CHAPTER VI

FUZZY SOFT SETS AND FUZZY SOFT SEMIGROUPS

Definition : 6.1

Let X be a non-empty set. A **fuzzy set** μ of X is a mapping given by $\mu : X \to [0,1]$. The set of all fuzzy subsets of X is called the **fuzzy power set** of X and is denoted by $FP(X)$.

Definition : 6.2

Let $\mu,\upsilon \in FP(X)$. Then the **product** $\mu \circ \upsilon$ is defined as follows,

$$(\mu \circ \upsilon)(x) = \begin{cases} \vee_{x=yz} (\mu(y) \wedge \upsilon(z)) & \text{if for every } y, z \in S, \ni x = yz \\ 0 & \text{otherwise} \end{cases}$$

for all $x \in S$. The operation $\circ$ is associative.

Definition : 6.3

Let μ, $\upsilon \in FP(X)$. If $\mu(x) \leq u(x)$ for all $x \in X$, then μ is said to be **contained in** υ.

we write $\mu \subseteq \upsilon$.

clearly, the inclusion relation $\subseteq$ is a partial order on $FP(X)$.

Definition : 6.4

Let μ, $\upsilon \in$ FP(X). Then $\mu \vee \upsilon$ and $\mu \wedge \upsilon$ are fuzzy subsets of X, defined as follows

$$(\mu \vee \upsilon)(x) = \mu(x) \vee \upsilon(x)$$

$$(\mu \wedge \upsilon)(x) = \mu(x) \wedge \upsilon(x) \qquad \text{for all } x \in X$$

The fuzzy subsets $\mu \vee \upsilon$ and $\mu \wedge \upsilon$ are called the **fuzzy union** and **fuzzy intersection** of μ and υ.

Definition : 6.5

A fuzzy subset μ of a semigroup S is called a **fuzzy subsemigroup** of S if $\mu(ab) \geq \mu(a)\mu(b)$ for all a,b $\in S$

Definition : 6.6

A fuzzy set μ of a semigroup S is called a **fuzzy left (right) ideal** of S if

$$\mu(ab) \geq \mu(b) \quad (\mu(ab) \geq \mu(a)) \qquad \text{for all a,b} \in S$$

A fuzzy set μ of a semigroup S is called a **fuzzy ideal** of S if it is both a fuzzy left and a fuzzy right ideal of S.

Definition : 6.7

Let U be an initial universe and E be the set of parameters. Let A be a nonempty subset of E and $FP(U)$ be the collection of all fuzzy subsets of U then the pair $(\tilde{F}, A)$ is called a **fuzzy soft set** over U, where $\tilde{F}$ is a mapping given by $\tilde{F} : A \rightarrow FP(U)$.

Definition : 6.8

If $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two fuzzy soft sets over a common universe U. Then $(\tilde{F}, A)$ is a **fuzzy soft subset** of $(\tilde{G}, B)$ if

1) $A \subseteq B$

2) For all $e \in A$, $\tilde{F}(e)$ is a fuzzy subset of $\tilde{G}$ (e).

we write $(\tilde{F}, A) \subseteq (\tilde{G}, B)$.

Definition : 6.9

If $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two fuzzy soft sets over a common universe U. Then $(\tilde{F}, A)$ is said to be a **fuzzy soft super set** of $(\tilde{G}, B)$, if $(\tilde{G}, B)$ is a fuzzy soft subset of $(\tilde{F}, A)$.

we write $(\tilde{F}, A) \supseteq (\tilde{G}, B)$.

Definition : 6.10

If $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two fuzzy soft sets over a common universe U.

Then $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are **fuzzy soft equal** if $(\tilde{F}, A)$ is a fuzzy soft subset of $(\tilde{G}, B)$ and $(\tilde{G}, B)$ is a fuzzy soft subset of $(\tilde{F}, A)$.

Definition : 6.11

If $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two fuzzy soft sets over a common universe U, then $(\tilde{F}, A)\mathbf{AND}(\tilde{G}, B)$ is a fuzzy soft set denoted by $(\tilde{F}, A) \wedge (\tilde{G}, B)$ and defined as, $(\tilde{F}, A) \wedge (\tilde{G}, B) = (\tilde{H}, A \times B)$, where $\tilde{H}(a, b) = \tilde{F}(a) \wedge \tilde{G}(b)$ for all $(a, b) \in A \times B$.

Definition : 6.12

If $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two fuzzy soft sets over a common universe U then $(\tilde{F}, A)\mathbf{OR}(\tilde{G}, B)$ is a fuzzy soft set denoted by $(\tilde{F}, A) \vee (\tilde{G}, B)$ and is defined as, $(\tilde{F}, A) \vee (\tilde{G}, B) = (\tilde{H}, A \times B)$, where $\tilde{H}(a, b) = \tilde{F}(a) \vee \tilde{G}(b)$ for all $(a, b) \in A \times B$.

Definition : 6.13

The **extended union** of two fuzzy soft sets $(\tilde{F}, A)$ and $(\tilde{G}, B)$ over a common universe U is denoted as the fuzzy soft set $(\tilde{H}, C) = (\tilde{F}, A) \cup_E (\tilde{G}, B)$, where $C = A \cup B$ and for all $e \in C$,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \tilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e) \vee \tilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

Definition : 6.14

The **extended intersection** of two fuzzy soft sets $(\tilde{F}, A)$ and $(\tilde{G}, B)$ over a common universe U is denoted as the fuzzy soft set $(\tilde{H}, C) = (\tilde{F}, A) \cap_E (\tilde{G}, B)$, where $C = A \cup B$ and for all $e \in C$,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \tilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e) \wedge \tilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

Definition : 6.15

If $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two fuzzy soft sets over a common universe U then their **restricted intersection** is a fuzzy soft set $(\tilde{H}, C)$ denoted by,

$$(\tilde{F}, A) \cap_R (\tilde{G}, B) = (\tilde{H}, C) \text{ where } C = A \cap B \neq \phi$$

and $\tilde{H}$ is a function from C to $FP(U)$, defined as $\tilde{H}(e) = \tilde{F}(e) \wedge \tilde{G}(e)$ and for all $e \in C$.

Definition : 6.16

If $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two fuzzy soft sets over a common universe U then their **restricted union** is a fuzzy soft set $(\tilde{H}, C)$ denoted by $(\tilde{F}, A) \cup_R (\tilde{G}, B) = (\tilde{H}, C)$ where $C = A \cap B \neq \phi$ and $\tilde{H}$ is a function from C to $FP(U)$, defined as

$$\tilde{H}(e) = \tilde{F}(e) \vee \tilde{G}(e) \text{ and for all } e \in C.$$

Definition : 6.17

Let U be an initial universe set, E be the set of parameters and A be the non-empty subset of E. Then $(\tilde{F}, A)$is called a **relative null fuzzy soft set** (with respect to the parameter set A), denoted by Φ_A, if $\tilde{F}(e) = \emptyset$ for all $e \in A$.

Definition : 6.18

Let U be an initial universe set, E be the set of parameters and A be the non-empty subset of E. Then $(\tilde{F}, A)$ is called a **relative whole fuzzy soft set** (with respect to the parameter set A), denoted by U_A, if $\tilde{F}(e) = U$ for all $e \in A$. The relative whole fuzzy soft set U_E with respect to the universe set of parameters E is called the **absolute fuzzy soft set** over U. We write (U, E).

Definition : 6.19

If $A \cap B \neq \emptyset$ then the **restricted product** $(\tilde{H}, C)$ of two fuzzy soft sets $(\tilde{F}, A)$ and $(\tilde{G}, B)$ over a semigroup S is defined as the fuzzy soft set, $(\tilde{H}, A \cap B)$ denoted by $(\tilde{F}, A)\ \hat{o}(\tilde{G}, B)$ where $\tilde{H}$ is a function from $A \cap B$ to $FP(S)$ defined by, $\tilde{H}$ (e) = $\tilde{F}$ (e) o $\tilde{G}$ (e) , for all $e \in A \cap B$. Here $\tilde{F}(e)\ o\tilde{G}(e)$ is the product of two fuzzy subsets of the semigroup S.

Definition : 6.20

The **extended product** of two fuzzy soft sets $(\tilde{F}, A)$ and $(\tilde{G}, B)$ over S is defined as the fuzzy soft set, $(\tilde{H}, C) = (\tilde{F}, A)\ \tilde{o}_E\ (\tilde{G}, B)$ where $C = A \cup B$ and for all $e \in C$.

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \tilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e)\ o\ \tilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

Here $\tilde{F}$ (e) o $\tilde{G}$(e) is the product of two fuzzy subsets of the semigroup S.

Theorem : 6.21

i) The operation $\tilde{o}_E$ is associative.

ii) Let $(\tilde{F}, A), (\tilde{G}, B)$ and $(\tilde{H}, C)$ be any fuzzy soft sets over S, where A, B and C are subsets of E. Then,

$$(\tilde{F}, A)\tilde{o}((\tilde{G}, B) \hat{\cup} (\tilde{H}, C)) = ((\tilde{F}, A)\tilde{o}(\tilde{G}, B)) \hat{\cup} ((\tilde{F}, A)\tilde{o}(\tilde{H}, C));$$

$$(\tilde{F}, A)\tilde{o}((\tilde{G}, B) \cup_R (\tilde{H}, C)) = ((\tilde{F}, A)\tilde{o}(\tilde{G}, B)) \cup_R ((\tilde{F}, A)\tilde{o}(\tilde{H}, C));$$

$$(\tilde{F}, A)\tilde{o}_E((\tilde{G}, B) \cup_R (\tilde{H}, C)) \hat{\subset} ((\tilde{F}, A)\tilde{o}_E(\tilde{G}, B)) \cup_R ((\tilde{F}, A)\tilde{o}_E(\tilde{H}, C));$$

$$(\tilde{F}, A)\tilde{o}_E((\tilde{G}, B) \hat{\cup} (\tilde{H}, C)) \hat{\subset} (\tilde{F}, A)\tilde{o}_E(\tilde{G}, B)) \cup_R ((\tilde{F}, A)\tilde{o}_E(\tilde{H}, C));$$

$$((\tilde{F}, A) \hat{\cup} (\tilde{G}, B))\tilde{o}(\tilde{H}, C) = ((\tilde{F}, A)\tilde{o}(\tilde{H}, C)) \hat{\cup} (\tilde{G}, B)\tilde{o}(\tilde{H}, C));$$

$$((\tilde{F}, A) \cup_R (\tilde{G}, B))\tilde{o}(\tilde{H}, C) = ((\tilde{F}, A)\tilde{o}(\tilde{H}, C)) \cup_R ((\tilde{G}, B)\tilde{o}(\tilde{H}, C));$$

$$((\tilde{F}, A) \cup_R ((\tilde{G}, B)\tilde{o}_E(\tilde{H}, C) \hat{\subset} ((\tilde{F}, A)\ \tilde{o}_E(\tilde{H}, C)) \cup_R ((\tilde{G}, B)\tilde{o}_E(\tilde{H}, C));$$

$$((\tilde{F}, A) \hat{\cup} (\tilde{G}, B))\tilde{o}_E(\tilde{H}, C) \hat{\subset} ((\tilde{F}, A)\tilde{o}_E(\tilde{H}, C)) \hat{\cup} (\tilde{G}, B)\tilde{o}_E(\tilde{H}, C)).$$

Theorem : 6.22

Let $(\tilde{F}, A), (\tilde{G}, B)$ and $(\tilde{H}, C)$ be any fuzzy soft sets over S, where A, B and C are subsets of E. Then,

1. $(\tilde{F}, A)\tilde{o}((\tilde{G}, B) \cap_R (\tilde{H}, C)) \hat{\subset} ((\tilde{F}, A)\tilde{o}(\tilde{G}, B))\cap_R((\tilde{F}, A)\tilde{o}(\tilde{H}, C));$
2. $(\tilde{F}, A)\tilde{o}((\tilde{G}, B) \cap_E (\tilde{H}, C)) \hat{\subset} ((\tilde{F}, A)\tilde{o}(\tilde{G}, B))\cap_E((\tilde{F}, A)\tilde{o}(\tilde{H}, C));$
3. $(\tilde{F}, A) \cap_R((\tilde{G}, B)\tilde{o}(\tilde{H}, C)) \hat{\subset} ((\tilde{F}, A)\tilde{o}(\tilde{H}, C))\cap_R((\tilde{G}, B)\tilde{o}(\tilde{H}, C));$
4. $(\tilde{F}, A) \cap_E((\tilde{G}, B)\tilde{o}(\tilde{H}, C)) \hat{\subset} (\tilde{F}, A)\tilde{o}(\tilde{H}, C))\cap_E ((\tilde{G}, B)\tilde{o}(\tilde{H}, C)).$

Proof :

Let $\qquad (\tilde{G}, B) \cap_R (\tilde{H}, C) = (\tilde{F}_1,\ B \cap C)$

and $\qquad (\tilde{F}, A) \cap_R ((\tilde{G}, B)\tilde{o}(\tilde{H}, C)) = (\tilde{F}_2,\ A \cap (B \cap C))$

then $\tilde{F}_1(e) = \tilde{G}(e) \wedge \tilde{H}(e)$ for all $e \in B \cap C$

and

$$\tilde{F}_2(e) = \tilde{F}(e) \wedge \tilde{F}_1(e)$$

$$= \tilde{F}(e) o \left(\tilde{G}(e) \wedge \tilde{H}(e)\right)$$

$$\subseteq \left(\tilde{F}(e) o \tilde{G}(e)\right) \wedge \left(\tilde{F}(e) o \tilde{G}(e)\right)$$

$$= \tilde{G}_1(e)$$

for all $e \in A \cap (B \cap C) = (A \cap B) \cap (A \cap C)$

where $((\tilde{F}, A)\tilde{o}(\tilde{G}, B)) \cap_R \left((\tilde{F}, A)\tilde{o}(\tilde{H}, C)\right) = (G_1, (A \cap B) \cap (A \cap C))$

Thus $(\tilde{F}, A)\tilde{o}((\tilde{G}, B) \cap_R (\tilde{H}, C))((\tilde{F}, A)\tilde{o}(\tilde{G}, B)) \cap_R ((\tilde{F}, A)\tilde{o}(\tilde{H}, C))$. Similarly, we can prove 2, 3 and 4.

Theorem : 6.23

Let $(\tilde{F}, A), (\tilde{G}, B)$ and $(\tilde{H}, C)$ be any fuzzy soft sets over S. If $(\tilde{F}, A) \hat{\subset} ((\tilde{G}, B)$.Then,

1. $(\tilde{H}, C)\tilde{o}((\tilde{F}, A) \hat{\subset} (\tilde{H}, C)\tilde{o}(\tilde{G}, B)$;
2. $(\tilde{H}, C)\tilde{o}_E(\tilde{F}, A) \hat{\subset} (\tilde{H}, C)\tilde{o}_E(\tilde{G}, B)$;
3. $(\tilde{F}, A)\tilde{o}(\tilde{H}, C) \hat{\subset} (\tilde{G}, B)\tilde{o}(\tilde{H}, C)$;
4. $(\tilde{F}, A)\tilde{o}_E(\tilde{H}, C) \hat{\subset} (\tilde{G}, B)\tilde{o}_E(\tilde{H}, C)$.

Proof :

Let $(\widetilde{H}, C)\tilde{o}_E(\tilde{F}, A) = (\tilde{F}_1, A \cup C)$,

then

$$\tilde{F}_1(e) = \begin{cases} \widetilde{H}(e) & \text{if } e \in C - A \\ \tilde{F}(e) & \text{if } e \in A - C \\ \widetilde{H}(e) \text{ o } \tilde{F}(e) & \text{if } e \in C \cap A \end{cases}$$

$$\subseteq \begin{cases} \widetilde{H}(e) & \text{if } e \in C - B \\ \widetilde{G}(e) & \text{if } e \in B - C \\ \widetilde{H}(e) \text{ o } \widetilde{G}(e) & \text{if } e \in C \cap B \end{cases}$$

because $\tilde{F}(e) \subseteq \widetilde{G}(e)$ for all $e \in A \subseteq B$.

This implies that $(\widetilde{H}, C)\tilde{o}_E(\tilde{F}, A) \mathbin{\widehat{\subset}} (\widetilde{H}, C)\tilde{o}_E(\widetilde{G}, B)$.

Similarly we can prove 2, 3 and 4.

Definition : 6.24

Let E, E' be the sets of parameters and A, B be the non-empty subsets of E and E' respectively. The **external product** of two fuzzy soft sets $(\tilde{F}, A)$ and $(\widetilde{G}, B)$ over the semigroups S and T respectively, is the fuzzy soft set $(\widetilde{H}, C)$ over $S \times T$ denoted by $(\tilde{F}, A) \times (\widetilde{G}, B)$ and defined as

$$(\widetilde{H}, C) = (\tilde{F}, A) \times (\widetilde{G}, B)$$

where $C = A \times B$ and $\widetilde{H}((a, b)) = \tilde{F}(a)\widehat{\otimes}\widetilde{G}(b)$ for all $(a, b) \in A \times B$. Here ' $\widehat{\otimes}$ ' is defined as follows:

$$\tilde{F}(a)\widehat{\otimes}\widetilde{G}(b)(e, e') = \min\{(\tilde{F}(a))(e), (\widetilde{G}(b))(e')\}$$

Definition : 6.25

The **internal product** of two fuzzy soft sets $(\tilde{F}, A)$ and $(\widetilde{G}, B)$ over the semigroup S is the fuzzy soft set $(\widetilde{H}, C)$ over S, denoted by $(\tilde{F}, A)\tilde{o}_x(\widetilde{G}, B)$ and defined as $(\widetilde{H}, C) = (\tilde{F}, A)\tilde{o}_x(\widetilde{G}, B)$ where $C = A \times B$ and $\widetilde{H}((a, b)) = \tilde{F}(a) o \widetilde{G}(b)$ for all $(a, b) \in A \times B$.

The cartesian product is not associative because $(A \times B) \times C \neq A \times (B \times C)$ but if we take

$$(A \times B) \times C = A \times B \times C = A \times (B \times C)$$

Then $((\tilde{F}, A)\tilde{o}_x(\widetilde{G}, B))\tilde{o}_x(\widetilde{H}, C) = (\tilde{F}, A)\tilde{o}_x((\widetilde{G}, B)\tilde{o}_x(\widetilde{H}, C))$

Definition : 6.26

A fuzzy soft set $(\tilde{F}, A)$ over a semigroup S is called a **fuzzy soft semigroup** over S, if $\tilde{F}(e)$ is a fuzzy subsemigroup of S for all $e \in A$.

Theorem : 6.27

A non-empty fuzzy soft set $(\tilde{F}, A)$ over S is a fuzzy soft semigroup over S if and only if $(\tilde{F}, A)\tilde{o}(\tilde{F}, A) \mathrel{\hat{\subset}} (\tilde{F}, A)$

Proof:

Let $(\tilde{F}, A)$ be a fuzzy soft semigroup over S and let $(\tilde{F}, A)\tilde{o}(\tilde{F}, A) = (\widetilde{H}, A)$. Then $\widetilde{H}$ (e) = $\tilde{F}(e) o \tilde{F}(e) \subseteq \tilde{F}(e)$ for all $e \in A$, because $\tilde{F}(e)$ is a fuzzy subsemigroup of S.

Thus, $(\tilde{F}, A) \tilde{o} (\tilde{F}, A) \mathrel{\hat{\subset}} (\tilde{F}, A)$.

Conversely, assume that $(\tilde{F}, A) \tilde{o} (\tilde{F}, A) \mathrel{\hat{\subset}} (\tilde{F}, A)$.

Then $\tilde{F}(e) o \tilde{F}(e) \subseteq \tilde{F}(e)$ for all $e \in A$. This implies that $\tilde{F}(e)$ is a fuzzy subsemigroup of S. Thus $(\tilde{F}, A)$ is a fuzzy soft semigroup over S.

Definition : 6.28

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two fuzzy soft semigroups over S. Then $(\tilde{G}, B)$ is said to be a **fuzzy soft subsemigroup** of $(\tilde{F}, A)$ over S, if $(\tilde{G}, B)$ is a fuzzy soft subset of $(\tilde{F}, A)$ over S.

Theorem : 6.29

Let $\{S_i : i \in I\}$ be a non-empty family of semigroups and $\{(\tilde{F}_i, A_i) : i \in I\}$ be a non-empty family of fuzzy soft sets, such that $(\tilde{F}_i, A_i)$ is a fuzzy soft semigroup over S_i for each $i \in I$. Then direct (external) product $\prod_{i \in I}(\tilde{F}_i, A_i)$ is a fuzzy soft semigroup over $\prod_{i \in I} S_i$.

Proof:

By definition, for all

$$(e_i) \in \prod_{i \in I} A_i,\ (\prod_{i \in I} \tilde{F}_i(e_i))(x) = \wedge_{i \in I}((\tilde{F}_i(e_i))(x_i)) \text{ where } x = (x_i) \in \prod_{i \in I} s_i$$

and $\tilde{F}_i(e_i)$ is a fuzzy subsemigroup of S_i for all $i \in I$.

Let $\prod_{i \in I}(\tilde{F}_i, A_i) = (\tilde{H}, \prod_{i \in I} A_i)$ where, $\tilde{H}(e_i) = \prod_{i \in I}(\tilde{F}_i(e_i))$ for all $(e_i) \in \prod_{i \in I} A_i$.

Then for any $(x_i), (y_i) \in \prod_{i \in I} S_i$.

$$\{\tilde{H}(e_i)\}((x_i)(y_i)) = (\prod_{i \in I} \tilde{F}_i(ei))((x_i y_i))$$

$= \wedge_{i \in I}((\tilde{F}_i (e_i))(x_i y_i))$

$\geq \wedge_{i \in I}\{(\tilde{F}_i (e_i))(x_i) \wedge (\tilde{F}_i (e_i))(y_i)\}$

$\because \tilde{F}_i(e_i)$ is a fuzzy subsemigroup of S_i.

$= [\wedge_{i \in I}\{\tilde{F}_i (e_i))(x_i)\}] \wedge [\wedge_{i \in I}\{(\tilde{F}_i (e_i))(y_i)\}]$

$= (\prod_{i \in I} \tilde{F}_i(e_i))(x_i) \wedge (\prod_{i \in I} \tilde{F}_i(e_i))(y_i)$

$= \{\tilde{H}(e_i)\}(x_i) \wedge \{\tilde{H}(e_i)\}(y_i)$.

This implies that $\tilde{H}(e_i)$ is a fuzzy subsemigroup of $\prod_{i \in I} S_i$ for all $(e_i) \in \prod_{i \in I} A_i$.

Thus $\prod_{i \in I}(\tilde{F}_i, A_i)$ is a fuzzy soft semigroup over $\prod_{i \in I} S_i$.

Remark : 6.30

Let $\{(\tilde{F}_i, A_i) : i \in I\}$ be a non-empty family of fuzzy soft subsemigroups of fuzzy soft semigroup $(\tilde{F}, A)$ over S. Then direct (external) product $\prod_{i \in I}(\tilde{F}_i, Ai)$ is a fuzzy soft subsemigroup of $\prod_{i \in I}(\tilde{F}_i, A_i)$ over $\prod_{i \in I} S_i$.

Theorem : 6.31

Let $\{(\tilde{F}_i, A_i) : i \in I\}$ be a non-empty family of fuzzy soft semigroups over S. Then $\hat{\cup}_{i \in I}(\tilde{F}_i, A_i)$ is a fuzzy soft semigroup over S, provided $A_i \cap A_j = \emptyset$ for all $i, j \in I$ and $i \neq j$, $\cap_{E\, i \in I}(\tilde{F}_i, A_i)$ is a fuzzy soft semigroup over S, $\cap_{R\, i \in I}(\tilde{F}_i, A_i)$ is a fuzzy soft semigroup over S provided $\cap_{i \in I} A_i \neq \emptyset$, $\wedge_{i \in I}(\tilde{F}_i, A_i)$ is a fuzzy soft semigroup over S.

Proof :

Since A_i 's are non-empty, so $\underset{i \in I}{\cup} A_i$ is non-empty.

Let $\underset{i \in I}{\hat{\cup}} (\tilde{F}_i, A_i) = (\tilde{H}, \underset{i \in I}{\cup} A_i)$. Then for any $e \in \underset{i \in I}{\cap} A_i \Rightarrow e \in A_j$ for some $j \in I$ and $\tilde{H}(e) = F_j(e)$. This implies that $\tilde{H}(e)$ is a fuzzy subsemigroup of S for all $e \in \underset{i \in I}{\cup} A_i$. Thus $\underset{i \in I}{\hat{\cup}} (\tilde{F}_i, A_i)$ is a fuzzy soft semigroup over S.

Let $\underset{i \in I}{\cap_E} (\tilde{F}_i, A_i)$ where $\tilde{H}(e) = \underset{j \in \wedge(e)}{\wedge} \tilde{F}_j(e)$, for all $e \in \underset{i \in I}{\cup} A_i$. We define the set $\wedge(e) = \{j : e \in A_j\}$. Then for any $x, y \in S$.

$$\{\tilde{H}(e)\}(xy) = \left\{\underset{j \in \wedge(e)}{\wedge} \tilde{F}_j(e)\right\}(xy)$$

$$= \underset{j \in \wedge(e)}{\wedge} \{(\tilde{F}_j(e))(xy)\}$$

$$\geq \underset{j \in \wedge(e)}{\wedge} \{(\tilde{F}_j(e))(x) \wedge (\tilde{F}_j(e))(y)\}$$

$\because \tilde{F}_j(e)$ is a fuzzy sub semigroup of S

$$= \left(\underset{j \in \wedge(e)}{\wedge} \{(\tilde{F}_j(e))(x)\}\right) \wedge \left(\underset{j \in \wedge(e)}{\wedge} \{(\tilde{F}_j(e))(y)\}\right)$$

$$= \left(\underset{j \in \wedge(e)}{\wedge} \left\{\left(\tilde{F}_j(e)\right)\right\}(x)\right) \wedge \left(\underset{j \in \wedge(e)}{\wedge} \left\{\left(\tilde{F}_j(e)\right)(y)\right\}\right)$$

$$= \{(\tilde{H}(e))(x)\} \wedge \{(\tilde{H}(e))(y)\}$$

Hence $\tilde{H}(e)$ is a fuzzy subsemigroup of S for all $e \in \underset{i \in I}{\cup} A$. Thus $\underset{i \in I}{\cap_E} (\tilde{F}_i, A_i)$ is a fuzzy soft subsemigroup over S.

Let $\bigcap_{R\, i \in I} (\tilde{F}_i, A_i) = (\widetilde{H}, \bigcap_{i \in I} A_i)$ where $\widetilde{H}(e) = \bigwedge_{i \in I} \tilde{F}_i(e)$ for all $e \in \bigcap_{i \in I} A_i$.

Then for any $x, y \in S$.

$$\{\widetilde{H}(e)\}(xy) = \left\{\bigwedge_{i \in I} \tilde{F}_i(e)\right\}(xy)$$

$$= \bigwedge_{i \in I} \{(\tilde{F}_i(e))(xy)\}$$

$$\geq \bigwedge_{i \in I} \{(\tilde{F}_i(e))(x) \wedge (\tilde{F}_i(e))(y)\}$$

$\because \tilde{F}_i(e)$ is a fuzzy subsemigroup of S

$$= \left(\bigwedge_{i \in I} \{(\tilde{F}_i(e))(x)\}\right) \wedge \left(\bigwedge_{i \in I} \{(\tilde{F}_i(e))(y)\}\right)$$

$$= \left(\left\{\bigwedge_{i \in I} \left(\tilde{F}_i(e)\right)\right\}(x)\right) \wedge \left(\left\{\bigwedge_{i \in I} \left(\tilde{F}_i(e)\right)\right\}(y)\right)$$

$$= \{(\widetilde{H}(e))(x)\} \wedge \{(\widetilde{H}(e))(y)\}$$

Hence $\widetilde{H}(e)$ is a fuzzy subsemigroup of S for all $e \in \bigcap_{i \in I} A$. Thus $\bigcap_{R\, i \in I} (\tilde{F}_i, A_i)$ is a fuzzy soft subsemigroup over S.

Let $(\bigwedge_{i \in I} \tilde{F}_i, A_i) = (\widetilde{H}, \prod_{i \in I} A_i)$ where $\widetilde{H}((e_i)) = \bigwedge_{i \in I} \tilde{F}_i(e_i)$ for all $(e_i) \in \prod_{i \in I} A_i$.

Then for any $x, y \in S$,

$$\{\widetilde{H}\ ((e_i))\}(xy) = \left\{\bigwedge_{i \in I} \tilde{F}_i(e_i)\right\}(xy)$$

$$= \bigwedge_{i \in I} \{(\tilde{F}_i(e_i))(xy)\}$$

$$\geq \bigwedge_{i \in I}\{(\tilde{F}_i(e_i))(x) \wedge (\tilde{F}_i(e_i))(y)\}$$

$\because \tilde{F}_i(e)$ is a fuzzy subsemigroup of S

$$= \left(\bigwedge_{i \in I}\{(\tilde{F}_i(e_i))(x)\}\right) \wedge \left(\bigwedge_{i \in I}\{(\tilde{F}_i(e_i))(y)\}\right)$$

$$= \left(\left\{\bigwedge_{i \in I}\left(\tilde{F}_i(e_i)\right)\right\}(x)\right) \wedge \left(\left\{\bigwedge_{i \in I}\left(\tilde{F}_i(e_i)\right)\right\}(y)\right)$$

$$= \{\tilde{H}((e_i))(x)\} \wedge \{\tilde{H}((e_i))(y)\}$$

Hence $\tilde{H}((e_i))$ is a fuzzy subsemigroup of S for all $(e_i) \in \prod_{i \in I} A_i$. Thus $\cap_{R\, i \in I}(\tilde{F}_i, A_i)$ is a fuzzy soft semigroup over S.

Theorem : 6.32

Let $\{(\tilde{F}_i, A_i) : i \in I\}$ be a non-empty family of fuzzy soft subsemigroups of the fuzzy soft semigroup $(\tilde{F}, A)$ over S. Then $\hat{\cup}_{i \in I}(\tilde{F}_i, A_i)$ is a fuzzy soft subsemigroup of $(\tilde{F}, A)$ over S, provided $A_i \cap A_j = \emptyset$ for all i, j ∈ I, where $i \neq j$, $\cap_{E\, i \in I}(\tilde{F}_i, A_i)$ is a fuzzy soft subsemigroup of $(\tilde{F}, A)$ over S, $\cap_{R\, i \in I}(\tilde{F}_i, A_i)$ is a fuzzy soft subsemigroup of $(\tilde{F}, A)$ over S, provided $\bigcap_{i \in I} A_i \neq \emptyset$, $\bigwedge_{i \in I}(\tilde{F}_i, A_i)$ is a fuzzy soft subsemigroup of $\bigwedge_{i \in I}(\tilde{F}, A)$ over S.

Remark : 6.33

For any two fuzzy soft semigroups $(\tilde{F}, A)$ and $(\tilde{G}, B)$ over S, $(\tilde{F}, A) \cup_R (\tilde{G}, B)$ and $(\tilde{F}, A) \cup (\tilde{G}, B)$ may not be fuzzy soft semigroups over S.

Definition : 6.34

A fuzzy soft set $(\tilde{F}, A)$ over S is said to be a **fuzzy soft left (right) ideal** over S if $\tilde{F}(e)$ is a fuzzy left (right) ideal of S and for all $e \in A$.

A fuzzy soft set $(\tilde{F}, A)$ over S is said to be a **fuzzy soft ideal** if it is both a fuzzy soft left and a fuzzy soft right ideal over S.

Theorem : 6.35

A fuzzy soft set $(\tilde{F}, A)$ over S is a fuzzy soft left (right) ideal over S if and only if $(S, E)\ \tilde{o}\ (\tilde{F}, A) \mathrel{\widehat{\subset}} (\tilde{F}, A)\ \left((\tilde{F}, A)\ \tilde{o}\ (S, E) \mathrel{\widehat{\subset}} (\tilde{F}, A)\right)$.

Proof:

Suppose $(\tilde{F}, A)$ is a fuzzy soft left ideal over S. Let$(S, E)\tilde{o}(\tilde{F}, A) = (\tilde{H}, A)$. Then $\tilde{H}$(e) $=$ S(e) o $\tilde{F}$(e) $=$ S o $\tilde{F}$(e)$\subseteq$ $\tilde{F}$(e) for all e $\in$ A, because $\tilde{F}$(e) is a fuzzy left ideal of S. This implies that $(S, E)\ \tilde{o}\ (\tilde{F}, A) \mathrel{\widehat{\subset}} (\tilde{F}, A)$.

Conversely, if $(S, E)\ \tilde{o}\ (\tilde{F}, A) \mathrel{\widehat{\subset}} (\tilde{F}, A)$ then, $S(e)$ o $\tilde{F}$(e) $\subseteq$ $\tilde{F}$(e) for all $e \in A$. so, S o $\tilde{F}$(e) $\subseteq$ $\tilde{F}$(e) for all $e \in A$, implies that $\tilde{F}$(e) is a fuzzy left ideal of S. Similarly we can prove for the fuzzy soft right ideals.

Theorem : 6.36

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two fuzzy soft left (right, two-sided) ideals over S and T respectively. Then their cartesian product$(\tilde{F}, A) \times (\tilde{G}, B)$ is a fuzzy soft left (right, two-sided) ideal over $S \times T$.

Proof:

Let $(\tilde{F}, A) \times (\tilde{G}, B) = (\tilde{H}, A \times B)$ where $\tilde{F}(a) \otimes \tilde{G}(b) = \tilde{H}((a, b))$ for all $(a, b) \in A \times B$. Then for any $(x, y), (z, t) \in$ S×T

$$\{\tilde{H}((a, b))\}((x, y)(z, t)) = \{\tilde{H}((a, b))\}(xz, yt))$$

$$= \{\tilde{F}(a) \otimes \tilde{G}(b)\}(xz, yt))$$

$$= \{\tilde{F}(a)(xz)\} \wedge \{\tilde{G}(b)(yt)\}$$

$$\geq \left\{\left(\tilde{F}(a)\right)(z)\right\} \wedge \{\left(\tilde{G}(b)\right)(t)\}$$

$$= \{\tilde{F}(a)\widehat{\otimes}\tilde{G}(b)\}((z, y))$$

$$= \{\tilde{H}((a, b))\}((z, t))$$

From the above, $\tilde{H}((a, b))$ is a fuzzy left ideal of S for all $(a, b) \in A \times B$. Thus $\left(\tilde{F}, A\right) \times (\tilde{G}, B)$ is a fuzzy soft left ideal over $S \times T$.

Similarly we can prove for fuzzy soft right ideals and hence for fuzzy soft two-sided ideals over $S \times T$.

Theorem : 6.37

Let $\{S_i : i \in I\}$ be a non-empty family of semigroups and $\{(\tilde{F}_i, A_i) : i \in I\}$ be a non-empty family of fuzzy soft sets, such that $(\tilde{F}_i, A_i)$ is a fuzzy soft left (right, two-sided) ideal over S_i for each $i \in I$. Then $(\prod_{i \in I}(\tilde{F}_i, A_i)$ is a fuzzy soft left (right, two-sided) ideal over $\prod_{i \in I} S_i$.

Proof :

For all $i \in I$, $\tilde{F}_i(e_i)$ is a fuzzy left ideal of S_i. By definition, for all $(e_i) \in \prod_{i \in I} A_i$, $(\widehat{\prod_{i \in I}} \tilde{F}_i (e_i))(x) = \wedge_{i \in I}((\tilde{F}(e_i))(x_i))$ where $x = (x_i) \in \prod_{i \in I} S_i$.

Let $\prod_{i \in I}(F_i, A_i) = (\tilde{H}, \prod_{i \in I} A_i)$ where, $\tilde{H}((e_i)) = \widehat{\prod_{i \in I}} \tilde{F}_i(e_i)$ for all $(e_i) \in \prod_{i \in I} A_i$.

Then for any $(x_i), (y_i) \in \prod_{i \in I} S_i$.

$$\{\tilde{H}((e_i))\}((x_i)(y_i)) = (\widehat{\prod_{i \in I}} \tilde{F}_i(e_i)((x_i y_i))$$

$$= \wedge_{i \in I}((\tilde{F}_i(e_i))\ (x_i y_i))$$

$$\geq \wedge_{i \in I}((\tilde{F}_i(e_i))(y_i)) \qquad \because \tilde{F}_i(e_i) \text{ is a fuzzy left ideal of } S_i.$$

$$= \widehat{\prod_{i \in I}} \tilde{F}_i(e_i))((y_i))$$

This implies that $\tilde{H}((e_i))$ is a fuzzy left ideal of the product semigroup $\prod_{i \in I} S_i$ for all $(e_i) \in \prod_{i \in I} A_i$. Thus $\prod_{i \in I} (\tilde{F}_i, A_i)$ is a fuzzy soft left ideal over $\prod_{i \in I} S_i$.

Similarly we can prove for fuzzy soft right ideals and hence for fuzzy soft two-sided ideals.

Theorem : 6.38

If $(\tilde{F}, A)$ is a non-empty fuzzy soft set over S, then $(S, E)\tilde{o}(\tilde{F}, A)((\tilde{F}, A)\tilde{o}(S, E))$ is a fuzzy soft left (right) ideal over S.

Theorem : 6.39

If $(\tilde{F}, A)$ is a fuzzy soft right (left) ideal over S, then $(\tilde{F}, A)\ \hat{\cup}\ ((S, E)\tilde{o}(\tilde{F}, A))\ ((\tilde{F}, A)\ \hat{\cup}\ ((\tilde{F}, A)\tilde{o}(S, E)))$ is a fuzzy soft ideal over S.

Proof :

Suppose $(\tilde{F}, A)$ is a fuzzy soft right ideal over S.

Then

$$(S,E)\tilde{o}\left((\tilde{F},A)\ \hat{\cup}\ \left((S,E)\tilde{o}(\tilde{F},A)\right)\right)$$

$$=\left((S,E)\tilde{o}(\tilde{F},A)\right)\hat{\cup}\left((S,E)\tilde{o}(\tilde{F},A)\right)\hat{\cup}\ ((S,E)\tilde{o}(\tilde{F},A))$$

$$=((S,E)\tilde{o}(\tilde{F},A))\ \hat{\cup}\ (\left((S,E)\tilde{o}(\tilde{F},A)\right)\hat{\cup}\left(((S,E))\tilde{o}(\tilde{F},A)\right)$$

$$\hat{\subset}\left((S,E)\tilde{o}(\tilde{F},A)\right)\hat{\cup}\ ((S,E)\tilde{o}(\tilde{F},A))$$

$$=(S,E)\ \tilde{o}\ (\tilde{F},A)\ \hat{\subset}\ (\tilde{F},A)\ \hat{\cup}\ ((S,E)\tilde{o}(\tilde{F},A))$$

So, $(\tilde{F},A)\ \hat{\cup}\ (S,E)\ \tilde{o}\ (\tilde{F},A))$ is a fuzzy soft left ideal over S.

Now,

$$\left((\tilde{F},A)\ \hat{\cup}\ \left((S,E)\tilde{o}(\tilde{F},A)\right)\right)\ \tilde{o}\ (S,E)=\left((\tilde{F},A)\tilde{o}(S,E)\right)\hat{\cup}\left(\left((S,E)\tilde{o}(\tilde{F},A)\right)\tilde{o}(S,E)\right)$$

$$\hat{\subset}\ (\tilde{F},A)\ \tilde{o}\ \left((S,E)\ \tilde{o}\ \left((\tilde{F},A)\ \tilde{o}\ (S,E)\right)\right)$$

$$\hat{\subset}\ (\tilde{F},A)\ \hat{\cup}\ ((S,E)\tilde{o}(\tilde{F},A)$$

because is a fuzzy soft right ideal over S.

Thus $(\tilde{F},A)\ \hat{\cup}\ ((S,E)\tilde{o}(\tilde{F},A))$ is a fuzzy soft ideal over S.

CHAPTER VII

FUZZY SOFT LA - SEMIGROUPS

Definition : 7.1

If $(\tilde{F}, A)$ be a fuzzy soft set of a LA-semigroup S. Then for any $a \in A$, $\tilde{F}(a)$ is called a **fuzzy LA-semigroup** if

$$\tilde{F}(a) \circ \tilde{F}(a) \leq \tilde{F}(a)$$

Definition : 7.2

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be any two fuzzy soft sets over an LA-semigroup S. Then the **restricted product** of $(\tilde{F}, A)$ and $(\tilde{G}, B)$ is defined as the fuzzy soft set $(\tilde{H}, C) = (\tilde{F}, A) \odot (\tilde{G}, B)$, where $C = A \cap B$ and $\tilde{H}$ is a set valued function from C to $FP(S)$ defined as,

$$\tilde{H}(e) = \tilde{F}(e) \circ \tilde{G}(e) \text{ for all } e \in C$$

Definition : 7.3

A fuzzy soft set $(\tilde{F}, A)$ over S is called a **fuzzy soft LA-semigroup** over S if

$$(\tilde{F}, A) \odot (\tilde{F}, A) \subseteq (\tilde{F}, A)$$

Theorem : 7.4

A fuzzy soft set $(\tilde{F}, A)$ over S is a fuzzy soft LA-semigroup iff each $\tilde{F}(a) \neq \phi$ is an fuzzy LA-semigroup of S for all $a \in A$.

Proof:

Assume $(\tilde{F}, A)$ is a fuzzy soft LA-semigroup. By definition (7.3),

$$(\tilde{F}, A) \odot (\tilde{F}, A) \subseteq (\tilde{F}, A)$$

$$\Rightarrow \quad \tilde{F}(a) \circ \tilde{F}(a) \leq \tilde{F}(a) \text{ for all } a \in A$$

Therefore $\tilde{F}(a)$ is an fuzzy LA-semigroup of S.

Conversely, assume $\tilde{F}(a)$ is an fuzzy LA-semigroup of S.

By definition, $\tilde{F}(a) \circ \tilde{F}(a) \leq \tilde{F}(a)$ for all $a \in A$

$\Rightarrow$ $(\tilde{F}, A) \circledcirc (\tilde{F}, A) \subseteq (\tilde{F}, A)$

Therefore $(\tilde{F}, A)$ is a fuzzy soft LA-semigroup.

Theorem : 7.5

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two fuzzy soft LA-semigroups over S. Then, $(\tilde{F}, A) \wedge (\tilde{G}, B)$ is also a fuzzy soft LA-semigroup over S.

Proof:

By definition,

$\left(\tilde{F}, A\right) \wedge \left(\tilde{G}, B\right) = \left(\tilde{H}, A \times B\right)$, where $\tilde{H}(a, b) = \tilde{F}(a) \wedge \tilde{G}(b)$ for all $(a, b) \in A \times B$.

Since $\tilde{F}(a)$ and $\tilde{G}(b)$ are fuzzy LA-semigroups over S.

Therefore, either $\tilde{F}(a) \wedge \tilde{G}(b) = \emptyset$ (or) $\tilde{F}(a) \wedge \tilde{G}(b)$ is an fuzzy LA-semigroup of S.

which implies, $\tilde{H}(a, b)$ is an fuzzy LA-semigroup of S.

$\therefore$ $\left(\tilde{H}, A \times B\right)$ is a fuzzy soft LA-semigroup over S.

Hence $(\tilde{F}, A) \wedge (\tilde{G}, B)$ is a fuzzy soft LA-semigroup over S.

Theorem : 7.6

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two fuzzy soft LA-semigroups over S. Then $\left(\tilde{F}, A\right) \cap_E \left(\tilde{G}, B\right)$ is also a fuzzy soft LA-semigroup over S.

Proof:

Assume $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two fuzzy soft LA-semigroups over S.

By definition,

$(\tilde{F}, A) \cap_E (\tilde{G}, B) = (\tilde{H}, C)$, where $C = A \cup B$ and for all $e \in C$,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \tilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e) \wedge \tilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

Then for all $e \in C \Rightarrow$ either $e \in A - B$ (or) $e \in B - A$ (or) $e \in A \cap B$.

If $e \in A - B \Rightarrow \tilde{H}(e) = \tilde{F}(e)$

If $e \in B - A \Rightarrow \tilde{H}(e) = \tilde{G}(e)$

If $e \in A \cap B \Rightarrow \tilde{H}(e) = \tilde{F}(e) \wedge \tilde{G}(e)$

From the above, we have

$\tilde{F}(e) \wedge \tilde{G}(e)$ is an fuzzy LA-semigroup of S.

$\Rightarrow \tilde{H}(e)$ is an fuzzy LA-semigroup of S. Therefore $(\tilde{H}, C)$ is a fuzzy soft LA-semigroup over S.

Hence $(\tilde{F}, A) \cap_E (\tilde{G}, B)$ is a fuzzy soft LA-semigroup over S.

Theorem : 7.7

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two fuzzy soft LA-semigroups over S such that $A \cap B \neq \emptyset$. Then $(\tilde{F}, A) \cap_R (\tilde{G}, B)$ is a fuzzy soft LA-semigroup over S.

Proof:

Assume $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are fuzzy soft LA-semigroups over S.

By definition,

$(\tilde{F}, A) \cap_R (\tilde{G}, B) = (\tilde{H}, C)$ where $C = A \cap B \neq \emptyset$ and $\tilde{H}(e) = \tilde{F}(e) \wedge \tilde{G}(e)$ for all $e \in C$.

In the above, $\tilde{F}(e)$ and $\tilde{G}(e)$ are fuzzy LA-semigroups of S and also $\tilde{F}(e) \wedge \tilde{G}(e)$ is an fuzzy LA-semigroup of S is either empty or an fuzzy LA-semigroup of S.

Hence, $(\tilde{F}, A) \cap_R (\tilde{G}, B) = (\tilde{H}, C)$ is a fuzzy soft LA-semigroup over S.

Theorem : 7.8

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two fuzzy soft LA-semigroups over S such that $A \cap B = \emptyset$. Then $(\tilde{F}, A) \cup_E (\tilde{G}, B)$ is also a fuzzy soft LA-semigroup over S.

Proof:

Assume $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two fuzzy soft LA-semigroups.

By definition,

$(\tilde{F}, A) \cup_E (\tilde{G}, B) = (\tilde{H}, C)$, where $C = A \cup B$ and for all $e \in C$,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \tilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e) \vee \tilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

Then for all $e \in C$, either $e \in A - B$ (or) $e \in B - A$

If $e \in A - B \Rightarrow \tilde{H}(e) = \tilde{F}(e)$ and

If $e \in B - A \Rightarrow \tilde{H}(e) = \tilde{G}(e)$

From both the cases $\tilde{H}(e)$ is an fuzzy LA-semigroup of S.

Therefore $(\tilde{H}, C)$ is a fuzzy soft LA-semigroup over S.

Hence $(\tilde{F}, A) \cup_E (\tilde{G}, B)$ is a fuzzy soft LA-semigroup over S.

Theorem : 7.9

A fuzzy soft set $(\tilde{F}, A)$ over S is a fuzzy soft ideal over S iff $\tilde{F}(a) \neq \emptyset$ is an fuzzy ideal of S for all $a \in A$.

Proof:

From the definition it follows that.

Theorem : 7.10

Let $(\tilde{F}, A)$ and $(\tilde{G}, A)$ are two fuzzy soft ideals over S. Then $(\tilde{F}, A) \wedge (\tilde{G}, A)$ is also a fuzzy soft ideal over S.

Proof:

By definition,

$(\tilde{F}, A) \wedge (\tilde{G}, B) = (\tilde{H}, A \times B)$, where $\tilde{H}(a, b) = \tilde{F}(a) \wedge \tilde{G}(b)$ for all $(a, b) \in A \times B$.

Since $(\tilde{F}, A)$ and $(\tilde{G}, A)$ are fuzzy soft ideals over S. Then $\tilde{F}(a)$ and $\tilde{G}(b)$ are fuzzy ideals of S for every $a \in A$ and $b \in B$.

$\Rightarrow \tilde{F}(a) \wedge \tilde{G}(b)$ is an fuzzy ideal of S. Therefore $(\tilde{H}, A \times B)$ is a fuzzy soft ideal over S.

Hence $(\tilde{F}, A) \wedge (\tilde{G}, B)$ is a fuzzy soft ideal over S.

Theorem : 7.11

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two fuzzy soft ideals over S. Then $(\tilde{F}, A) \vee (\tilde{G}, B)$ is also a fuzzy soft ideal over S.

Proof:

By definition,

$(\tilde{F}, A) \vee (\tilde{G}, B) = (\tilde{H}, A \times B)$, where $\tilde{H}(a, b) = \tilde{F}(a) \vee \tilde{G}(b)$ for all $(a, b) \in A \times B$.

Since $(\tilde{F}, A)$ and $(\tilde{G}, A)$ are fuzzy soft ideals over S. Then $\tilde{F}(a)$ and $\tilde{G}(b)$ are fuzzy ideals of S for every $a \in A$ and $b \in B$.

$\Rightarrow \tilde{F}(a) \vee \tilde{G}(b)$ is an fuzzy ideal of S. Therefore $(\tilde{H}, A \times B)$ is a fuzzy soft ideal over S.

Hence $(\tilde{F}, A) \vee (\tilde{G}, B)$ is a fuzzy soft ideal over S.

Theorem : 7.12

Let $(\tilde{F}, A)$ and $(\tilde{G}, A)$ are two fuzzy soft ideals over S, such that $A \cap B \neq \emptyset$. Then $(\tilde{F}, A) \cap_R (\tilde{G}, A)$ is also a fuzzy soft ideal over S.

Proof:

By definition, $(\tilde{F}, A) \cap_R (\tilde{G}, B) = (\tilde{H}, C)$ where $C = A \cap B$ and $\tilde{H}(e) = \tilde{F}(e) \wedge \tilde{G}(e)$ for all e$\in C$.

Then $\tilde{F}(e) \wedge \tilde{G}(e)$ is an fuzzy ideal of S.

$\Rightarrow \tilde{H}(e)$ is an fuzzy ideal of S. Therefore $(\tilde{H}, C)$ is a fuzzy soft ideal over S.

Hence $(\tilde{F}, A) \cap_R (\tilde{G}, B)$ is a fuzzy soft ideal over S.

Theorem : 7.13

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two fuzzy soft ideals over S. Then $(\tilde{F}, A) \cap_E (\tilde{G}, B)$ is also a fuzzy soft ideal over S.

Proof:

By definition,

$(\tilde{F}, A) \cap_E (\tilde{G}, B) = (\tilde{H}, C)$, where $C = A \cup B$ and for all $e \in C$,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \tilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e) \wedge \tilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

Then for all $e \in C \Rightarrow$ either $e \in A - B$ (or) $e \in B - A$ (or) $e \in A \cap B$.

If $e \in A - B \Rightarrow \tilde{H}(e) = \tilde{F}(e)$

If $e \in B - A \Rightarrow \widetilde{H}(e) = \widetilde{G}(e)$

If $e \in A \cap B \Rightarrow \widetilde{H}(e) = \tilde{F}(e) \wedge \widetilde{G}(e)$

From the above, we have

$\tilde{F}(e) \wedge \widetilde{G}(e)$ is an fuzzy ideal of S.

$\Rightarrow \widetilde{H}(e)$ is an fuzzy ideal of S. Therefore $(\widetilde{H}, C)$ is a fuzzy soft ideal over S.

Hence $(\tilde{F}, A) \cap_E (\widetilde{G}, B)$ is a fuzzy soft ideal over S.

Theorem : 7.14

Let $(\tilde{F}, A)$ and $(\widetilde{G}, B)$ are two fuzzy soft ideals over S. Then $(\tilde{F}, A) \cup_E (\widetilde{G}, B)$ is also a fuzzy soft ideal over S.

Proof:

By definition,

$(\tilde{F}, A) \cup_E (\widetilde{G}, B) = (\widetilde{H}, C)$, where $C = A \cup B$ and for all $e \in C$,

$$\widetilde{H}(e) = \begin{cases} \tilde{F}(e) & \text{if } e \in A - B \\ \widetilde{G}(e) & \text{if } e \in B - A \\ \tilde{F}(e) \vee \widetilde{G}(e) & \text{if } e \in A \cap B \end{cases}$$

Then for all $e \in C$, either $e \in A - B$ (or) $e \in B - A$ (or) $e \in A \cap B$.

If $e \in A - B \Rightarrow \widetilde{H}(e) = \tilde{F}(e)$

If $e \in B - A \Rightarrow \widetilde{H}(e) = \widetilde{G}(e)$

If $e \in A \cap B \Rightarrow \widetilde{H}(e) = \tilde{F}(e) \vee \widetilde{G}(e)$

From the above, we have

$\tilde{F}(e) \vee \widetilde{G}(e)$ is an fuzzy ideal of S.

$\Rightarrow \widetilde{H}(e)$ is an fuzzy ideal of S. Therefore $(\widetilde{H}, C)$ is a fuzzy soft ideal over S.

Hence $(\tilde{F}, A) \cup_E (\tilde{G}, B)$ is a fuzzy soft ideal over S.

Theorem : 7.15

Let $(\tilde{F}, A)$ and $(\tilde{G}, A)$ are two fuzzy soft ideals over S. Then $(\tilde{F}, A) \cup_R (\tilde{G}, A)$ is also a fuzzy soft ideal over S.

Proof:

By definition,

$(\tilde{F}, A) \cup_R (\tilde{G}, B) = (\tilde{H}, C)$ where $C = A \cap B \neq \emptyset$ and $\tilde{H}(e) = \tilde{F}(e) \vee \tilde{G}(e)$ for all e $\in$ C

Then $\tilde{F}(a) \vee \tilde{G}(b)$ is an fuzzy ideal of S.

$\Rightarrow$ $\tilde{H}(e)$ is an fuzzy ideal of S. Therefore $(\tilde{H}, C)$ is a fuzzy soft ideal over S.

Hence $(\tilde{F}, A) \cup_R (\tilde{G}, B)$ is a fuzzy soft ideal over S.

SUMMARY AND CONCLUSION

Soft set theory and Fuzzy Soft set theory became a very good source of research for many mathematicians of recent years because of its wide range of applicability. The development in the fields of Soft sets and Fuzzy Soft sets had been taking place in a rapid pace.

The algebraic structure of Soft set theories has been explored in recent years.

Studies of Soft semigroups and Fuzzy Soft semigroups are carried out by several Researchers.

This thesis is devoted to the study of

1) Soft sets and Soft semigroups
2) Soft Regular semigroups
3) Soft Γ-semigroups
4) Soft ternary semigroups
5) Soft LA-semigroups
6) Fuzzy Soft sets and Fuzzy Soft semigropus
7) Fuzzy Soft LA-semigroups.

In the first chapter preliminary definitions, operations and properties regarding Soft sets and Soft semigroups are introduced with interesting examples. Soft semi-prime ideals and Soft prime ideals over a semigroup are defined and some of their properties are studied.

In the second chapter Regular semigroups are classified by considering its Soft right ideals and Soft left ideals. An important characterization regarding Soft Regular semigroups is proved.

The definition and properties of Soft Γ-semigroups are studied in chapter III.

In the fourth chapter the concept of Soft ternary semigroups, Soft left (right, lateral, quasi) ideals of ternary semigroups are defined and their properties are studied.

Chapter V deals with Soft LA-semigroups. In this chapter the definition of Soft LA-semigroup is given and some of its characteristics are discussed.

Fuzzy Soft sets and Fuzzy Soft semigroups are studied in Chapter VI. Fundamental definitions, operations and properties regarding Fuzzy Soft sets are given. Fuzzy Soft sets over a semigroup are defined and some of their properties are studied. Fuzzy Soft ideals are defined over a semigroup and their properties are investigated.

The author of this thesis introduced the concept of Fuzzy Soft LA-semigroups and studied their basic properties. Chapter VII deals with this Fuzzy Soft LA-semigroups. The definition of Fuzzy Soft LA-semigroups is given and their properties are discussed in detail.