGOS in X for every IFγGOS B in Y,
(iii) $f^{-1}(\gamma int(B)) \subseteq \gamma int(f^{-1}(B))$ for every IFS B in Y,
(iv) $\gamma cl(f^{-1}(B)) \subseteq f^{-1}(\gamma cl(B))$ for every IFS B in Y.

Proof: (i) ⟺ (ii) is obvious from Proposition 3.5.5.

(ii) ⇒ (iii) Let B be an IFS in Y. Since γint(B) is an IFγOS in Y, it is an IFγGOS in Y. Therefore f^{-1}(γint(B)) is an IFγGOS in X, by hypothesis. Since X is an IFγ_γ $T_{1/2}$ space, f^{-1}(γint(B)) is an IFγOS in X. Hence f^{-1}(γint(B)) = γint(f^{-1}(γint(B))) ⊆ γint(f^{-1}(B)).

(iii) ⇒ (iv) is obvious by taking complement in (iii).

(iv) ⇒ (i) Let B be an IFγGCS in Y. Since Y is an IFγ_γ $T_{1/2}$ space, B is an IFγCS in Y and γcl(B) = B. Hence f^{-1}(B) = f^{-1}(γcl(B)) ⊇ γcl(f^{-1}(B)) ⊇ f^{-1}(B). Therefore γcl(f^{-1}(B)) = f^{-1}(B). This implies f^{-1}(B) is an IFγCS and hence it is an IFγGCS in X. Thus f is an IFγG irresolute mapping.

Proposition 3.5.11: Let f: (X, τ) → (Y, σ) be an IFγG irresolute mapping and g: (Y, σ) → (Z, δ) is an IF almost γG continuous mapping then g o f : (X, τ) → (Z, δ) is an IF almost γG continuous mapping.

Proof: Let A be an IFRCS in Z. Then g^{-1}(A) is an IFγGCS in Y. Since f is an IFγG irresolute mapping, f^{-1}(g^{-1}(V)) is an IFγGCS in X. Hence g o f is an IF almost γG continuous mapping.

Proposition 3.5.12: Let f: (X, τ) → (Y, σ) be an IFγG irresolute mapping. Then f^{-1}(B) ⊆γint(f^{-1}(int(cl(B)))) for every IFγGOS B in Y, if X and Y are IF$\gamma_p T_{1/2}$ spaces.

Proof: Let B be an IFγGOS in Y. Then by hypothesis f^{-1}(B) is an IFγGOS in X. Since X is an IF$\gamma_p T_{1/2}$ space, f^{-1}(B) is an IFPOS in X. Since every IFPOS is an IFγOS, γint(f^{-1}(B)) = f^{-1}(B). Since Y is an IF$\gamma_p T_{1/2}$ space, B is an IFPOS in Y and B ⊆ int(cl(B)). Hence f^{-1}(B) = γint(f^{-1}(B)) ⊆ γint(f^{-1}(int(cl(B)))).

Proposition 3.5.13: Let f: (X, τ) → (Y, σ) be an IFγG irresolute mapping. Then f is an IF pre irresolute mapping, if (X, τ) is an IFγ_p $T_{1/2}$ space.

Proof: Let B be an IFPCS in Y. Then B is an IFγGCS in Y. Since f is an IFγG irresolute mapping, f^{-1}(B) is an IFγGCS in X. Since X is an IFγ_p $T_{1/2}$ space, f^{-1}(B) is an IFPCS in X. Hence f is an IF pre irresolute mapping.

Proposition 3.5.14: Let f: (X, τ) → (Y, σ) be an IFγG irresolute mapping. Then $f^{-1}(B) \subseteq \gamma int(f^{-1}(int(cl(B)) \cup cl(int(B))))$ for every IFγGOS B in Y, if X and Y are $IF\gamma_{\gamma}T_{1/2}$ spaces.

Proof: Let B be an IFγGOS in Y. Then by hypothesis $f^{-1}(B)$ is an IFγGOS in X. Since X is an $IF\gamma_{\gamma}T_{1/2}$ space, $f^{-1}(B)$ is an IFγOS in X. Therefore $\gamma int(f^{-1}(B)) = f^{-1}(B)$. Since Y is an $IF\gamma_{\gamma}T_{1/2}$ space, B is an IFγOS in Y and $B \subseteq int(cl(B)) \cup cl(int(B))$. Now $f^{-1}(B) = \gamma int(f^{-1}(B)) \subseteq \gamma int(f^{-1}(int(cl(B)) \cup cl(int(B))))$.

Proposition 3.5.15: Let f: (X, τ) → (Y, σ) be an IFγG irresolute mapping. Then $f^{-1}(B) \subseteq \gamma int(f^{-1}(int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B)))))$ for every IFγGOS B in Y, if X and Y are $IF\gamma_{\gamma}T_{1/2}$ spaces.

Proof: Let B be IFγGOS in Y. Then by hypothesis $f^{-1}(B)$ is an IFγGOS in X. Since X is $IF\gamma_{\gamma}T_{1/2}$ space, $f^{-1}(B)$ is an IFγOS in X. Therefore $\gamma int(f^{-1}(B)) = f^{-1}(B)$ and $f^{-1}(B) \subseteq (int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B))))$. Hence $f^{-1}(B) = \gamma int(f^{-1}(B)) \subseteq \gamma int(f^{-1}(int(cl(f^{-1}(B))) \cup cl(int(f^{-1}(B)))))$.

Definition 3.5.16: A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy contra γ generalized*** (IF contra γG) ***irresolute mapping*** if $f^{-1}(V)$ is an IFγGCS in (X, τ) for every IFγGOS V of (Y, σ).

Example 3.5.17: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG irresolute mapping.

Proposition 3.5.18: Every IF contra γG irresolute mapping is an IF contra γG continuous mapping but not conversely in general.

Proof: Let A be an IFCS in Y. Since every IFCS is an IFγGCS, then A is an IFγGCS in Y. By hypothesis, $f^{-1}(A)$ is an IFγGOS in X. Hence f is IF contra γG continuous mapping.

Example 3.5.19: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then

$\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF contra γG continuous mapping but not an IF contra γG irresolute mapping. For, let $A = \langle y, (0.4_u, 0.5_v), (0.6_u, 0.5_v)\rangle$ is an IFγGOS in Y, but $f^{-1}(A) = \langle x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)\rangle$ is not an IFγGCS in (X, τ) as $f^{-1}(A) \subseteq G_1$ and $\gamma cl(f^{-1}(A)) = 1_\sim \not\subseteq G_1$.

Proposition 3.5.20: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, δ) be any two mappings. Then

i. g o f : (X, τ) → (Z, δ) is an IFγG irresolute mapping if f and g are IF contra γG irresolute mapping,
ii. g o f : (X, τ) → (Z, δ) is an IFγG continuous mapping if f is an IF contra γG irresolute mapping and g is an IF contra γG continuous mapping,
iii. g o f : (X, τ) → (Z, δ) is an IF contra γG irresolute mapping if f is an IFγG irresolute mapping and g is an IF contra γG irresolute mapping,
iv. g o f : (X, τ) → (Z, δ) is an IF contra γG continuous mapping if f is an IFγG irresolute mapping and g is an IF contra γG continuous mapping.

Proof: (i) Let A be an IFγGCS in Z. Then $g^{-1}(A)$ is an IFγGOS in Y. Since f is an IF contra γG irresolute mapping, $f^{-1}(g^{-1}(A)$ is an IFγGCS in X. Hence g o f : (X, τ) → (Z, δ) is an IFγG irresolute mapping.

(ii) Let A be an IFCS in Z. Then by hypothesis, $g^{-1}(A)$ is an IFγGOS in Y. Since f is an IF contra γG irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IFγGCS in X. Hence g o f is an IFγG continuous mapping.

(iii) Let A be an IFγGCS in Z. Then by hypothesis, $g^{-1}(A)$ is an IFγGOS in Y. Since f is an IFγG irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IFγGOS in X. Hence g o f is an IF contra γG irresolute mapping.

(iv) Let A be an IFOS in Z. Since g is an IF contra γG continuous mapping, $g^{-1}(A)$ is an IFγGCS in Y. Since f is an IFγG irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IFγGCS in X. Hence g o f is an IF contra γG continuous mapping.

Proposition 3.5.21: Let f: (X, τ) → (Y, σ) and g: (Y, σ) →(Z, δ) be any two mappings. If the mapping g ∘ f is an IF contra γG irresolute mapping and X is an IF$\gamma_c T_{1/2}$ space. Then

(i) $(g \circ f)^{-1}(B)$ is an IFγGOS in X for each IFγGCS B in Z

(ii) $cl(g \circ f)^{-1}(int(B)) \subseteq (g \circ f)^{-1}(B)$ for each IFS B of Z

Proof: (i) Let B be an IFγGCS in Z. Then B^c is an IFγGOS in Z. By hypothesis, $(g \circ f)^{-1}(B^c)$ is an IFγGCS in X. This implies $(g \circ f)^{-1}(B)$ is an IFγGOS in X.

(ii) Let B be any IFS in Z and int(B) ⊆ B. Then $(g \circ f)^{-1}(int(B)) \subseteq ((g \circ f)^{-1}(B))$. Since int(B) is an IFOS in Z, int(B) is an IFγGOS in Z. Therefore by hypothesis $(g \circ f)^{-1}(int(B))$ is an IFγGCS in X. Since X is an IF$\gamma_c T_{1/2}$ space, $(g \circ f)^{-1}(int(B))$ is an IFCS in X. Hence $cl((g \circ f)^{-1}(int(B))) = (g \circ f)^{-1}(int(B)) \subseteq (g \circ f)^{-1}(B)$.

3.6 Completely γ generalized continuous mappings in intuitionistic fuzzy topological spaces

In this section we have introduced intuitionistic fuzzy completely γ generalized continuous mappings and studied some of their properties.

Definition 3.6.1: A mapping f: (X, τ) → (Y, σ) is said to be an ***intuitionistic fuzzy completely γ generalized*** (IF completely γG) ***continuous mapping*** if $f^{-1}(V)$ is an IFRCS in X for every IFγGCS V in Y.

Proposition 3.6.2: Every IF completely γG continuous mapping is an IF continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFCS, $f^{-1}(V)$ is an IFCS in X. Hence f is an IF continuous mapping.

Example 3.6.3: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.2_a, 0.3_b), (0.8_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF continuous mapping but not an IF completely γG continuous mapping, since G_3^c is an IFγGCS in Y, but $f^{-1}(G_3^c) = \langle x, (0.8_a, 0.7_b), (0.2_a, 0.3_b)\rangle$ is not an IFRCS in X, as $cl(int(f^{-1}(G_3^c))) = cl(G_1) = G_1^c \neq f^{-1}(G_3^c)$.

Proposition 3.6.4: Every IF completely γG continuous mapping is an IF semi continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFSCS, $f^{-1}(V)$ is an IFSCS in X. Hence f is an IF semi continuous mapping.

Example 3.6.5: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.2_a, 0.3_b), (0.8_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF semi continuous mapping but not an IF completely γG continuous mapping.

Proposition 3.6.6: Every IF completely γG continuous mapping is an IF pre continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFPCS, $f^{-1}(V)$ is an IFPCS in X. Hence f is an IF pre continuous mapping.

Example 3.6.7: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.2_a, 0.3_b), (0.8_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF pre continuous mapping but not an IF completely γG continuous mapping.

Proposition 3.6.8: Every IF completely γG continuous mapping is an IFα continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFαCS, $f^{-1}(V)$ is an IFαCS in X. Hence f is an IFα continuous mapping.

Example 3.6.9: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.2_a, 0.3_b), (0.8_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFα continuous mapping but not an IF completely γG continuous mapping.

Proposition 3.6.10: Every IF completely γG continuous mapping is an IFγ continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFγCS, $f^{-1}(V)$ is an IFγCS in X. Hence f is an IFγ continuous mapping.

Example 3.6.11: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.2_a, 0.3_b), (0.8_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$.Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγ continuous mapping but not an IF completely γG continuous mapping.

Proposition 3.6.12: Every IF completely γG continuous mapping is an IF semipre continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an

IFRCS in X. Since every IFRCS is an IFSPCS, $f^{-1}(V)$ is an IFSPCS in X. Hence f is an IF semipre continuous mapping.

Example 3.6.13: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.2_a, 0.3_b), (0.8_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF semipre continuous mapping but not an IF completely γG continuous mapping.

Proposition 3.6.14: Every IF completely γG continuous mapping is an IFγG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping. Let V be an IFCS in Y. Since every IFCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFγGCS, $f^{-1}(V)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping.

Example 3.6.15: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$ and $G_4 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG continuous mapping but not an IF completely γG continuous mapping.

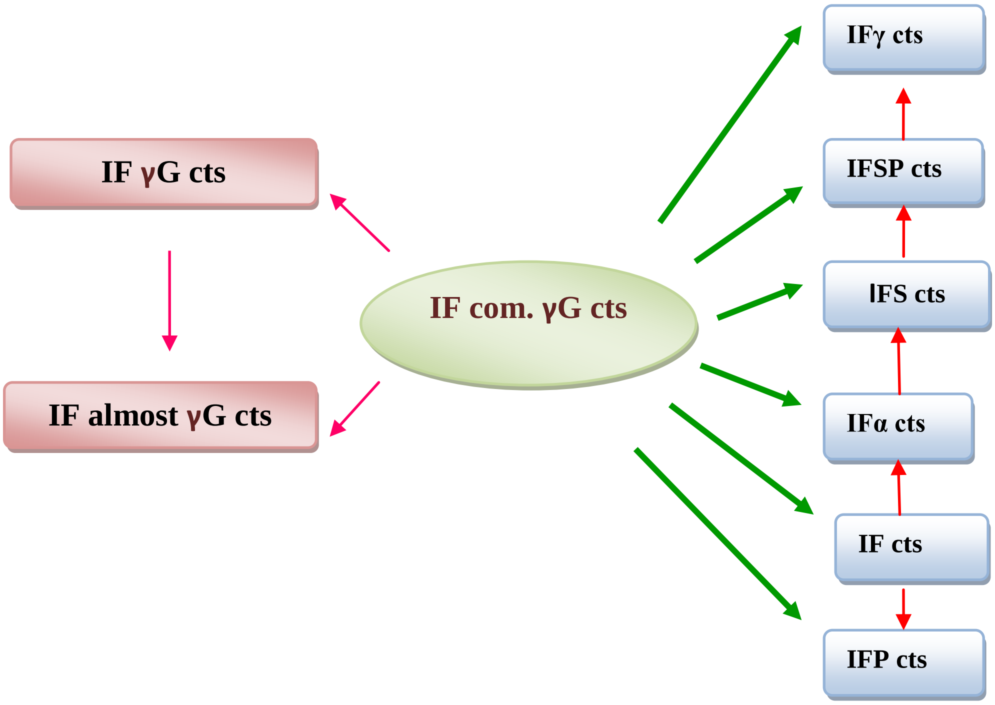
Proposition 3.6.16: Every IF completely γG continuous mapping is an IF almost γG continuous mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF completely γG continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFγGCS, V is an IFγGCS in Y. Then $f^{-1}(V)$ is an IFRCS and hence is an IFγGCS in X. Thus f is an IF almost γG continuous mapping.

Example 3.6.17: Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.2_u, 0.3_v), (0.8_u, 0.7_v)\rangle$ and $G_4 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ are

IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost γG continuous mapping but not an IF completely γG continuous mapping.

The relation between various types of intuitionistic fuzzy continuous mappings with intuitionistic fuzzy completely γG continuous mapping is given in the following diagram. In this diagram 'cts.' means continuous and IF com. γG cts. means IF completely γG continuous.



The reverse implications are not true in general in the above diagram.

Proposition 3.6.18: If f: (X, τ) → (Y, σ) is an IF completely γG continuous mapping, then $\gamma cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every IFγOS A ⊆ Y.

Proof: Let A be an IFγOS in Y. Then cl(A) is an IFRCS in Y. Hence cl(A) is an IFγGCS in Y. By hypothesis, $f^{-1}(cl(A))$ IFRCS in X and thus an IFγCS in X. Therefore $\gamma cl(f^{-1}(A)) \subseteq \gamma cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$.

Proposition 3.6.19: Let f: (X, τ) → (Y, σ) be a mapping. Then the following are equivalent:

(i) f is an IF completely γG continuous mapping,

(ii) $f^{-1}(V)$ is an IFROS in X for every IFγGOS V in Y,

(iii) for every IFP $p_{(\alpha,\beta)} \in X$ and for every IFγGOS B in Y such that $f(p_{(\alpha,\beta)}) \in B$ there exists an IFROS in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

Proof: (i) ⇔ (ii) is obvious.

(ii) ⇒ (iii) Let $p_{(\alpha,\beta)} \in X$ and $B \subseteq Y$ such that $f(p_{(\alpha,\beta)}) \in B$. This implies $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since B is an IFγGOS in Y, by hypothesis $f^{-1}(B)$ is an IFROS in X. Let $A = f^{-1}(B)$. Then $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \in f^{-1}(B) = A$. Therefore $p_{(\alpha,\beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(iii) ⇒ (ii) Let $B \subseteq Y$ be an IFγGOS. Let $p_{(\alpha,\beta)} \in X$ and $f(p_{(\alpha,\beta)}) \in B$. By hypothesis, there exists an IFROS C in X such that $p_{(\alpha,\beta)} \in C$ and $f(C) \subseteq B$. This implies $C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B)$. Therefore $p_{(\alpha,\beta)} \in C \subseteq f^{-1}(B)$. That is $f^{-1}(B) = \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} \{ p_{(\alpha,\beta)}\} \subseteq \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} C \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} C$. Since the union IFROSs is an IFROS, $f^{-1}(B)$ is an IFROS in X. Hence f is an IF completely γG continuous mapping.

Proposition 3.6.20: If a mapping $f: (X, \tau) \to (Y, \sigma)$ is an IF completely γG continuous mapping, then for every IFP $p_{(\alpha,\beta)} \in X$ and for every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFROS $B \subseteq X$ such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.

Proof: Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha,\beta)}) \in C \subseteq A$. Since every IFOS is an IFγGOS, C is an IFγGOS in Y. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \subseteq f^{-1}(C) \subseteq f^{-1}(A)$ and therefore $p_{(\alpha,\beta)} \in f^{-1}(C)$. Now let $f^{-1}(C) = B$. Therefore $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.

Proposition 3.6.21: A mapping $f: (X, \tau) \to (Y, \sigma)$ is an IF completely γG continuous mapping then for every IFP $p_{(\alpha,\beta)} \in X$ and for every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFROS $B \subseteq X$ such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$.

Proof: Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha,\beta)}) \in C \subseteq A$. Since every IFOS is an IFγGOS, C is an IFγGOS in Y. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p_{(\alpha,\beta)} \in f^{-1}(C)$. Now let $f^{-1}(C) = B$. Therefore $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$. Thus $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

Proposition 3.6.22: A mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ is an IF completely γG continuous mapping, then $int(cl(f^{-1}(int(B)))) \subseteq f^{-1}(B)$ for every IFS B in Y.

Proof: Let $B \subseteq Y$. Then int(B) is an IFOS in Y and hence an IFγGOS in Y. By hypothesis, $f^{-1}(int(B))$ is an IFROS in X. Hence $int(cl(f^{-1}(int(B)))) = f^{-1}(int(B)) \subseteq f^{-1}(B)$.

Proposition 3.6.23: For any two IF completely γG continuous mappings $f_1, f_2 : (X, \tau) \rightarrow (Y, \sigma)$, the mapping $(f_1, f_2) : (X, \tau) \rightarrow (Y \times Y, \sigma \times \sigma)$ is also an IF completely γG continuous mapping where $(f_1, f_2)(x) = (f_1(x), f_2(x))$ for every $x \in X$.

Proof: Let A x B be an IFγGOS in Y x Y. Then

$$(f_1, f_2)^{-1}(A \times B)(x) = (A \times B)(f_1(x), f_2(x))$$

$$= \langle x, \min(\mu_A(f_1(x)), \mu_B(f_2(x))), \max(\nu_A(f_1(x)), \nu_B(f_2(x)))\rangle$$

$$= \langle x, \min(f_1^{-1}(\mu_A)(x), f_2^{-1}(\mu_B)(x)), \max(f_1^{-1}(\nu_A)(x), f_2^{-1}(\nu_B)(x))\rangle$$

$$= (f_1^{-1}(A) \cap f_2^{-1}(B))(x).$$

Since f_1 and f_2 are IF completely γG continuous mappings, $f_1^{-1}(A)$ and $f_2^{-1}(B)$ are IFROSs in X. Since intersection of IFROSs is an IFROS, $f_1^{-1}(A) \cap f_2^{-1}(B)$ is an IFROS in X. Hence (f_1, f_2) is an IF completely γG continuous mappings.

Proposition 3.6.24: A mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ is an IF completely γG continuous mapping then the following are equivalent:

(i) For any IFγGOS A in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$, then $p_{(\alpha,\beta)}\ _q\ int(f^{-1}(A))$,

(ii) For any IFγGOS A in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$, then there exists an IFOS B such that $p_{(\alpha,\beta)}\ _q\ B$ and $f(B) \subseteq A$.

Proof: (i) ⇒ (ii) Let $A \subseteq Y$ be an IFγGOS and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})\ _q\ A$. Then $p_{(\alpha,\beta)}\ _q\ f^{-1}(A)$. (i) implies that $p_{(\alpha,\beta)}\ _q\ int(f^{-1}(A))$, where $int(f^{-1}(A))$ is an IFOS in X. Let $B = int(f^{-1}(A))$. Since $int(f^{-1}(A)) \subseteq f^{-1}(A)$, $B \subseteq f^{-1}(A)$. Then $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(ii) ⇒ (i) Let A ⊆ Y be an IFγGOS and let $p_{(\alpha,\beta)}$ ∈ X. Suppose $f(p_{(\alpha,\beta)})\ _q$ A, then by (ii) there exists an IFOS B in X such that $p_{(\alpha,\beta)}\ _q$ B and f(B) ⊆ A. Now B ⊆ f^{-1}(f(B)) ⊆ f^{-1}(A). That is B = int(B) ⊆ int(f^{-1}(A)). Therefore $p_{(\alpha,\beta)}\ _q$ B implies $p_{(\alpha,\beta)}\ _q$ int(f^{-1}(A)).

Proposition 3.6.25: The composition of any two IF completely γG continuous mapping is an IF completely γG continuous mapping in general.

Proof: Let f: X → Y and g: Y → Z be any two IF completely γG continuous mappings. Let B be an IFγGOS in Z. Since g is an IF completely γG continuous mapping, g^{-1}(B) is an IFROS in Y. Since every IFROS is an IFγGOS, g^{-1}(B) is an IFγGOS in Y. Since f is an IF completely γG continuous mapping, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is an IFROS in X. Hence g ο f is an IF completely γG continuous mapping.

Proposition 3.6.26: Let f: X → Y and g: Y → Z be any two mappings. Then

(i) g ο f is an IF completely γG continuous mapping if f is an IF completely γG continuous mapping and g is an IFγG irresolute mapping,

(ii) g ο f is an IFγG continuous mapping if f is an IF completely γG continuous mapping and g is an IFγG continuous mapping.

Proof:

(i) Let B be an IFγGOS in Z. Since g is an IFγG irresolute mapping, g^{-1}(B) is an IFγGOS in Y. Also, since f is an IF completely γG continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFROS in X. Since $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, g ο f is an IF completely γG continuous mapping.

(ii) Let B be an IFOS in Z. Since g is an IFγG continuous mapping, g^{-1}(B) is an IFγGOS in Y. Also, since f is an IF completely γG continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFROS in X. Hence $f^{-1}(g^{-1}(B))$ is an IFγGOS in X. From the fact that $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, it follows that (g ο f) is an IFγG continuous mapping.