Chapter III

CHAPTER III

FUZZY COGNITIVE BIMAPS AND NEUTROSOPHIC COGNITIVE BIMAPS.

SECTION 3.1

FUZZY COGNITIVE BIMAPS

Definition 3.1.1

A **Fuzzy cognitive map** (FCM) is a directed graph with concepts like policies, events etc. as nodes and causalities as edges. It represents causal relationship between concepts.

We illustrate this by the example 3.1.2:

Example 3.1.2

In Tamil Nadu (a southern state in India) in the last decade several new engineering colleges have been approved and started. The resultant increase in the production of engineering graduates in these years is disproportionate with the need of engineering graduates. This has resulted in thousands of unemployed and underemployed graduate engineers. Using an expert's opinion we study the effect of such unemployed people on the society. An expert spells out the five major concepts relating to the unemployed graduated engineers as

E_1 – Frustration

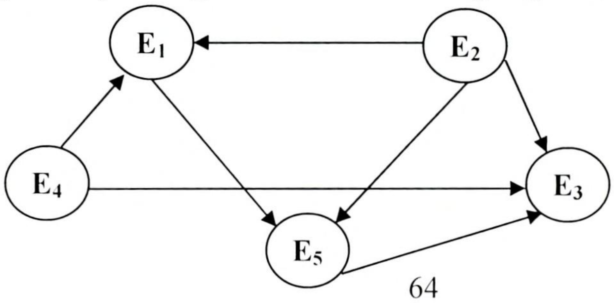
E_2 – Unemployment

E_3 – Increase of educated criminals

E_4 – Under employment

E_5 – Taking up drugs etc.

The directed graph where E_1, …, E_5 are taken as the nodes and causalities as edges as given by an expert is given in the following Figure:



According to this expert, increase in unemployment increases frustration. Increase in unemployment, increases the educated criminals. Frustration increases the graduates to take up to evils like drugs etc. Unemployment also leads to the increase in number of persons who take up to drugs, drinks etc. to forget their worries and unoccupied time. Under-employment forces then to do criminal acts like theft (leading to murder) for want of more money and so on. Thus one cannot actually get data for this but can use the expert's opinion for this unsupervised data to obtain some idea about the real plight of the situation. This is just an illustration to show how FCM is described by a directed graph.

{If increase (or decrease) in one concept leads to increase (or decrease) in another, then we give the value 1. If there exists no relation between two concepts the value 0 is given. If increase (or decrease) in one concept decreases (or increases) another, then we give the value –1. Thus FCMs are described in this way.}

Definition 3.1.3

When the nodes of the FCM are fuzzy sets then they are called as **fuzzy nodes**.

Definition 3.1.4

FCMs with edge weights or causalities from the set {–1, 0, 1} are called **simple FCMs**.

Definition 3.1.5

Consider the nodes / concepts $C_1, \ldots, C_n$ of the FCM. Suppose the directed graph is drawn using edge weight $e_{ij} \in \{0, 1, -1\}$. The matrix E be defined by $E = (e_{ij})$ where e_{ij} is the weight of the directed edge $C_i C_j$. E is called the **adjacency matrix** of the FCM, also known as the **connection matrix** of the FCM.

Note 3.1.6

It is important to note that all matrices associated with an FCM are always square matrices with diagonal entries as zero.

Definition 3.1.7

Let $C_1, C_2, \ldots, C_n$ be the nodes of an FCM. $A = (a_1, a_2, \ldots, a_n)$ where

$a_i \in \{0, 1\}$. A is called the **instantaneous state vector** and it denotes the on-off position of the node at an instant.

$a_i = 0$ if a_i is off and

$ai = 1$ if a_i is on for i = 1, 2, …, n.

Definition 3.1.8

Let $C_1, C_2, \ldots C_n$ be the nodes of an FCM. Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$, …, $\overrightarrow{C_iC_j}$ be the edges of the FCM ($i \neq j$). Then the edges form a directed cycle. An FCM is said to be **cyclic** if it possesses a directed cycle. An FCM is said to be **acyclic** if it does not possess any directed cycle.

Definition 3.1.9

An FCM with cycles is said to have a **feedback**.

Definition 3.1.10

When there is a feedback in an FCM, i.e., when the causal relations flow through a cycle in a revolutionary way, the FCM is called a **dynamical system**.

Definition 3.1.11

Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$, …, $\overrightarrow{C_{n-1}C_n}$ be a cycle. When Ci is switched on and if the causality flows through the edges of a cycle and if it again causes C_i, we say that the dynamical system goes round and round. This istrue for any node C_i, for i = 1, 2, … , n. The equilibrium state for this dynamical system is called the **hidden pattern**.

Definition 3.1.12

If the equilibrium state of a dynamical system is a unique state vector, then it is called a **fixed point**.

Example 3.1.13

Consider a FCM with $C_1, C_2, \ldots, C_n$ as nodes. For example let us start the dynamical system by switching on C_1. Let us assume that the FCM settles down with C_1 and C_n on i.e. the state vector remains as (1, 0, 0,…, 0, 1) this state vector (1, 0, 0, …, 0, 1) is called the **fixed point**.

Definition 3.1.14

If the FCM settles down with a state vector repeating in the form $A_1 \rightarrow A_2 \rightarrow \ldots \rightarrow A_i \rightarrow A_1$ then this equilibrium is called a **limit cycle**.

METHODS OF FINDING THE HIDDEN PATTERN:

Definition 3.1.15

Finite number of FCMs can be combined together to produce the joint effect of all the FCMs. Let $E_1, E_2, \ldots, E_p$ be the adjacency matrices of the FCMs with nodes $C_1, C_2, \ldots, C_n$ then the **combined FCM** is got by adding all the adjacency matrices $E_1, E_2, \ldots, E_p$.

We denote the combined FCM adjacency matrix by $E = E_1 + E_2 + \ldots + E_p$.

Notation 3.1.16

Suppose $A = (a_1, \ldots, a_n)$ is a vector which is passed into a dynamical system E. Then $AE = (a'_1, \ldots, a'_n)$ after thresholding and updating the vector suppose we get $(b_1, \ldots, b_n)$ we denote that by $(a'_1, a'_2, \ldots, a'_n) \rightarrow (b_1, b_2, \ldots, b_n)$. Thus the symbol '$\rightarrow$' means the resultant vector has been thresholded and updated.

Definition 3.1.17

Fuzzy Cognitive bimaps (FCBMs) are fuzzy signed directed bigraphs with feed back. The directed edge e^p_{ij} from causal concept c^p_i to concept c^p_j measures how much c^p_i causes c^p_j, (p = 1, 2). The time varying concept function c^p_j (t) measures the non negative occurrence of some fuzzy event, perhaps the strength of a political statement, in medical analysis or so on. The edge values e^p_{ij} takes values in the fuzzy causal interval [-1, 1], $e^p_{ij} = 0$ indicates no causality, $e^p_{ij} > 0$ indicates causal increase, c^p_j increases as c^p_i increases (or c^p_j decreases as c^p_i decreases); $e^p_{ij} < 0$ indicates causal decrease or negative causality c^p_i decreases as c^p_j increases (and or c^p_i increases or c^p_j decreases) (p = 1, 2).

Simple FCBMs have edge values $e^p_{ij} \in \{-1, 0, 1\}$, (p = 1, 2).

SECTION 3.2

NEUTROSOPHIC COGNITIVE BIMAPS

Definition 3.2.1

A **Neutrosophic Cognitive Map** (NCM) is a neutrosophic directed graph with concepts like policies, events etc. as nodes and causalities or indeterminates as edges. It represents the causal relationship between concepts.

Let C_1, C_2, …, C_n denote n nodes, further we assume each node is a neutrosophic vector from neutrosophic vector space V. So a node C_i will be represented by $(x_1, \ldots, x_n)$ where x_k's are zero or one or I where I is the indeterminate and $x_k = 1$ means that the node C_k is in the on state and $x_k = 0$ means the node is in the off state and $x_k = I$ means the nodes state is an indeterminate at that time or in that situation.

Let C_i and C_j denote the two nodes of the NCM. The directed edge from C_i to C_j denotes the causality of C_i on C_j called **connections**. Every edge in the NCM is weighted with a number in the set {-1, 0, 1, I}. Let e_{ij} be the weight of the directed edge C_iC_j, $e_{ij} \in \{-1, 0, 1, I\}$. $e_{ij} = 0$ if C_i does not have any effect on C_j, $e_{ij} = 1$ if increase (or decrease) in C_i causes increase (or decreases) in C_j, $e_{ij} = -1$ if increase (or decrease) in C_i causes decrease (or increase) in C_j . $e_{ij} = I$ if the relation or effect of C_i on C_j is an indeterminate.

Definition 3.2.2

NCMs with edge weight from {-1, 0, 1,I} are called **simple NCMs**.

Definition 3.2.3

Let C_1, C_2, …, C_n be nodes of a NCM. Let the neutrosophic matrix N(E) be defined as N(E) = (e_{ij}) where e_{ij} is the weight of the directed edge C_i C_j, where $e_{ij} \in \{0, 1, -1, I\}$. N(E) is called the **neutrosophic adjacency matrix of the NCM**.

Definition 3.2.4

Let C_1, C_2, …, C_n be the nodes of the NCM. Let A = $(a_1, a_2, \ldots, a_n)$ where

$a_i \in \{0, 1, I\}$. A is called the **instantaneous state neutrosophic vector** and it denotes the on – off – indeterminate state position of the node at an instant

$a_i = 0$ if ai is off (no effect)

$a_i = 1$ if ai is on (has effect)

$a_i = I$ if ai is indeterminate(effect cannot be determined)

for i = 1, 2,..., n.

Definition 3.2.5

Let $C_1, C_2, \ldots, C_n$ be the nodes of the FCM. Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$, ... , $\overrightarrow{C_iC_j}$ be the edges of the NCM. Then the edges form a directed cycle. An NCM is said to be **cyclic** if it possesses a directed cyclic. An NCM is said to be **acyclic** if it does not possess any directed cycle.

Definition 3.2.6

An NCM with cycles is said to have a **feedback**. When there is a feedback in the NCM i.e. when the causal relations flow through a cycle in a revolutionary manner the NCM is called a **dynamical system**.

Definition 3.2.7

Let $\overrightarrow{C_1C_2}$, $\overrightarrow{C_2C_3}$, $\overrightarrow{C_3C_4}$,..., $\overrightarrow{C_{n-1}C_n}$ be cycle, when C_i is switched on and if the causality flow through the edges of a cycle and if it again causes C_i, we say that the dynamical system goes round and round. This is true for any node C_i, for i = 1, 2,..., n. the equilibrium state for this dynamical system is called the **hidden pattern**.

Definition 3.2.8

If the equilibrium state of a dynamical system is a unique state vector, then it is called a **fixed point**. Consider the NCM with $C_1, C_2, \ldots, C_n$ as nodes. For example let us start the dynamical system by switching on C_1. Let us assume that the NCM settles down with C_1 and C_n on, i.e. the state vector remain as (1, 0,..., 1) this neutrosophic state vector (1,0,..., 0, 1) is called the **fixed point**.

Definition 3.2.9

If the NCM settles with a neutrosophic state vector repeating in the form

$$A_1 \rightarrow A_2 \rightarrow \ldots \rightarrow A_i \rightarrow A_1,$$

then this equilibrium is called a **limit cycle** of the NCM.

METHODS OF DETERMINING THE HIDDEN PATTERN:

Let $C_1, C_2, \ldots, C_n$ be the nodes of an NCM, with feedback. Let E be the associated adjacency matrix. Let us find the hidden pattern when C_1 is switched on when an input is given as the vector $A_1 = (1, 0, 0, \ldots, 0)$, the data should pass through the neutrosophic matrix N(E), this is done by multiplying A_1 by the matrix N(E). Let $A_1N(E) = (a_1, a_2, \ldots, a_n)$ with the threshold operation that is by replacing a_i by 1 if $a_i \geq k$ and a_i by 0 if $a_i < k$ (k – a suitable positive integer) and a_i by I if a_i is not a integer. We update the resulting concept, the concept C_1 is included in the updated vector by making the first coordinate as 1 in the resulting vector. Suppose $A_1N(E) \rightarrow A_2$ then consider $A_2N(E)$ and repeat the same procedure. This procedure is repeated till we get a limit cycle or a fixed point.

Definition 3.2.10

Finite number of NCMs can be combined together to produce the joint effect of all NCMs. If $N(E_1), N(E_2), \ldots, N(E_p)$ be the neutrosophic adjacency matrices of a NCM with nodes $C_1, C_2, \ldots, C_n$ then the **combined NCM** is got by adding all the neutrosophic adjacency matrices $N(E_1), \ldots, N(E_p)$. We denote the combined NCMs adjacency neutrosophic matrix by

$$N(E) = N(E_1) + N(E_2) + \ldots + N(E_p).$$

Notation 3.2.11

Let $(a_1, a_2, \ldots, a_n)$ and $(a'_1, a'_2, \ldots, a'_n)$ be two neutrosophic vectors. We say $(a_1, a_2, \ldots, a_n)$ is equivalent to $(a'_1, a'_2, \ldots, a'_n)$ denoted by

$$((a_1, a_2, \ldots, a_n) \sim (a'_1, a'_2, \ldots, a'_n))$$

If $(a'_1, a'_2, \ldots, a'_n)$ is got after thresholding and updating the vector $(a_1, a_2, \ldots, a_n)$ after passing through the neutrosophic adjacency matrix N(E).

Note 3.2.12

The nodes C_1, C_2, …, C_n are not indeterminate nodes because they indicate the concepts which are well known. But the edges connecting C_i and C_j may be indeterminate i.e. an expert may not be in a position to say that C_i has some causality on C_j either will he be in a position to state that C_i has no relation with C_j in such cases the relation between C_i and C_j which is indeterminate is denoted by I.

Note 3.2.13

The nodes when sent will have only ones and zeros i.e. on and off states, but after the state vector passes through the neutrosophic adjacency matrix the resultant vector will have entries from {0, 1, I} i.e. they can be neutrosophic vectors. The presence of I in any of the coordinate implies the expert cannot say the presence of that node i.e. on state of it after passing through N(E) nor can we say the absence of the node i.e. off state of it the effect on the node after passing through the dynamical system is indeterminate so only it is represented by I. Thus only in case of NCMs we can say the effect of any node on other nodes can also be indeterminates. Such possibilities and analysis is totally absent in the case of FCMs.

Note 3.2.14

In the neutrosophic matrix N(E), the presence of I in the a_{ij}^{th} place shows, that the causality between the two nodes i.e. the effect of C_i on C_j is indeterminate. Such chances of being indeterminate is very possible in case of unsupervised data and that too in the study of FCMs which are derived from the directed graphs.

Thus only NCMs helps in such analysis.

Definition 3.2.15

A **neutrosophic cognitive bimap** (NCBM) is a neutrosophic directed bigraph with concepts like policies or events etc as nodes and causalities and indeterminate as edges.

It represents the casual relationship between concepts.

Let $\{C_1^1,\ldots,C_n^1\}$ and $\{C_1^2,\ldots,C_m^2\}$ be a set of n and m nodes, further we assume each node is a neutrosophic vector from a neutrosophic vector space V. So a node $C = C_1 \cup C_2$ will be represented by $\{X_1^1,\ldots,X_n^1\} \cup \{X_1^2,\ldots,X_m^2\}$ where X_i^t are zero or one or I (I is the indeterminate) (t = 1, 2) and $X_i^t = 1$ means the node C_i is in the on state, $X_i^t = 0$ means the node is in the off state $X_i^t = I$ means the node is in the indeterminate state at that time or in that situation or circumstance.

Example 3.2.16

Suppose we are studying about the behaviour of a naughty child in the presence of a stern teacher surrounded by his classmates, the node naughtiness at that movement in that situation is an indeterminate for by the nice behaviour of the child in that circumstance the expert cannot make conclusions that the child is not naughty, he can only say indeterminate without fully knowing about the child's nature from his parents or relatives. Thus the coordinate naughtiness cannot be given 1 as in that circumstance at that time he is so good, cannot be given 0 for the expert is not fully aware of the fact that he is naughty so the expert can say only indeterminate for in the presence of that particular teacher and that room he may be behaving good, might be his behaviour would be very different in the play ground with his group of friends…

Definition 3.2.17

Let C_i^t and C_j^t denote the a pair of two nodes of the NCBM (t = 1, 2). The directed edges from C_i^1 to C_j^1 and C_i^2 to C_j^2 denotes the causality of C_i^t on C_j^t (t = 1, 2) called **connections**. Every edge in the NCBM is weighted with a number in the set {-1 0 1 I}. Let e_{ij}^t ,(t =1,2) be the weight of the directed edges $C_i^t\, C_j^t$ (t = 1, 2) $e_{ij}^t \in \{1,0,-1,I\}$, e_{ij}^t is 0 if C_i^t does not have any effect on C_j^t , e_{ij}^t =1 if increase (or decrease) in C_i^t causes increase (or decrease) on C_j^t. $e_{ij}^t = -1$ if increase (or decrease) on C_i^t causes decrease (or increase) in C_j^t. $e_{ij}^t = I$ if the relation or effect of C_i^t on C_j^t is an indeterminate.

Definition 3.2.18

NCBMs with edge weight from {-1, 0, 1, I} are called **simple NCBMs.**

Definition 3.2.19

Let $C_1^t, C_2^t, \ldots, C_{k_t}^t$ (t = 1, 2) be nodes of a NCBM. Let the neutrosophic bimatrix N(E) be defined as N(E) = (e_{ij}^t) (t = 1, 2) where e_{ij}^t is the weight of the directed edge C_i^t, C_j^t where $e_{ij}^t \in \{1, 0, -1, I\}$, (t = 1, 2) N(E) is called the **neutrosophic adjacency bimatrix of the NCBM**.

Definition 3.2.20

Let $C_1^t, C_2^t, \ldots, C_{n_t}^t$ be nodes of the NCBM. Let A = $(a_1^t, a_2^t, \ldots, a_{n_t}^t)$ where $a_i^t \in \{0, 1, I\}$, A is called the **instantaneous state neutrosophic bivector** and it denotes the on-off-indeterminate state / position of the node at an instant

$a_i^t = 0$ if a_i^t is off (no effect)

$a_i^t = 1$ if a is on (has effect)

$a_i^t = I$ if a is indeterminate (effect cannot be determined)

(t = 1, 2) i = 1, 2,…, n.

Definition 3.2.21

Let $C_1^t, C_2^t, \ldots, C_{n_t}^t$ be the nodes of the NCBM.

Let $C_1^t C_2^t, C_2^t C_3^t, \ldots, C_i^t C_j^t$ = (t = 1, 2) be the edge of the NCBM. Then the edges form a directed cycle An NCBM is said to be **cyclic** if it possesses a directed cycle. An NCBM is said to be **acyclic** if it does not possess any directed cycle.

Definition 3.2.22

An NCBM with cycles is said to have a **feed back**. When there is a feed back in the NCBM i.e. when the causal relation flow through a cycle in a revolutionary manner the NCBM is called a **dynamical system**.

Definition 3.2.23

Let $C_1^tC_2^t, C_2^tC_3^t, \ldots, C_{n-1}^tC_n^t$ be a cycle when C_i^t (t = 1, 2) is switched on and if the causality flow through the edges of a cycle and it again causes C_i^t we say that the dynamical system goes round and round. This is true for any node C_i^t (t = 1, 2) for i = 1, 2,…, n_t the equilibrium state for this dynamical system is called the **bihidden pattern**.

Definition 3.2.24

If the equilibrium state of a dynamicalsystem is a unique state bivector, then it is called a **fixed bipoint**. Consider the NCBM with $C_1, C_2, \ldots, C_{n_t}$ as nodes. For instance let us start the dynamical system by switching on C_i. Let us assume that the NCBM settles down with C_1^t and $C_{m_t}^t$ on i.e. the state vector remains as (1, 0,…,0 10) ∪ (1, 0,…, 0 1 0) this neutrosophic bivector is called the **fixed bipoint**.

Definition 3.2.25

If the NCBM settles with a neutrosophic state bivector repeating in the form $A_1^t \to A_2^t \to, \ldots, \to A_i^t \to A_i$ (t = 1, 2) then this equilibrium is called a **limit bicycle** of the NCBM.

Methods of determining the hidden pattern:

Let $C_1^t, C_2^t, \ldots, C_{n_t}^t$ (t = 1, 2) be the nodes of the NCBM with feed back. Let $N(E_B) = E_1 \cup E_2$ be the associated adjacency bimatrix. Let us find the bihidden pattern when C_1^t is switched on (t = 1, 2) when an input is given as the bivector $A = A_1 \cup A_2$ = (1 0 0…0) ∪ (1 0 …0) the data should pass through the neutrosophic bimatrix N(EB) this is done by multiplying the bivector A by $N(E_B)$.

$$\text{Now} \quad AN(E_B) = (A_1 \cup A_2)(E_1 \cup E_2)$$

$$= A_1 E_1 \cup A_2 E_2$$

$$= (a_1^1, \ldots, a_{n_1}^1) \cup (a_1^2, \ldots, a_{n_2}^2),$$

with the threshold operation that is by replacing a_i^t by 1 (t = 1, 2) if $a_i^t \geq K$ and $a_i^t = 0$ if $a_i^t < K$ (K – a suitable positive integer) and a_i^t by I if a_i^t is not an integer. We update the resulting concept, the concept C_1 is included in the updated bivector by making the first coordinate as 1 in the resulting bivector for we started the operation in this case with the first coordinate in the on state. Suppose

$AN(E_B) \rightarrow B = B_1 \cup B_2$ then consider

$$\begin{aligned} BN(B_B) &= (B_1 \cup B_2)(E_1 \cup E_2) \\ &= B_1 E_1 \cup B_2 E_2 \text{ and} \\ &= (b_1^1,\ldots,b_{n_1}^1) \cup (b_1^2,\ldots,b_{n_1}^2); \end{aligned}$$

and repeat the same procedure. This procedure is repeated till we get a **limit bicycle** or **a fixed bipoint**.

Definition 3.2.26

Finite number of NCBM can be combined together to produce the joint effect of all NCBMs. If $N(E_B^1),\ldots, N(E_B^p)$ be the neutrosophic adjacency bimatrices of a NCBM with nodes $C_1^1, C_2^1,\ldots,C_{n_1}^1$ then the combined NCBM is got by adding all the neutrosophic adjacency bimatrices $N(E_B^1),\ldots, N(E_B^p)$. We denote the combined NCBMs adjacency neutrosophic bimatrices by

$$N(E_B) = N(E_B^1), + \ldots +, N(E_B^p)$$