CHAPTER - V

CHAPTER V

SEMIOPEN AND SEMICLOSED FUZZY SOFT SETS IN FUZZY SOFT TOPOLOGICAL SPACES

Definition 5.1

In a fuzzy soft topological space (U, E, τ), a fuzzy soft set

i. G_A is said to be **semiopen fuzzy soft set** if there exist an open fuzzy soft set H_A such that $H_A \tilde{\subseteq} G_A \tilde{\subseteq} Cl(H_A)$.

ii. P_A is said to be **semiclosed fuzzy soft set** if there exist a closed fuzzy soft set K_A such that $Int(K_A) \tilde{\subseteq} P_A \tilde{\subseteq} K_A$.

Remark 5.2

Every open (closed) fuzzy soft set is semiopen (semiclosed) fuzzy soft set but not conversely.

Remark 5.3

$\tilde{0}$ and $\tilde{1}$ are always semiclosed and semiopen fuzzy soft sets.

Notation 5.4

We shall denote the family of all semiopen fuzzy soft sets and semiclosed fuzzy soft sets of a fuzzy soft topological space (U, E, τ) by $SOFSS(U, E)$ and $SCFSS(U, E)$ respectively.

Theorem 5.5

Arbitrary union of semiopen fuzzy soft sets is a semiopen fuzzy soft set.

Proof

Let $\{(G_A)_\lambda : \lambda \in \Lambda\}$ be a collection of semiopen fuzzy soft sets of a fuzzy soft topological space(U, E, τ). Then there exists an open sets $(H_A)_\lambda$ such that $(H_A)_\lambda \tilde{\subseteq} (G_A)_\lambda \tilde{\subseteq} Cl(H_A)_\lambda$ for eachλ, hence $\tilde{\cup} (H_A)_\lambda \tilde{\subseteq} \tilde{\cup} (G_A)_\lambda \tilde{\subseteq} Cl(\tilde{\cup} (H_A)_\lambda)$ and $\tilde{\cup} (H_A)_\lambda$ is open fuzzy soft set. So $\tilde{\cup} (G_A)_\lambda$ is semi open fuzzy soft set.

Remark 5.6

Arbitrary intersection of semiopen fuzzy soft sets is a semiopen fuzzy soft set.

Theorem 5.7

If a semiopen fuzzy soft set G_A is such that $G_A \tilde{\subseteq} K_A \tilde{\subseteq} Cl(G_A)$ then K_A is also semiopen.

Proof

As G_A is semiopen fuzzy soft set there exists an open fuzzy soft set H_A such that $H_A \tilde{\subseteq} G_A \tilde{\subseteq} Cl(H_A)$ then by hypothesis $H_A \tilde{\subseteq} K_A$ and $Cl(G_A) \tilde{\subseteq} Cl(H_A)$ $\Rightarrow K_A \tilde{\subseteq} Cl(G_A) \tilde{\subseteq} Cl(H_A)$ i.e.,$H_A \tilde{\subseteq} K_A \tilde{\subseteq} Cl(H_A)$, hence K_A is a semi open fuzzy soft set.

Theorem 5.8

If a semiclosed fuzzy soft set M_A is such that $Int(M_A) \tilde{\subseteq} K_A \tilde{\subseteq} Cl(M_A)$ then K_A is also semiclosed.

Theorem 5.9

A fuzzy soft set $G_A \tilde{\in} SOFSS(U, E) \Leftrightarrow$ for every fuzzy soft point $e(G_A) \tilde{\in} G_A$, there exists a fuzzy set $H_A \tilde{\in} SOFSS(U, E)$ such that $e(G_A) \tilde{\in} H_A \tilde{\subseteq} G_A$.

Proof

Take $H_A = G_A$, this shows that the condition is necessary.

For sufficiency, we have $G_A = \tilde{\cup}_{e(G_A)\tilde{\in} G_A} e(G_A) \tilde{\subseteq} \tilde{\cup}_{e(G_A)\tilde{\in} G_A} H_A \tilde{\subseteq} G_A$.

Theorem 5.10

If G_A is any fuzzy soft set in a fuzzy soft topological space(U, E, τ) then following are equivalent

i. G_A is semi closed fuzzy soft set
ii. $Int(Cl(G_A)) \tilde{\subseteq} G_A$
iii. $Cl(Int(G_A^C)) \tilde{\supseteq} G_A^c$
iv. G_A^C is semiopen fuzzy soft set.

Proof

(i)⇒(ii)

If G_A is semiclosed fuzzy soft set, then there exists a closed fuzzy soft set H_A such that $Int(H_A) \tilde{\subseteq} G_A \tilde{\subseteq} H_A \Rightarrow Int(H_A) \tilde{\subseteq} G_A \tilde{\subseteq} Cl(G_A) \tilde{\subseteq} H_A$. By the property of interior we have $Int(Cl(G_A)) \tilde{\subseteq} Int(H_A) \tilde{\subseteq} G_A$.

(ii) ⇒(iii)

$$Int(Cl(G_A)) \tilde{\subseteq} G_A \Rightarrow G_A^C \tilde{\subseteq} Int(Cl(G_A))^C = Cl(Int(G_A^C)) \tilde{\supseteq} G_A^C$$

(iii) ⇒(iv)

$H_A = Int(G_A^C)$ is an open fuzzy soft set such that $Int(G_A^C) \tilde{\subseteq} G_A^C \tilde{\subseteq} Cl(Int(G_A^C))$, hence G_A^C is semiopen.

(iv) ⇒(i)

As G_A^C is semiopen there exists an open fuzzy soft set H_A such that $H_A \tilde{\subseteq} G_A^C \tilde{\subseteq} Cl(H_A) \Rightarrow H_A^C$ is a closed fuzzy soft set such that $G_A \tilde{\subseteq} H_A^C$ and $G_A^C \tilde{\subseteq} Cl(H_A) \Rightarrow Int(H_A^C) \tilde{\subseteq} G_A$, hence G_A is semiclosed fuzzy soft set.

Definition 5.11

Let (U, E, τ) be a fuzzy soft topological space and G_A be a fuzzy soft set over (U, E).

i. The **fuzzy soft semi closure** of G_A is a fuzzy soft set $FSSCl(G_A) = \tilde{\cap} \{ S_A : G_A \tilde{\subseteq} S_A \text{ and } S_A \in SCFSS(U, E)\}$
ii. The **fuzzy soft semi interior** of G_A is a fuzzy soft set $FSSInt(G_A) = \tilde{\cup} \{ S_A : S_A \tilde{\subseteq} G_A \text{ and } S_A \in SOFSS(U, E)\}$

$FSSCl(G_A)$ is the smallest semiclosed fuzzy soft set containing G_A and $FSSInt(G_A)$ is the largest semiopen fuzzy soft set contained in G_A.

Theorem 5.12

Let (U, E, τ) be a fuzzy soft topological space and G_A and K_A be two fuzzy soft set over (U, E) then

i. $G_A \tilde{\in} SCFSS(U, E) \Leftrightarrow G_A = FSSCl(G_A)$
ii. $G_A \tilde{\in} SOFSS(U, E) \Leftrightarrow G_A = FSSInt(G_A)$
iii. $(FSSCl(G_A))^C = FSSInt(G_A^C)$
iv. $(FSSInt(G_A))^C = FSSCl(G_A^C)$
v. $G_A \tilde{\subseteq} K_A \Rightarrow FSSInt(G_A) \tilde{\subseteq} FSSInt(K_A)$
vi. $G_A \tilde{\subseteq} K_A \Rightarrow FSSCl(G_A) \tilde{\subseteq} FSSCl(K_A)$
vii. $FSSCl(G_A \tilde{\cup} K_A) = FSSCl(G_A) \tilde{\cup} FSSCl(K_A)$
viii. $FSSInt(G_A \tilde{\cap} K_A) = FSSInt(G_A) \tilde{\cap} FSSInt(K_A)$
ix. $FSSCl(G_A \tilde{\cap} K_A) \tilde{\subset} FSSCl(G_A) \tilde{\cap} FSSCl(K_A)$
x. $FSSInt(G_A \tilde{\cup} K_A) \tilde{\subset} FSSInt(G_A) \tilde{\cup} FSSInt(K_A)$
xi. $FSSCl(FSSCl(G_A)) = FSSCl(G_A)$
xii. $FSSInt(FSSInt(G_A)) = FSSInt(G_A)$