



Avinashilingam Institute for Home Science and Higher Education for Women
 Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD [now MoE]
 Re-accredited with 'A++' Grade by NAAC. CGPA 3.65/4, Category I by UGC
 Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test II – April 2025
IV/VI Semester

Class : II UG / III UG

Time : 2 Hours

Branch : Special Education & Mathematics / Mathematics

Max.Marks : 60

23BSMC10 / 21BMAC25 Special Functions and Fourier series

Course Outcomes:

- CO1: Understand the basic concepts of Beta and Gamma functions
 CO2: Use the Beta and Gamma functions to evaluate integrals.
 CO3: Apply Bessel functions in Physics and Engineering.
 CO4: Establish Bessel integrals for Bessel coefficients occurring in Astronomy problems.
 CO5: Express periodic function as a Fourier series.

Part A

6 x 1 = 6

Choose the Correct Answer

- $J_{1/2}(x) =$ CO3K2
 a. $\sqrt{\frac{2\pi}{x}} \sin x$ b. $\sqrt{\frac{2\pi}{x}} \cos x$ c. $\sqrt{\frac{2}{\pi x}} \sin x$ d. $\sqrt{\frac{2}{\pi x}} \cos x$
- Which one of the following is correct? CO3K1
 a. $J_n(0) = 0$ b. $J_n(0) = 1$ c. $J_0(0) = 0$ d. $J_{-n}(0) = 1$
- The value of $[J_0(x)]^2 + 2[J_1(x)]^2 + 2[J_2(x)]^2 + \dots$ is CO4K1
 a. 1 b. π c. $\frac{1}{2}$ d. 0
- Which of the following is correct? CO4K2
 a. $x/2 = J_1 + 3J_3 + 5J_5 + \dots$ b. $x = J_1 + 3J_3 + 5J_5 + \dots$
 c. $x/2 = J_1 - 3J_3 + 5J_5 - \dots$ d. $x = J_1 - 3J_3 + 5J_5 - \dots$
- The constant term in the Fourier cosine expansion for $f(x) = x$ in $(0, \pi)$ is CO5K2
 a. π b. $-\pi$ c. 2π d. $\pi/2$
- The function $f(x) = e^{-x}$ is said to be ____ function. CO5K2
 a. an odd b. an even c. neither even nor odd d. none

Part B

3 x 6 = 18

Answer ALL questions

- a. Find the value of $J_{-1/2}(x)$. CO3K3
 (or)
- b. When n is an integer, show that $J_{-n}(x) = (-1)^n J_n(x)$. CO3K3
- a. Express $J_{5/2}(x)$ in finite form. CO4K3
 (or)
- b. Find $\int x^2 J_2(x) dx$. CO4K4
- a. Find the Fourier series of $f(x) = e^x$ in $(-\pi, \pi)$. CO5K4
 (or)
- b. Express $f(x) = x$ in $-\pi < x < \pi$ as a Fourier series with period 2π . CO5K3

Part C

3 x 12 = 36

Answer ALL questions

- a. Find series solution of Bessel's equation when $m = n$. CO3K3
 (or)
- b. Solve in terms of Bessel functions, the equation $x^2 y'' + xy' + (\lambda^2 x^2 - n^2) y = 0$. CO3K3
- a. Derive the Recurrence formula for Bessel functions. CO4K4
 (or)
- b. Prove that $\int_0^\infty e^{-ax} J_0(bx) dx = \frac{1}{\sqrt{a^2 + b^2}}$ CO4K4
- a. Show that $x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^\infty (-1)^n \frac{\cos nx}{n^2}$ in the interval $(-\pi \leq x \leq \pi)$. CO5K4
 Hence deduce that (i) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$ (ii) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$.
 (or)
- b. Expand $f(x) = x \sin x$ as a cosine series in the range $0 < x < \pi$ and deduce CO5K5
 $\frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots = \frac{\pi}{4}$

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