Introduction

Topology is a branch of mathematics that plays an important role in a wide range of fields. It is concerned with the properties of a geometric object that are preserved despite continuous deformations such as stretching, twisting, crumpling, and bending, without ripping, gluing, or passing through itself. A topology describes how members of a set are related spatially. Set theory is the foundation of most other fields of topology, including differential geometry, geometric topology and algebraic topology, hence modern topology is heavily reliant on it. General topology, often known as point-set topology, is a branch of topology that deals with abstract objects.

The term "topology" has two meanings: it refers to both a mathematical discipline and a mathematical structure. A mathematical structure called a topology on a set is a collection of open subsets that meet particular relations on their intersections, unions and complements. Differentiable equations, dynamical systems, knot theory and Reimann surfaces in complex analysis are all applications of this topology. It's also employed in physics' string theory to describe the universe's space-time structure. Biology, Computer Science, Physics and Robotics are among the domains where it can be used. A special sort of function called as a homeomorphism is used in an intrinsic definition of topological equivalence. The ease with which continuity of functions can be defined in extended topological spaces typically provides expansions of the traditional perception of continuity explored in analysis as well as a clear generalisation of the notion of homeomorphism. As a result, topology does not stand alone; it serves as a foundation for the wide application-oriented areas stated above.

The concept of open sets is a strong tool for establishing a topological space in point set topology. Stone [1937] proposed regular open sets, which are a stronger form of open sets. Semi open sets in topological spaces were presented by Levine [1963] as a weaker form of open sets. Njastad [1965] developed the concept of α-sets in topological spaces and investigated various essential characteristics. Levine [1970] published the first investigation of the generalisation to the closed set. He also looked into the attributes of the proposed g-closed sets, paving the path for others to build on his work and apply it to various closed sets and other topological spaces. The concept of generalised closed sets in point set topology has sparked substantial interest in the field of research activity among general topologists during the last few decades, leading to the development of various new and interesting concepts. By relying on the work of Njastad [1965], Mashhour et al. [1983] identified the complement of α-sets, called α-closed sets and established a number of other properties. Maki [1986] later proposed the Λ-set, which is "equivalent to its kernel [saturated set], i.e., the intersection of all open supersets of the subsets of the topological spaces." By involving Λ-sets and closed sets, Francisco G. Arenas et al. [1997] proposed and examined the notion of λ-closed sets and λ-open sets. By utilizing the notion of λ-closed sets defined by Francisco G Arenas et al. [1997], Caldas et al. [2007 b] introduced the notion of λ-closure of a set.

In the flow of concepts, topology provide its notions inevitably to many areas of mathematics one such concept is continuity. Continuity is the very initial concept in the study of topological spaces which was defined under the name continuous mappings by Hausdorff [1978]. On defining and analysing a concept the next process of extending it is generalising the concept so, several researchers put forth a wide interest in developing the continuous mappings with their respective closed sets. In its way, Mashhour et al. [1983] introduced and studied α-continuous maps in topological spaces. Generalization to continuity was initiated and defined by Balachandran et al. [1991]. Before the introduction of generalized continuity, the study of irresoluteness came into existence by Crossley and Hildebrand [1972], who defined irresolute maps in topological spaces and analyzed the association related to the existing maps. It has been demonstrated that irresolute maps are stronger than semi continuous maps and are independent of continuous maps. Later, Francisco G. Arenas et al. [1997] investigated the key fundamental continuity idea of λ-continuous maps in topological spaces, which is a weaker form of continuous maps.

Continuing this, λ-irresolute maps were defined and studied by Caldas et al. [2007 b], they have contributed the study with productive theorems and results.

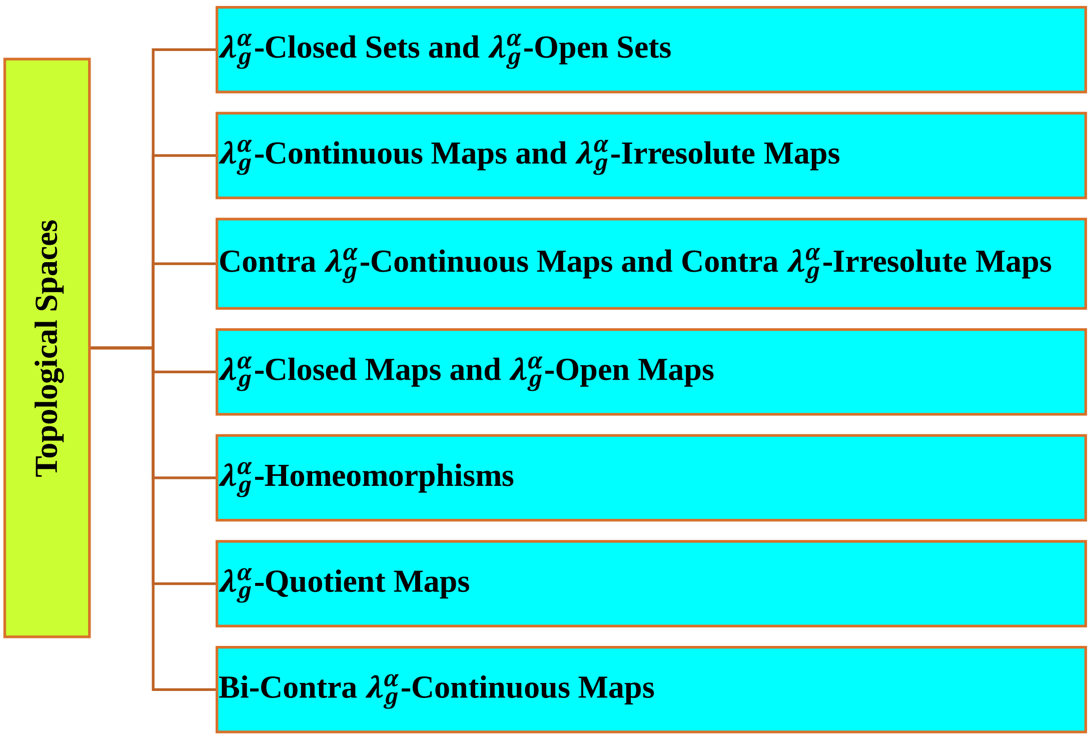
Having resolved with the work of pre-images of mappings in the study of topology, the next concept is the study of images of mappings. For that the introduction of closed mappings and open mappings in association to the respective closed and open sets are superimposed. Malghan [1982] initiated this study by defining generalized closed maps utilizing g-closed sets in topological spaces and studied their properties and derived fundamental theorems. The analysis in this area will be completed only with the extension of closed maps and open maps to homeomorphisms, as this forms the base for the topology. In this regard, Maki et al. [1991] defined g-homeomorphsims and gc-homeomorphisms with respect to g-continuous maps and g-irresolute maps. They had also explored the concept to a greater extent. General topologists were inspired by these ideas of homeomorphisms and were motivated to work on this area and provided fruitful results. Quotient maps are an important form of mappings which are defined with stronger conditions to continuity. With this as the base, α-quotient maps were defined and studied by Lellis Thivagar [1991], which gave productive results.

The word "Contra" means opposite; inverse; negative; such a contra case is associated with the continuities of topological spaces. The discussion in defining the idea of contra continuity provided many results and theorems in association with the respective continuous maps. Dontchev [1996] introduced the contra continuous maps and analyzed their associations. In view of this initiation, Dontchev and Noiri [1999] studied the fundamental properties of contra semi continuous mappings. Later, Saeid Jafari and Takashi Noiri [2002] analyzed the idea of contra pre continuous maps and provided the basic theorems. Contra λ-continuous maps were studied by Caldas et al. [2006 a]. Later on, Caldas et al. [2007 c] presented the contra g-continuous mappings in view of Levine's generalization to the closed sets and these were much useful in defining maps which were correlated to contra conditions.

With these in hand, an outstanding ideology was ascertained by Caldas et al. [2008 b]. That particular idea was given the name bi-contra continuity, wherein they have defined bi-contra continuous maps and bi-contra α-continuous maps in topological spaces. These arised as a motivation from the contra continuous maps. In bi-contra continuity, the two-way contra conditions or the stronger contra conditions were provided in the definition to get results regarding negative effects in real time applications. Following this, Lellis Thivagar et al. [2017] also realized that the definition of bi-contra continuous maps, when extended to nano topological space have a bright and realistic application in the field of biotechnology. They have applied the contra conditions and the bi-contra conditions between a set of viruses to a set of anti-viruses as a treatment for the disease-causing viruses. This have become a highly effective article in the field of biotechnology.

These existing research work induced the author to define a new form of generalized closed sets called λ_g^α-closed sets in topological spaces, through which all the above mentioned basic concepts are derived and improved to a greater extent. We have implemented the definition of λ_g^α-closed sets by using α-open sets and λ-closed sets to initiate the properties of continuity, irresoluteness, homeomorphisms, quotient mappings, contra continuity and bi-contra continuity in topological spaces.

The Profile of the present work is presented in the following diagram.



In **Chapter 1**, preliminary definitions which are used in the research work are aligned accordingly. Fundamental definitions like closed set, open set, generalizations to closed set, continuity, generalized continuity, irresoluteness, quotient mappings, homeomorphisms, contra continuity, bi-contra continuity are arranged and the related results of the fundamental definitions are also exhibited.

Chapter 2 exemplifies the definition of λ_g^α-closed sets and the fundamental properties of the defined closed set are derived sequentially. The dependency and independency relationships with the newly defined set and the previously existing sets are also exhibited diagrammatically. Moreover, the essential closure and interior operators of the proposed set is defined and the familiar preservation properties, union, intersection properties and the idempotent properties have also been discussed and derived.

Chapter 3 promptly explains the concept of continuity and irresoluteness of the newly defined λ_g^α-closed set in topological spaces. The definition of λ_g^α-continuous maps and λ_g^α-irresolute maps are written precisely to understand the concept of continuity in relation to the λ_g^α-closed set. Examples have been provided for varied understanding of the idea. Associations of the previously defined maps in relation with the proposed maps have been derived.

Also, we have defined the quasi λ_g^α-continuous maps and analyzed its properties. Following it, with a minor variation in the condition we have defined perfectly λ_g^α-continuous, totally λ_g^α-continuous and strongly λ_g^α-continuous maps. Properties of the defined various forms of λ_g^α-continuity have been derived and investigated.

Contra conditions play a dynamic role in general topology for the need of negative cases in real time applications. Contra λ_g^α-continuous maps and contra λ_g^α-irresolute maps in topological spaces are defined in **Chapter 4**. Moreover, the basic properties and associations of the proposed definitions are also analyzed.

Chapter 5 deals with the study of the λ_g^α-closed maps and λ_g^α-open maps in topological spaces. The variations of λ_g^α-closed maps like quasi λ_g^α-closed map and strongly

λ_g^{α}-closed map are also defined. Interrelationships, properties and fundamental theorems are derived and established.

Chapter 6 is devoted to the study of two new types of homeomorphisms called λ_g^{α}-homeomorphisms using the concept of λ_g^{α}-continuity and $\lambda_g^{\alpha}r$-homeomorphisms using the concept of λ_g^{α}-irresoluteness. Fundamental properties and basic theorems of the proposed definitions are presented.

Chapter 7 predominantly deals with λ_g^{α}-quotient mappings and its properties. Quotient mappings are a very interesting form of defining the maps in topological spaces, in which the stronger conditions are implemented in addition to continuity. Essential properties are derived and investigated with examples. Moreover, the special forms of λ_g^{α}-quotient maps such as strongly λ_g^{α}-quotient maps and completely λ_g^{α}-quotient maps are defined. Properties of such mappings are also investigated.

In Chapter 8, the concept of bi-contra λ_g^{α}-continuity is introduced and examined. We have defined bi-contra λ_g^{α}-continuous maps and its special forms like strongly bi-contra λ_g^{α}-continuous maps and completely bi-contra λ_g^{α}-continuous maps are defined, their interrelationships and basic properties are also analyzed.

Throughout the research period, we are ought to produce many counter examples to accompany the results obtained. Such examples in topological spaces with three, four and five elements are derived for various closed sets and open sets and the collections are provided as Appendix I, Appendix II and Appendix III.