Chapter 4

Contra λ_g^α-Continuous Maps and Contra λ_g^α-Irresolute Maps in Topological Spaces

4.1 Introduction

It is indeed a significant theme in general topology that various modified forms of continuities were identified and explored. Likewise, Dontchev [1996] introduced contra-continuous maps and strongly S-continuous maps in topological spaces. Contra continuity was found to be the stronger form of Gangster and Reilly's [1989] LC-continuous maps. Many investigations were carried out after the formulation of contra continuity. Dontchev and Noiri [1999] introduced a new weaker form of contra continuous maps called contra semi continuous maps and derived its properties. Saeid Jafari and Takashi Noiri [2001] defined and investigated the properties of contra α-continuous maps. Caldas et al. [2006 a] defined contra λ-continuous maps and examined the fundamental properties. Caldas [2000] proposed a new generalization of irresoluteness called contra irresoluteness in topological spaces which are the stronger form of ap-irresoluteness.

This chapter is dealt with new forms of the contra maps called as contra λ_g^α-continuous maps and contra λ_g^α-irresolute maps in topological spaces. In addition to it, the dependency and independency relationships are also derived accordingly. Further the remarkable properties and theorems have also been examined.

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4.2 Contra λ_g^α-Continuous Maps

This section deals with the definition of contra λ_g^α-continuous maps in topological spaces. Interrelationships are derived and interesting results are obtained.

Definition 4.2.1 A map $u:(M,\mu) \to (N,\nu)$ is called contra λ_g^α-continuous if $u^{-1}(T)$ is λ_g^α-open in (M,μ) for every closed set T of (N,ν).

Example 4.2.2 Let $M = N = \{i,j,k,l\}$, $\mu = \{\phi,\{i,j\},M\}$ and $\nu = \{\phi,\{i,j,k\},N\}$. Let $u:(M,\mu) \to (N,\nu)$ be a map defined by $u(i) = j, u(j) = k, u(k) = l$ and $u(l) = i$. Then u is a contra λ_g^α-continuous map.

Proposition 4.2.3 Every contra λ-continuous map $u:(M,\mu) \to (N,\nu)$ is a contra λ_g^α-continuous map.

Proof: Let T be any closed set in (N,ν). Then $u^{-1}(T)$ is λ-open in (M,μ). Now using Proposition 2.4.3, $u^{-1}(T)$ is also λ_g^α-open in (M,μ). Hence u is a contra λ_g^α-continuous map.

The subsequent example shows that the converse of Proposition 4.2.3 may not hold good.

Example 4.2.4 Consider M,N,μ,ν and u as in Example 4.2.2. Then u is a contra λ_g^α-continuous map but not a contra λ-continuous map, since for the closed set $\{l\}$ in (N,ν), $u^{-1}(\{l\}) = \{k\}$ is not λ-open in (M,μ).

Proposition 4.2.5 Every continuous map $u:(M,\mu) \to (N,\nu)$ is a contra λ_g^α-continuous map.

Proof: Let S be an open set in (N,ν). Then $u^{-1}(S)$ is open in (M,μ). Now using Proposition 2.2.7, $u^{-1}(S)$ is λ_g^α-closed in (M,μ). Thus u is contra λ_g^α-continuous.

The subsequent example shows that the converse of Proposition 4.2.5 may not hold good.

Example 4.2.6 Consider M, N, μ, ν and u as in Example 4.2.2. Then u is a contra λ_g^α-continuous map but not a continuous map, since for the closed set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{k\}$ is not closed in (M, μ).

Proposition 4.2.7 Every contra continuous map $u: (M, \mu) \to (N, \nu)$ is a contra λ_g^α-continuous map.

Proof: Evident from Propositions 2.2.7 and 2.4.3.

The subsequent example shows that the converse of Proposition 4.2.7 may not hold good.

Example 4.2.8 Consider M, N, μ, ν and u as in Example 4.2.2. Then u is a contra λ_g^α-continuous map but not a contra continuous map, since for the closed set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{k\}$ is not open in (M, μ).

Proposition 4.2.9 Every λ_g^α-irresolute map $u: (M, \mu) \to (N, \nu)$ is a contra λ_g^α-continuous map.

Proof: Let S be an open set in (N, ν). Then by Proposition 2.2.7, S is also λ_g^α-closed in (N, ν). Since u is a λ_g^α-irresolute map, $u^{-1}(S)$ is λ_g^α-closed in (M, μ). Thus u is a contra λ_g^α-continuous map.

The subsequent example shows that the converse of Proposition 4.2.9 may not hold good.

Example 4.2.10 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i, j\}, M\}$ and $\nu = \{\phi, \{i, j, k\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = k, u(j) = l, u(k) = i$ and $u(l) = j$. Then the u is a contra λ_g^α-continuous map but not a λ_g^α-irresolute map, since for the λ_g^α-closed set $\{i, k\}$ in (N, ν), $u^{-1}(\{i, k\}) = \{i, k\}$ is not a λ_g^α-closed set in (M, μ).

Remark 4.2.11 The following examples show that λ-continuous maps and contra λ_g^α-continuous maps are independent generally.

Example 4.2.12 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i, j, k\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = j, u(j) = i, u(k) = l$ and $u(l) = k$. Then u is a contra λ_g^α-continuous map but not a λ-continuous map, since for the closed set $\{k, l\}$ in (N, ν), $u^{-1}(\{k, l\}) = \{k, l\}$ is not λ-closed in (M, μ).

Example 4.2.13 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i, j, k\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = k, u(j) = l, u(k) = i$ and $u(l) = j$. Then u is a λ-continuous map but not a contra λ_g^α-continuous map, since for the closed set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{j\}$ is not λ_g^α-closed in (M, μ).

Proposition 4.2.14 If the domain (M, μ) is a $T_{1/2}$ space, then the map $u: (M, \mu) \to (N, \nu)$ is contra λ_g^α-continuous.

Proof: Evident from Proposition 2.2.31.

Proposition 4.2.15 Every quasi λ_g^α-continuous map $u: (M, \mu) \to (N, \nu)$ is a contra λ_g^α-continuous map.

Proof: Let S be any open set of (N, ν). Using Proposition 2.2.7, S is also a λ_g^α-closed set. Since u is a quasi λ_g^α-continuous map, $u^{-1}(S)$ is closed in (M, μ). Using Proposition 2.2.5, $u^{-1}(S)$ is a λ_g^α-closed set in (M, μ). Hence u is a contra λ_g^α-continuous map.

Example 4.2.16 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{l\}, \{i, j\}, \{i, j, l\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = k, u(j) = l, u(k) = i$ and $u(l) = j$. Then u is a contra λ_g^α-continuous map but not a quasi λ_g^α-continuous map, since for the λ_g^α-open set $\{i, k\}$ in (N, ν), $u^{-1}(\{i, k\}) = \{i, k\}$ is not open in (M, μ).

Proposition 4.2.17 Every totally λ_g^α-continuous map $u: (M, \mu) \to (N, \nu)$ is a contra λ_g^α-continuous map.

Proof: Let S be any open set in (N,ν). Then $u^{-1}(S)$ is λ_g^α-clopen in (M,μ). Hence u is contra λ_g^α-continuous.

The subsequent example shows that the converse of Proposition 4.2.17 may not hold good.

Example 4.2.18 Consider M, N, μ, ν and u as in Example 4.2.16. Then u is a contra λ_g^α-continuous map but not a totally λ_g^α-continuous map, since for the open set $\{l\}$ in (N,ν), $u^{-1}(\{l\}) = \{j\}$ is not λ_g^α-clopen in (M,μ).

Proposition 4.2.19 Every strongly λ_g^α-continuous map $u: (M,\mu) \to (N,\nu)$ is a contra λ_g^α-continuous map.

Proof: Evident from the Definitions 4.2.1 and 3.4.23.

The subsequent example shows that the converse of Proposition 4.2.19 may not hold good.

Example 4.2.20 Consider M, N, μ, ν and u as in Example 4.2.16. Then u is a contra λ_g^α-continuous map but not a strongly λ_g^α-continuous map, since for the subset $\{i, j, k\}$ in (N,ν), $u^{-1}(\{i, j, k\}) = \{i, k, l\}$ is not a λ_g^α-clopen set in (M,μ).

Proposition 4.2.21 Every perfectly λ_g^α-continuous map $u: (M,\mu) \to (N,\nu)$ is a contra λ_g^α-continuous map.

Proof: Let T be a closed set in (N,ν). Using Proposition 2.2.5, T is λ_g^α-closed in (N,ν). As u is perfectly λ_g^α-continuous, $u^{-1}(T)$ is clopen in (M,μ). Using Proposition 2.4.3, $u^{-1}(T)$ is λ_g^α-open in (M,μ). Hence u is contra λ_g^α-continuous.

The subsequent example shows that the converse of Proposition 4.2.21 may not hold good.

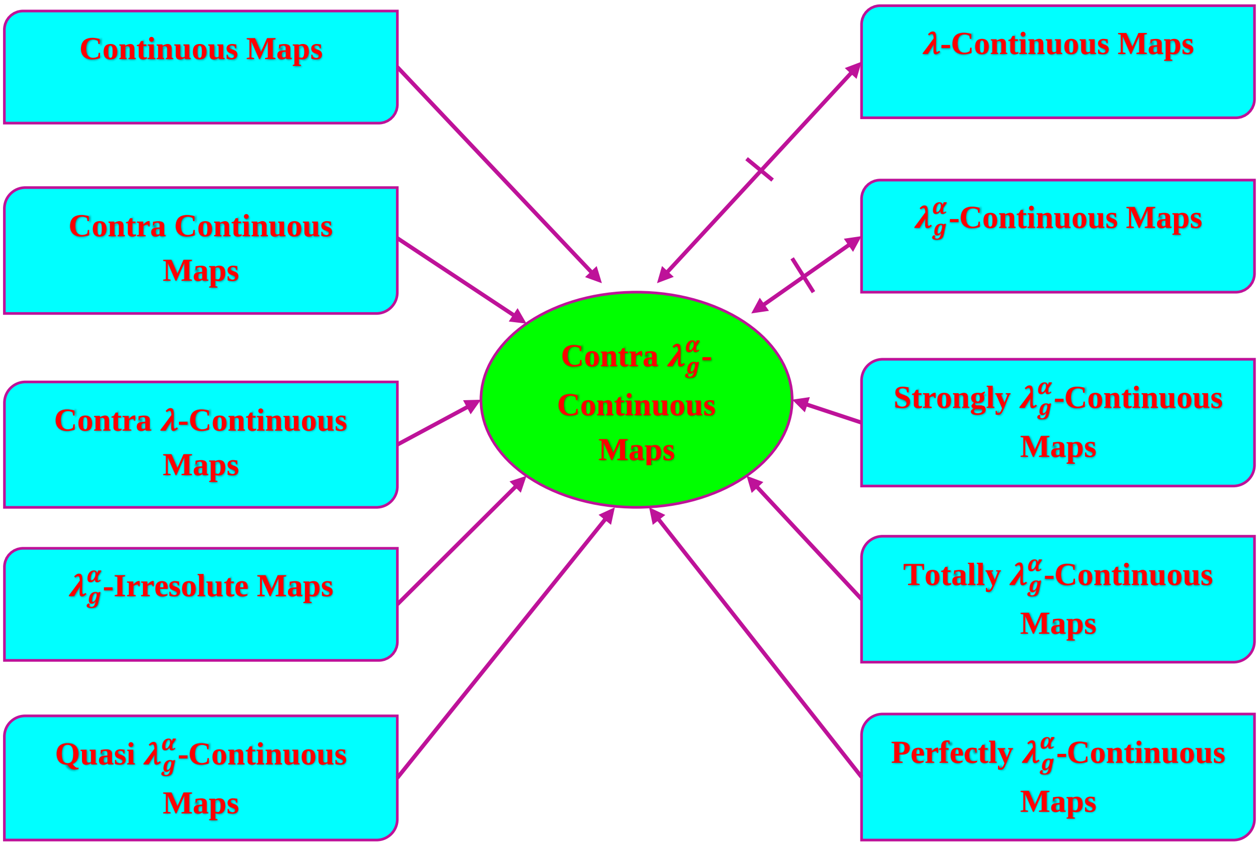
Example 4.2.22 Consider M, N, μ, ν and u as in Example 4.2.16. Then u is a contra λ_g^α-continuous map but not a perfectly λ_g^α-continuous map, since for the λ_g^α-closed set $\{j, l\}$ in (N,ν), $u^{-1}(\{j, l\}) = \{j, l\}$ is not a clopen set in (M,μ).

Remark 4.2.23 The following examples show that contra λ_g^α-continuous maps and λ_g^α-continuous maps are independent generally.

Example 4.2.24 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, \{i, k\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = j, u(j) = i$ and $u(k) = k$. Then u is a contra λ_g^α-continuous map but not a λ_g^α-continuous map, since for the closed set $\{j, k\}$ in (N, ν), $u^{-1}(\{j, k\}) = \{i, k\}$ is not λ_g^α-closed in (M, μ).

Example 4.2.25 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i, j, k\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = i, u(j) = l, u(k) = k$ and $u(l) = j$. Then u is a λ_g^α-continuous map but not a contra λ_g^α-continuous map, since for the closed set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{j\}$ is not λ_g^α-open in (M, μ).

Remark 4.2.26 The above associations are depicted in the following diagram.



Theorem 4.2.27 A map $u:(M,\mu)\to(N,\nu)$ is contra λ_g^α-continuous if $u^{-1}(S)$ is λ_g^α-closed in (M,μ) for every open set S in (N,ν) and vice versa.

Proof: Let S be any open set in (N,ν), then $N\setminus S$ is closed in (N,ν). Since u is contra λ_g^α-continuous, $u^{-1}(N\setminus S)=M\setminus u^{-1}(S)$ is λ_g^α-open in $(M,\mu)\Rightarrow u^{-1}(S)$ is λ_g^α-closed in (M,μ).

On the other side, let T be any closed set in (N,ν). Then $N\setminus T$ is open in (N,ν). By assumption, $u^{-1}(N\setminus T)=M\setminus u^{-1}(T)$ is λ_g^α-closed in $(M,\mu)\Rightarrow u^{-1}(T)$ is λ_g^α-open in (M,μ). Hence u is contra λ_g^α-continuous.

Theorem 4.2.28 If a map $u:(M,\mu)\to(N,\nu)$ is contra λ_g^α-continuous then u is contra continuous whenever (M,μ) is a T_1-space and λ-space.

Proof: Let T be a closed set in (N,ν). As u is a contra λ_g^α-continuous map, $u^{-1}(T)$ is λ_g^α-open in (M,μ). Since (M,μ) is a T_1-space and by Proposition 2.3.13, $u^{-1}(T)$ is λ-open in (M,μ). Also since (M,μ) is a λ-space, $u^{-1}(T)$ is open in (M,μ). Thus u is a contra continuous map.

Theorem 4.2.29 If a map $u:(M,\mu)\to(N,\nu)$ is contra λ_g^α-continuous then u is contra λ-continuous, whenever (M,μ) is T_1-space.

Proof: Let S be an open set in (N,ν). As u is a contra λ_g^α-continuous map, $u^{-1}(S)$ is λ_g^α-closed in (M,μ). Since (M,μ) is a T_1-space and by Proposition 2.3.13, $u^{-1}(S)$ is λ-closed. Thus u is a contra λ-continuous map.

Theorem 4.2.30 If a map $u:(M,\mu)\to(N,\nu)$ is λ-continuous then u is contra λ_g^α-continuous, whenever (M,μ) is a λ-space.

Proof: Let S be an open set in (N,ν). As u is λ-continuous, $u^{-1}(S)$ is λ-open in (M,μ). Since (M,μ) is a λ-space, $u^{-1}(S)$ is open in (M,μ). Using Proposition 2.2.7, $u^{-1}(S)$ is λ_g^α-closed in (M,μ). Thus u is a contra λ_g^α-continuous map.

Theorem 4.2.31 If a map $u: (M, \mu) \to (N, \nu)$ is contra λ-continuous and A is an open set of (M, μ), then the restriction $u_A: A \to N$ is contra λ_g^α-continuous.

Proof: Let S be an open set in (N, ν) and A be an open set in (M, μ). Using Lemma 1.1.8 (ii), A is λ-closed in (M, μ). Since u is contra λ-continuous, $u^{-1}(S)$ is λ-closed in (M, μ). Hence, we have $u^{-1}(S) \cap A$ is λ-closed in (M, μ) by Lemma 1.1.9. Using Proposition 2.2.3, $u^{-1}(S) \cap A$ is also a λ_g^α-closed set in (M, μ). Now, $[u^{-1}(S) \cap A] \subseteq A \subseteq M$ and $u^{-1}(S) \cap A = u_A^{-1}(S)$ is λ_g^α-closed in A. Thus, the restriction $u_A: A \to N$ is a contra λ_g^α-continuous map.

Proposition 4.2.32 The composition of two contra continuous maps is a contra λ_g^α-continuous map.

Proof: Let $u: (M, \mu) \to (N, \nu)$ and $w: (N, \nu) \to (K, \kappa)$ be contra continuous maps. Let S be an open set in (K, κ). Then $w^{-1}(S)$ is a closed set in (N, ν) as w is a contra continuous map and $u^{-1}(w^{-1}(S)) = (w \circ u)^{-1}(S)$ is an open set in (M, μ) as u is a contra continuous map. Now using Proposition 2.2.7, $(w \circ u)^{-1}(S)$ is a λ_g^α-closed set in (M, μ). Thus $(w \circ u): (M, \mu) \to (K, \kappa)$ is a contra λ_g^α-continuous map.

Proposition 4.2.33 The composition of two continuous maps is a contra λ_g^α-continuous map.

Proof: Evident from Definition 1.3.2 (i) and 4.2.1.

Proposition 4.2.34 If $u: (M, \mu) \to (N, \nu)$ is a contra λ_g^α-continuous map and $w: (N, \nu) \to (K, \kappa)$ is a continuous map, then $(w \circ u): (M, \mu) \to (K, \kappa)$ is a contra λ_g^α-continuous map.

Proof: Let T be a closed set in (K, κ). Then $w^{-1}(T)$ is closed in (N, ν) as w is continuous. Since u is contra λ_g^α-continuous, $(w \circ u)^{-1}(T) = u^{-1}(w^{-1}(T))$ is λ_g^α-open in (M, μ). Hence $(w \circ u)$ is a contra λ_g^α-continuous map.

Proposition 4.2.35 If $u: (M, \mu) \to (N, \nu)$ is a continuous map and $w: (N, \nu) \to (K, \kappa)$ is a contra continuous map, then $(w \circ u): (M, \mu) \to (K, \kappa)$ is a contra λ_g^α-continuous map.

Proof: Let S be an open set in (K,κ). Then $w^{-1}(S)$ is a closed set in (N,ν) as w is a contra continuous map and $u^{-1}(w^{-1}(S)) = (w \circ u)^{-1}(S)$ is a closed set in (M,μ) as u is a continuous map. Using Proposition 2.2.5, $(w \circ u)^{-1}(S)$ is a λ_g^α-closed set in (M,μ). Thus $(w \circ u)$ is a contra λ_g^α-continuous map.

Proposition 4.2.36 If $u: (M,\mu) \to (N,\nu)$ is a contra continuous map and $w: (N,\nu) \to (K,\kappa)$ is a continuous map, then $(w \circ u): (M,\mu) \to (K,\kappa)$ is a contra λ_g^α-continuous map.

Proof: Let S be an open set in (K,κ). Then $w^{-1}(S)$ is an open set in (N,ν) as w is a continuous map and $u^{-1}(w^{-1}(S)) = (w \circ u)^{-1}(S)$ is a closed set in (M,μ) as u is a contra continuous map. Now using Proposition 2.2.5, $(w \circ u)^{-1}(S)$ is a λ_g^α-closed set in (M,μ). Thus $(w \circ u)$ is a contra λ_g^α-continuous map.

Proposition 4.2.37 If $u: (M,\mu) \to (N,\nu)$ is a λ_g^α-continuous map and $w: (N,\nu) \to (K,\kappa)$ is a contra continuous map, then $(w \circ u): (M,\mu) \to (K,\kappa)$ is a contra λ_g^α-continuous map.

Proof: Let S be an open set in (K,κ). Then $w^{-1}(S)$ is a closed set in (N,ν) as w is contra continuous and $u^{-1}(w^{-1}(S)) = (w \circ u)^{-1}(S)$ is a λ_g^α-closed set in (M,μ) as u is a λ_g^α-continuous map. Thus $(w \circ u)$ is a contra λ_g^α-continuous map.

Proposition 4.2.38 If $u: (M,\mu) \to (N,\nu)$ is a contra λ-continuous map and $w: (N,\nu) \to (K,\kappa)$ is a continuous map, then $(w \circ u): (M,\mu) \to (K,\kappa)$ is a contra λ_g^α-continuous map.

Proof: Let S be an open set in (K,κ). Then $w^{-1}(S)$ is an open set in (N,ν) as w is a continuous map and $u^{-1}(w^{-1}(S)) = (w \circ u)^{-1}(S)$ is a λ-closed set in (M,μ) as u is a contra λ-continuous map. Using Proposition 2.2.3, $(w \circ u)^{-1}(S)$ is a λ_g^α-closed set in (M,μ). Thus $(w \circ u)$ is a contra λ_g^α-continuous map.

Proposition 4.2.39 The composition of two λ-irresolute maps is a contra λ_g^α-continuous map.

Proof: Follows from Lemma 1.1.8 (ii) and Proposition 2.2.3.

Proposition 4.2.40 Let $u: (M, \mu) \rightarrow (N, \nu)$ and $w: (N, \nu) \rightarrow (K, \kappa)$ be the bijective maps. Then the following are true:

(i) If $(w \circ u): (M, \mu) \rightarrow (K, \kappa)$ is a contra λ-continuous map and u is a λ-closed map then w is a contra λ_g^α-continuous map.

(ii) If $(w \circ u): (M, \mu) \rightarrow (K, \kappa)$ is a λ-irresolute map and u is a λ-closed map then w is a contra λ_g^α-continuous map.

(iii) If $(w \circ u): (M, \mu) \rightarrow (K, \kappa)$ is a contra continuous map and u is a λ-closed map then w is a contra λ_g^α-continuous map.

Proof: (i) Let S be an open set in (K, κ). Since $(w \circ u)$ is a contra λ-continuous map $(w \circ u)^{-1}(S) = u^{-1}(w^{-1}(S))$ is λ-closed in (M, μ). Now since u is a λ-closed map $u(u^{-1}(w^{-1}(S))) = w^{-1}(S)$ is a λ-closed set in (N, ν), which is also a λ_g^α-closed set by Proposition 2.2.3. Thus w is a contra λ_g^α-continuous map.

(ii) and (iii) can be proved similarly.

Proposition 4.2.41 Let $u: (M, \mu) \rightarrow (N, \nu)$ be a perfectly λ_g^α-continuous map and $w: (N, \nu) \rightarrow (K, \kappa)$ be a contra λ_g^α-continuous map then $(w \circ u): (M, \mu) \rightarrow (K, \kappa)$ is a totally λ_g^α-continuous map.

Proof: Follows from the Proposition 2.2.5 and Proposition 2.2.7.

Theorem 4.2.42 If $u: (M, \mu) \rightarrow (N, \nu)$ be a λ-irresolute map and $w: (N, \nu) \rightarrow (K, \kappa)$ be a contra λ-continuous map then their composition $(w \circ u): (M, \mu) \rightarrow (K, \kappa)$ is a contra λ_g^α-continuous map.

Proof: Let S be an open set in (K, κ). Since w is a contra λ-continuous map, $w^{-1}(S)$ is a λ-closed set in (N, ν) and since u is a λ-irresolute map, $u^{-1}(w^{-1}(S)) = (w \circ u)^{-1}(S)$ is a

λ-closed set in (M, μ). By Proposition 2.2.3, $(w \circ u)^{-1}(S)$ is a λ_g^α-closed set in (M, μ). Thus $(w \circ u)$ is a contra λ_g^α-continuous map.

Remark 4.2.43 The composition of two contra λ_g^α-continuous maps need not be a contra λ_g^α-continuous map as seen from the following example.

Example 4.2.44 Let $M = N = K = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$, $\nu = \{\phi, \{i, j\}, N\}$ and $\kappa = \{\phi, \{i, j, k\}, K\}$. Let $u: (M, \mu) \to (N, \nu)$ be the identity map and the map $w: (N, \nu) \to (K, \kappa)$ be defined by $w(i) = i, w(j) = l, w(k) = k$ and $w(l) = j$. Here u and w are both contra λ_g^α-continuous map but their composition $(w \circ u): (M, \mu) \to (K, \kappa)$ is not a contra λ_g^α-continuous map, since for the closed set $\{l\}$ in (K, κ), $(w \circ u)^{-1}(\{l\}) = u^{-1}(w^{-1}(\{l\}) = \{j\}$ is not a λ_g^α-open set in (M, μ).

Proposition 4.2.45 Let $u: (M, \mu) \to (N, \nu)$ be a λ_g^α-irresolute map and $w: (N, \nu) \to (K, \kappa)$ be a contra λ_g^α-continuous map, then $(w \circ u): (M, \mu) \to (K, \kappa)$ is a contra λ_g^α-continuous map.

Proof: Let T be a closed set in (K, κ). Since w is contra λ_g^α-continuous, $w^{-1}(T)$ is λ_g^α-open in (N, ν). Since u is λ_g^α-irresolute, $(w \circ u)^{-1}(T) = u^{-1}(w^{-1}(T))$ is λ_g^α-open in (M, μ). Hence $(w \circ u)$ is a contra λ_g^α-continuous map.

Proposition 4.2.46 The composition of two λ_g^α-irresolute maps is contra λ_g^α-continuous.

Proof: Let $u: (M, \mu) \to (N, \nu)$ and $w: (N, \nu) \to (K, \kappa)$ be λ_g^α-irresolute maps. Let S be an open set in (K, κ). Then S is a λ_g^α-closed set in (K, κ), by Proposition 2.2.7. Now as w is a λ_g^α-irresolute map, $w^{-1}(S)$ is a λ_g^α-closed set in (N, ν). Since u is a λ_g^α-irresolute map, $(w \circ u)^{-1}(S) = u^{-1}(w^{-1}(S))$ is a λ_g^α-closed set in (M, μ). Thus $(w \circ u): (M, \mu) \to (K, \kappa)$ is a contra λ_g^α-continuous map.

4.3 Contra λ_g^α-Irresolute Maps

In this section, contra λ_g^α-irresolute maps are defined with suitable examples. Some of the interesting facts and theorems have been analyzed and results have been furnished.

Definition 4.3.1 A map $u:(M,\mu) \to (N,\nu)$ is called contra λ_g^α-irresolute if $u^{-1}(S)$ is λ_g^α-closed in (M,μ) for every λ_g^α-open set S in (N,ν).

Example 4.3.2 Let $M = N = \{i,j,k\}$, $\mu = \{\phi,\{i\},\{j,k\},M\}$ and $\nu = \{\phi,\{i\},\{i,j\},\{i,k\},N\}$. Let u be the map defined by $u(i) = j, u(j) = i$ and $u(k) = k$. Then u is a contra λ_g^α-irresolute map.

Theorem 4.3.3 Let $u:(M,\mu) \to (N,\nu)$ be a map. Then the following statements are equivalent

(i) u is a contra λ_g^α-irresolute map.

(ii) The inverse image of every λ_g^α-closed (λ_g^α-open) set in (N,ν) is λ_g^α-open (λ_g^α-closed) in (M,μ).

Proof: Obvious from the Definition 4.3.1.

Proposition 4.3.4 Every contra λ_g^α-irresolute map $u:(M,\mu) \to (N,\nu)$ is a contra λ_g^α-continuous map.

Proof: Let S be any open set in (N,ν), using Proposition 2.4.3, S is λ_g^α-open in (N,ν). Since u is contra λ_g^α-irresolute, $u^{-1}(S)$ is λ_g^α-closed in (M,μ). Hence u is a contra λ_g^α-continuous map.

The subsequent example shows that the converse of Proposition 4.3.4 may not hold good.

Example 4.3.5 Let $M = N = \{i,j,k,l\}$, $\mu = \{\phi,\{i,j\},M\}$ and $\nu = \{\phi,\{i,j,k\},N\}$. Let $u:(M,\mu) \to (N,\nu)$ be the map defined by $u(i) = k, u(j) = l, u(k) = i$ and $u(l) = j$. Then

u is a contra λ_g^α-continuous map but not a contra λ_g^α-irresolute map, since for the λ_g^α-closed set $\{i,k\}$ in (N,ν), $u^{-1}(\{i,k\}) = \{i,k\}$ is not a λ_g^α-open set in (M,μ).

Proposition 4.3.6 Every quasi λ_g^α-continuous map $u:(M,\mu) \to (N,\nu)$ is a contra λ_g^α-irresolute map.

Proof: Let S be a λ_g^α-open set in (N,ν). As u is quasi λ_g^α-continuous, $u^{-1}(S)$ is open in (M,μ). Using Proposition 2.2.7, $u^{-1}(S)$ is λ_g^α-closed in (M,μ). Hence u is a contra λ_g^α-irresolute map.

The subsequent example shows that the converse of Proposition 4.3.6 may not hold good.

Example 4.3.7 Let $M = N = \{i,j,k,l\}, \mu = \{\phi, \{i,j\}, M\}$ and $\nu = \{\phi, \{i\}, \{i,j\}, N\}$. Then the identity map $u:(M,\mu) \to (N,\nu)$ is a contra λ_g^α-irresolute map but not a quasi λ_g^α-continuous map, since for the λ_g^α-open set $\{k,l\}$ in (N,ν), $u^{-1}(\{k,l\}) = \{k,l\}$ is not an open set in (M,μ).

Proposition 4.3.8 Every perfectly λ_g^α-continuous map $u:(M,\mu) \to (N,\nu)$ is a contra λ_g^α-irresolute map.

Proof: Let S be a λ_g^α-open set in (N,ν). As u is perfectly λ_g^α-continuous, $u^{-1}(S)$ is clopen in (M,μ) and using Propositions 2.2.5 and 2.2.7, $u^{-1}(S)$ is λ_g^α-closed in (M,μ). Hence u is a contra λ_g^α-irresolute map.

The subsequent example shows that the converse of Proposition 4.3.8 may not hold good.

Example 4.3.9 Consider M, N, μ, ν and u as in Example 4.3.7. Then u is a contra λ_g^α-irresolute map but not a perfectly λ_g^α-continuous map, since for the λ_g^α-closed set $\{j,k,l\}$ in (N,ν), $u^{-1}(\{j,k,l\}) = \{j,k,l\}$ is not a clopen set in (M,μ).

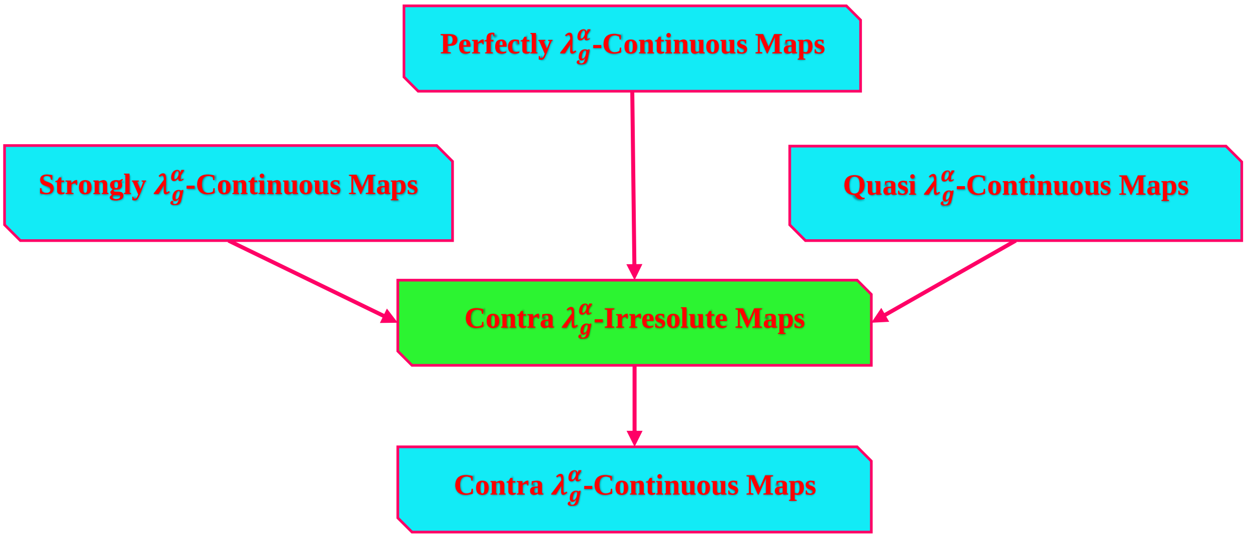
Proposition 4.3.10 Every strongly λ_g^α-continuous map $u:(M,\mu) \to (N,\nu)$ is a contra λ_g^α-irresolute map.

Proof: Let S be a λ_g^α-open set in (N, ν). As u is strongly λ_g^α-continuous, $u^{-1}(S)$ is λ_g^α-clopen in (M, μ). Therefore $u^{-1}(S)$ is λ_g^α-closed in (M, μ). Hence u is a contra λ_g^α-irresolute map.

The subsequent example shows that the converse of Proposition 4.3.10 may not hold good.

Example 4.3.11 Let $M = N = \{i, j, k, l\}, \mu = \{\phi, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, N\}$. Then the identity map $u: (M, \mu) \to (N, \nu)$ is a contra λ_g^α-irresolute map but not a strongly λ_g^α-continuous map, since for the subset $\{k\}$ in (N, ν), $u^{-1}(\{k\}) = \{k\}$ is not a λ_g^α-clopen set in (M, μ).

Remark 4.3.12 The above implications are depicted in the following diagram.



Proposition 4.3.13 Let $u: (M, \mu) \to (N, \nu)$ and $w: (N, \nu) \to (K, \kappa)$ be two maps. Then $(w \circ u): (M, \mu) \to (K, \kappa)$ is a contra λ_g^α-irresolute map if

(i) u is λ_g^α-irresolute and w is contra λ_g^α-irresolute

(ii) u is contra λ_g^α-irresolute and w is λ_g^α-irresolute

(iii) u is contra λ_g^α-continuous and w is quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous)

Proof: (i) Let S be any λ_g^α-open set in (K, κ). Since w is contra λ_g^α-irresolute, $w^{-1}(S)$ is λ_g^α-closed in (N, ν). Since u is λ_g^α-irresolute, $(w \circ u)^{-1}(S) = u^{-1}(w^{-1}(S))$ is λ_g^α-closed in (M, μ). Hence $(w \circ u)$ is a contra λ_g^α-irresolute map.

Proof of (ii) and (iii) are similar to the proof of (i).

Proposition 4.3.14 The composition of two contra λ_g^α-irresolute maps is a λ_g^α-continuous map.

Proof: Let $u: (M, \mu) \rightarrow (N, \nu)$ and $w: (N, \nu) \rightarrow (K, \kappa)$ be contra λ_g^α-irresolute maps. Let S be an open set in (K, κ). Then S is a λ_g^α-open set in (K, κ), by Proposition 2.4.3. As w is a contra λ_g^α-irresolute map, $w^{-1}(S)$ is λ_g^α-closed in (N, ν). Since u is a contra λ_g^α-irresolute map, $(w \circ u)^{-1}(S) = u^{-1}(w^{-1}(S))$ is λ_g^α-open in (M, μ). Thus $(w \circ u): (M, \mu) \rightarrow (K, \kappa)$ is a λ_g^α-continuous map.

Proposition 4.3.15 The composition of two contra λ_g^α-irresolute maps is contra λ_g^α-continuous.

Proof: Evident from Definitions 4.2.1, 4.3.1. and Proposition 2.2.7.