



# Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)

Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC

Coimbatore - 641 043, Tamil Nadu, India

## Continuous Internal Assessment Test I– August 2025

### Semester V

Class: III UG

Branch: Special Education & Mathematics

Time: 2 Hrs.

Max.Marks: 60

### 23BSMC15 –Complex Analysis I

#### Course Outcomes:

CO1: Operate complex derivatives of a function.

CO2: Determine analyticity of a function using C-R equations.

CO3: Construct analytic functions.

CO4: Analyze bilinear transformations.

CO5: Apply Cauchy's theorem and Cauchy's integral formula to evaluate integrals.

#### Part A

6x1=6

#### Choose the Correct Answer

- The value of  $\arg z$  which lies in the interval  $(-\pi, \pi]$  is called the \_\_\_\_\_ CO1K1
  - $\arg$  of  $z$
  - principal value of  $\arg$  of  $z$
  - principal value of  $z$
  - $\arg$  of  $\bar{z}$
- If a function is \_\_\_\_\_ at a point then the function is not differentiable at that point CO1K1
  - not continuous
  - continuous
  - analytic
  - not analytic
- The function  $f(z) = |z|^2$  is \_\_\_\_\_ CO2K2
  - differentiable at  $z=0$ , but not analytic at  $z=0$
  - not differentiable at  $z=0$ , but analytic at  $z=0$
  - analytic everywhere
  - none of the above
- A function which is analytic everywhere in the finite plane is called an \_\_\_\_\_ CO2K2
  - analytic function
  - entire function
  - regular function
  - continuous function
- The value of  $m$  such that  $2x - x^2 + my^2$  may be harmonic is \_\_\_\_\_ CO3K2
  - 1
  - 2
  - 3
  - 4
- Which of the following function is not harmonic? CO3K3
  - $e^x \cos y$
  - $e^x \sin y$
  - $x^3 - 3xy^2$
  - $3x^2y - y^2$

#### Part B

3x6=18

#### Answer ALL questions

- a. Prove that the limit of a function is unique. CO1K2  
(or)
- b. Prove that if  $f(z)$  is differentiable at  $z_0$ , then  $f(z)$  is continuous at  $z_0$ . CO1K3

8. a. Derive the condensed form of C-R equations. CO2K3  
 (or)
8. b. If  $f(z)=\sin z$ , find  $f'(z)$  CO2K2
9. a. If  $u$  and  $v$  are harmonic functions, prove that  $(u_y - v_x) + i(u_x + v_y)$  is an analytic function CO3K3  
 (or)
9. b. If  $z^3=u+iv$ , then show that  $u$  and  $v$  are harmonic functions. CO3K3

**Part C**

**3x12=36**

**Answer ALL questions**

10. a. State and prove the necessary conditions for differentiability of a function in terms of its real and imaginary parts  $u(x,y)$  and  $v(x,y)$ . CO1K4  
 (or)
10. b. State and prove the sufficient conditions for differentiability CO1K4
11. a. Suppose  $f(z)$  is analytic in a domain. Prove that, CO2K3
- (i) if its real part is a constant or
  - (ii) if its imaginary part is a constant or
  - (iii) if its modulus is a constant or
  - (iv) if its argument is a constant,

Then  $f(z)$  is a constant.

(or)

- 11.b. (i) If  $w=e^z$ , find its derivative. CO2K4
- (ii) Show that  $f(z) = \log z$  is analytic everywhere except at the origin and find its derivative.
12. a. If  $f(z)$  is analytic then show that (i)  $\nabla^2 |\operatorname{Re} f(z)|^2 = 2|f'(z)|^2$  CO3K4
- (ii)  $\nabla^2 |\operatorname{Im} f(z)|^2 = 2|f'(z)|^2$  (iii)  $\nabla^2 |f(z)|^2 = 4|f'(z)|^2$
- (iv)  $\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \log |f(z)| = 0.$

(or)

12. b. (i) If  $f(z) = u(x,y) + i v(x,y)$  is an analytic function then prove that  $u(x,y)$  and  $v(x,y)$  are Harmonic functions. CO3K3
- (ii) If  $f(z) = u(r,\theta) + i v(r,\theta)$  is an analytic function then prove that  $u(r,\theta)$  and  $v(r,\theta)$  are Harmonic functions.