

---

## Review of Literature

A review of literature of recent developments on generalized notions of closed and open sets, continuous mappings, closed mappings in topological spaces, fuzzy topological spaces and intuitionistic fuzzy topological spaces are given below.

### Topological Spaces

Closedness is the basic concept for the study and investigation in topological spaces. In the study of topological spaces many concepts of topology have been generalized by considering the concepts of semi open sets by Levine (1963) instead of open sets. Levine (1970) introduced the concept of generalized closed sets in topological spaces. Using this concept and Levine's idea, many researchers have introduced and studied various types of generalized closed sets. Abd El-Monsef et al. (1983, 1986) have introduced  $\beta$  open,  $\beta$  closure and  $\beta$  interior in topological spaces. Narmatha, Nagaveni and Noiri (2013) have introduced regular b-open sets in topological spaces. Palaniappan and Rao (1993) have introduced regular generalized closed sets in topological spaces. Maki et al. (1993, 1994) have introduced generalized  $\alpha$  closed sets and  $\alpha$  generalized closed sets in topological spaces. Fukutake, Nasef and El-Maghrabi (2003) introduced  $\gamma$  generalized closed sets in topological spaces. Ganster and Steiner (2007) investigated many relationships between b generalized closed sets with generalized notions of closed sets. Ahmad Al-Omari and Mohammed Salmi Md. Noorani (2009) introduced generalized b-closed sets. Kannan and Nagaveni (2012) have introduced  $\hat{\beta}$  generalized closed sets and open sets in topological spaces.

Continuity plays a vital role in the study of topological spaces. In addition to continuity, separation axioms also provide a wide area on the study of topological spaces. Mashhour, Abd El-Monsef and El-Deeb (1982) have investigated pre

continuous and weak precontinuous mappings in topology. Noiri (1984) introduced  $\alpha$ -continuous functions and  $\alpha$  irresolute functions in topology. Reilly and Vamanamurthy (1985) have introduced  $\alpha$  continuity in topological spaces. Yuksel and Noiri (1996) have defined the notion of  $\beta^*$  set and established the decomposition of continuity. Caldas and Navalagi (2003) have investigated weak forms of  $\beta$ -open and  $\beta$ -closed functions. Ali M. Mubarki et al. (2014) introduced and studied the notion of  $\beta^*$  open sets and  $\beta^*$  continuous functions in topological spaces. Tahiliani (2006) have introduced generalized  $\beta$ -closed functions. Almost continuous mapping in topology is introduced by Singal and Singal (1968). Paul and McGehee (1970) have investigated some properties of almost continuous functions. Thakur and Paik (1987) have introduced almost  $\alpha$ -continuous mappings. Jafari and Noiri (2001) have introduced contra  $\alpha$ -continuous, contra precontinuous mappings in topological spaces. El-Magbrabi (2010) has studied and analyzed some properties of contra continuous mappings in topological spaces.

Open maps and closed maps are very interesting concepts in topology. Noiri (1973) has introduced a class of mappings called semi-closed mappings which contain the class of closed mappings. Mashhour, Hasanein and El-Deeb (1983) have introduced  $\alpha$ -continuous and  $\alpha$ -open mapping in topological spaces. Semi generalized closed maps and generalized semi closed maps are introduced by Devi, Balachandran and Maki (1993). Devi, Balachandran and Maki (1998) have introduced generalized  $\alpha$  closed maps and  $\alpha$  generalized closed maps in topological spaces. Balachandran, Sundaram and Maki (1991) introduced GO connectedness in topological spaces.

### **Fuzzy Topological Spaces**

The notion of fuzzy set theory has caused great interest among both pure and applied mathematicians. It has also raised enthusiasm among some engineers, biologists, psychologists, economists and experts in other areas who use mathematical ideas and methods in their research. Many sets defined in topology take their new

form in fuzzy topology which was determined by Chang (1968). Lowen (1982) has introduced fuzzy neighbourhood spaces.

Sostak (1985) introduced the fundamental concept of a fuzzy topological structure, as an extension of both crisp topology and fuzzy topology, in the sense that not only the objects are fuzzified, but also the axiomatics. Ganguly and Saha (1986) has introduced fuzzy semiopen sets in fuzzy topological spaces. Singal and Niti Prakash (1991) have introduced fuzzy preopen sets. Thakur and Malviya (1995) have introduced generalized closed sets in fuzzy topology. Maki et al. (1998) has introduced generalized closed sets in fuzzy topological spaces. Fuzzy semipre open set is introduced by Thakur and Singh (1998).

Saraf and Meena Khanna (2003) have introduced gs closed sets in fuzzy topology. Saraf, Govindappa Navalagi and Meena Khanna (2005) have introduced fuzzy semipre generalized closed sets. Bayaz Daraby and Nimse (2007) have discussed fuzzy generalized alpha closed set and its applications.

Fuzzy semi continuity, fuzzy almost continuity and fuzzy weekly continuity are introduced by Azad (1981). Fuzzy  $\alpha$  sets and  $\alpha$  continuous mappings were introduced by Singal and Niti Rajvanshi (1992). Thakur and Singh (1998) have introduced semi pre continuous mappings in fuzzy topological spaces. Balasubramanian and Sundaram (1997) have investigated some generalizations of fuzzy continuous functions. Abd EI-Hakeim (1999) has introduced generalized semi continuous mappings in fuzzy topological spaces. Ekici and Kerre (2006) has introduced fuzzy contra continuities. Ahmad and Athar (2008) have introduced fuzzy almost continuous function in fuzzy topological space.

Mukherjee and Sinha (1989) have discussed irresolute and almost open functions between fuzzy topological spaces. Prasad, Thakur and Saraf (1994) introduced fuzzy  $\alpha$ -irresolute mappings. Bhaumik and Mukherjee (1993<sub>a</sub>, 1993<sub>b</sub>) have introduced fuzzy completely continuous mappings and fuzzy weakly completely continuous function in fuzzy topology.

Nanda (1986) studied fuzzy almost open mappings. Ghosh (1990) has introduced semi closed mappings in fuzzy setting. Park et al. (1997) has introduced fuzzy weakly open mappings in fuzzy topological spaces. Raja Sethupathy and Lakshmivarahan (1977) have introduced connectedness in fuzzy topological spaces. Zhao (1986) has contributed some beautiful results in connectedness on fuzzy topological spaces.

### **Intuitionistic Fuzzy Topological Spaces**

Atanassov (1986, 1988, 1989, 1994) has introduced intuitionistic fuzzy sets and also he gave new results in intuitionistic fuzzy sets and operations. Intuitionistic fuzzy points are introduced by Coker and Demirci (1995). Intuitionistic fuzzy open sets and intuitionistic fuzzy closed sets are introduced by Coker (1997). Intuitionistic fuzzy semi open sets, intuitionistic fuzzy pre open sets, intuitionistic fuzzy  $\alpha$  closed sets are introduced by Joung Kon Jeon et al. (2005). Young Bae Jun and Seok-Zun Song (2005) have introduced intuitionistic fuzzy semi-pre open sets. Thakur and Rekha Chaturvedi (2008) have introduced generalized closed sets in intuitionistic fuzzy topological spaces. Hanafy (2009) studied the properties of intuitionistic fuzzy gamma closed sets and intuitionistic fuzzy gamma open sets. Santhi and Jayanthi (2009) have introduced intuitionistic fuzzy generalized semi pre closed sets in topological spaces. Santhi and Arun Prakash (2010) have introduced intuitionistic fuzzy semi-generalized closed sets and their applications.

Rajarajeshwari and Senthil Kumar (2011) have introduced generalized pre-closed sets in intuitionistic fuzzy topological spaces. Thirumalaiswamy and Ramesh (2013) have introduced semi pre generalized closed sets in intuitionistic fuzzy topological spaces. Intuitionistic fuzzy generalized beta closed sets are introduced by Jayanthi (2014<sub>a</sub>). The concept nowhere dense in intuitionistic fuzzy topological space is introduced by Thakur and Dhavaseelan (2015). Saranya and Jayanthi (2016<sub>a</sub>) have introduced intuitionistic fuzzy  $\beta$ -generalized closed sets in intuitionistic fuzzy topological spaces.

Hur and Jun (2003) have studied intuitionistic fuzzy  $\alpha$  continuous mappings. Intuitionistic fuzzy continuous mappings and intuitionistic fuzzy irresolute mappings are studied by Young Bae Jun and Seok-Zun Song (2005). Joung Kon Jeon, Young Bae Jun and Jin Han Park (2005) have introduced intuitionistic fuzzy  $\alpha$  continuity and intuitionistic fuzzy pre continuity. Jun, Kang and Song (2005) have introduced intuitionistic fuzzy irresolute and continuous mappings. Thakur and Rekha Chaturvedi (2006<sub>b</sub>) have introduced generalized continuity in intuitionistic fuzzy topology. Krsteska and Salah Abbas (2007) have introduced intuitionistic fuzzy strongly irresolute pre continuous mappings in Coker's space. Hanafy (2009) has introduced intuitionistic fuzzy  $\gamma$  continuity in intuitionistic fuzzy topological space.

Krsteska and Ekici (2007) have introduced intuitionistic fuzzy contra strong pre continuity. Sakthivel (2010) has introduced intuitionistic fuzzy  $\alpha$  generalized continuous mappings and intuitionistic fuzzy  $\alpha$  generalized irresolute mappings. Roja, Uma and Dhavaseelan (2012) have introduced generalized intuitionistic fuzzy contra continuity. Generalized beta continuous mappings in intuitionistic fuzzy topological spaces are introduced by Jayanthi (2014<sub>b</sub>). Beta generalized continuous mappings in intuitionistic fuzzy topological spaces are introduced by Saranya and Jayanthi (2016<sub>b</sub>).

Hanafy and El-Arish (2003) have introduced completely continuous functions in intuitionistic fuzzy topological spaces. Shrivastava and Jyoti Gupta (2016) have introduced intuitionistic fuzzy almost continuity and weakly continuity.

Seok Jong Lee and Eun Pyo Lee (2000) have introduced intuitionistic fuzzy open mapping and intuitionistic fuzzy closed mapping in intuitionistic fuzzy topological spaces. Joung Kon Jeon, Young Bae Jun and Jin Han Park (2005) have introduced the notion of intuitionistic fuzzy pre open mappings, intuitionistic fuzzy  $\alpha$  open mappings in intuitionistic fuzzy topological spaces. Santhi and Sakthivel (2010) have introduced  $\alpha$  generalized closed mappings in intuitionistic fuzzy topological spaces. Jyoti Pandey and Thakur (2011) have introduced the notion

of intuitionistic fuzzy quasi rw-open and intuitionistic fuzzy rw-closed mappings in intuitionistic fuzzy topological spaces. Santhi and Arun Prakash (2012) have introduced intuitionistic fuzzy semi generalized closed mappings. Generalized beta closed mappings in intuitionistic fuzzy topological spaces is introduced by Jayanthi (2014<sub>c</sub>).

Intuitionistic fuzzy  $C_5$  connectedness is introduced by Coker (1997). Ozcag and Coker (1998) have introduced connectedness in intuitionistic fuzzy topological spaces. Intuitionistic fuzzy GO-connectedness in intuitionistic fuzzy topology is introduced by Turnali and Coker (2000). Hazra, Mandal and Samanta (2003) have introduced connectedness in topology of intuitionistic fuzzy sets. Young Chan Kim and Abbas (2005) have investigated connectedness in intuitionistic fuzzy topological spaces. Renuka and Seenivasan (2013) extended the concepts of  $\beta$  connectedness in intuitionistic fuzzy topological spaces.

In my research work, a new generalization of closed set called  $\beta^{**}$  generalized closed set in intuitionistic fuzzy topological space is introduced. Their basic properties, preservation propositions, interrelations, continuity, connectedness and their applications are established with necessary counter examples.

Some of the research articles which I refer for the thesis are given below.

## **1. GENERAL TOPOLOGY**

[Williard, S., 1970]

In this book, the author encompasses two broad areas of topology : continuous topology, represented by sections on convergence, compactness, metrization and complete metric spaces and function spaces.

**2. ON GENERALIZED b-CLOSED SETS**

[Ahamad Al Omari and Mohammed Salmi Md. Noorani, 2009]

In this article, the authors have studied the class of generalized b closed sets and use this notion to consider new weak and stronger forms of continuities associated with these sets. And they applied these notions to give new characterization of extremally disconnected spaces and  $T_{gs}$  spaces.

**3. FUZZY SETS**

[Zadeh, L.A., 1965]

In this article, the author has introduced a new class of sets namely fuzzy sets which are characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. Further the author has provided the notions of inclusion, union, intersection, complement etc., with respect to the fuzzy sets.

**4. FUZZY TOPOLOGICAL SPACES**

[Chang, C.L., 1968]

In this article, the author has introduced fuzzy topological spaces. This concept is considered to be the generalization of general topological spaces. In brief, the basic concepts such as fuzzy open set, fuzzy closed set, fuzzy neighbourhood, fuzzy continuity etc., are discussed in depth.

**5. ON FUZZY GENERALIZED ALPHA CLOSED SET AND ITS APPLICATIONS**

[Bayaz Daraby and Nimse, S.B., 2007]

In this article they have defined and studied fuzzy generalized alpha closed sets and r open sets, fuzzy alpha continuous functions and their applications.

**6. ON FUZZY b-NEIGHBOURHOODS AND FUZZY b-MAPPINGS IN FUZZY TOPOLOGICAL SPACES**

[Benchalli, S.S. and Jenifer Karnel, 2010]

In this article, the authors have introduced the concept of fuzzy b-neighbourhood and fuzzy b-continuous mappings in fuzzy topological spaces. The interrelationship of fuzzy b-continuous mappings with various fuzzy mappings is investigated.

**7. INTUITIONISTIC FUZZY SETS**

[Atanassov, K., 1986]

In this article, the author has provided the notion of intuitionistic fuzzy sets. This is considered to be the generalization on fuzzy sets. The highlight of this particular article is that some relations and operations concerning classical sets are extended to intuitionistic fuzzy sets.

**8. MORE ON INTUITIONISTIC FUZZY SETS**

[Atanassov, K., 1989]

In this article, author has introduced new results on intuitionistic fuzzy sets. Two news operators on intuitionistic fuzzy sets are defined and their basic properties are studied.

**9. AN INTRODUCTION TO INTUITIONISTIC FUZZY TOPOLOGICAL SPACES**

[Coker, D., 1997]

In this article, the author has introduced intuitionistic fuzzy topological space. The notions of intuitionistic fuzzy interior and intuitionistic fuzzy closure are being provided and this is followed by the discussion of some important properties concerning them. Furthermore, the notion of intuitionistic fuzzy continuity is provided.

**10. ON INTUITIONISTIC FUZZY  $\beta$ -GENERALIZED CLOSED SETS**

[Saranya, M. and Jayanthi, D., 2016<sub>a</sub>]

This article consists of the notion of intuitionistic fuzzy  $\beta$ -generalized closed sets. The authors have analyzed some of their properties and obtained some interesting theorems. Also, the relationship between this new class of sets and some of the previously existing sets are discussed.

**11. NOWHERE DENSE SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES**

[Thakur, S.S. and Dhavaseelan, R., 2015]

In this article, the authors have introduced the concept of nowhere dense subsets and investigated the characterizations of intuitionistic fuzzy nowhere dense sets.

**12. ON FUZZY CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES**

[Gurcay, H., Coker, D. and Haydar, Es.A., 1997]

In this article, the authors have introduced some definitions and features related to fuzzy continuity, fuzzy membership functions and fuzzy continuous functions in intuitionistic fuzzy topological spaces. Using these definitions and properties, some theorems about fuzzy continuous functions have been proved.

**13. INTUITIONISTIC FUZZY IRRESOLUTE AND CONTINUOUS MAPPING**

[Jun, Y.B., Kang, J.O. and Song, S.Z., 2005]

In this article, the notions of intuitionistic fuzzy irresolute mappings, intuitionistic fuzzy pre semi open mappings, intuitionistic fuzzy almost open mappings, intuitionistic fuzzy weakly continuous mappings, intuitionistic fuzzy H almost continuous mappings and intuitionistic fuzzy W almost open mappings are

introduced and their relations are investigated. A characterization of intuitionistic fuzzy irresolute mappings is also given.

**14. INTUITIONISTIC FUZZY ALPHA-CONTINUITY AND INTUITIONISTIC FUZZY PRECONTINUITY**

[Joung Kon Jeon, Young Bae Jun and Jin Han Park, 2005]

In this article, the authors have defined the notion of semi open mappings, pre open mappings and alpha open mappings and investigated the relation among them. They gave a characterization of intuitionistic fuzzy  $\alpha$  open set, intuitionistic fuzzy  $\alpha$  continuous mappings and intuitionistic fuzzy pre continuous mappings and provided conditions for a mapping of intuitionistic fuzzy topological spaces to be an intuitionistic fuzzy  $\alpha$  continuous mapping.

**15. COMPLETELY CONTINUOUS FUNCTIONS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES**

[Hanafy, I.M. and El-Arish, 2003]

In this article, after giving the basic results related to the product of the functions and the graph of functions in intuitionistic fuzzy topological spaces, the author introduced and study the concept of fuzzy completely continuous functions between intuitionistic fuzzy topological spaces.

**16. THE CATEGORY OF INTUITIONISTIC FUZZY TOPOLOGICAL SPACES**

[Seok Jong Lee and Eun Pyo Lee, 2000]

In this article, the authors have introduced intuitionistic fuzzy neighbourhood and investigated the properties of continuous, open and closed mappings in intuitionistic fuzzy topological spaces.

**17. QUASI COINCIDENCE FOR INTUITIONISTIC FUZZY POINTS**

[Luplianez, F.G., 2005]

In this article, the author has introduced the notion of quasi coincidence for intuitionistic fuzzy sets and the corresponding neighbourhood structure.

**18. GENERALIZED CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACE**

[Thakur, S.S. and Rekha Chaturvedi, 2006<sub>b</sub>]

In this article, the authors have introduced and studied the concept of generalized continuous mappings in intuitionistic fuzzy topological space.

**19. ON CONNECTEDNESS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES**

[Ozcag, S. and Coker, D., 1998]

This article consists of the basic concepts related to connectedness in intuitionistic fuzzy special topological spaces. They have introduced the concepts of  $C_5$ -connectedness,  $C_S$ -connectedness,  $C_M$ -connectedness, strong connectedness, super connectedness,  $C_i$ -connectedness ( $i = 1, 2, 3, 4$ ) and obtain several preservation properties and some characterizations concerning connectedness in these spaces.

---

## Notations

IFS	- Intuitionistic fuzzy set
IFSs	- Intuitionistic fuzzy sets
IFT	- Intuitionistic fuzzy topology
IFTS	- Intuitionistic fuzzy topological space
IFTSs	- Intuitionistic fuzzy topological spaces
IFP	- Intuitionistic fuzzy point
IFN	- Intuitionistic fuzzy neighbourhood
IFD	- Intuitionistic fuzzy dense
$A^c$	- The complement of $A$
$\text{int}(A)$	- Interior of $A$
$\text{cl}(A)$	- Closure of $A$
$\text{pint}(A)$	- pre interior of $A$
$\text{pcl}(A)$	- pre closure of $A$
$\beta\text{int}(A)$	- $\beta$ interior of $A$
$\beta\text{cl}(A)$	- $\beta$ closure of $A$
$\text{IFC}(X)$	- The family of all intuitionistic fuzzy closed sets of $X$
$\text{IFSC}(X)$	- The family of all intuitionistic fuzzy semi closed sets of $X$
$\text{IFPC}(X)$	- The family of all intuitionistic fuzzy pre closed sets of $X$
$\text{IF}\alpha\text{C}(X)$	- The family of all intuitionistic fuzzy $\alpha$ closed sets of $X$
$\text{IF}\beta\text{C}(X)$	- The family of all intuitionistic fuzzy $\beta$ closed sets of $X$
$\text{IFGC}(X)$	- The family of all intuitionistic fuzzy generalized closed sets of $X$
$\text{IFO}(X)$	- The family of all intuitionistic fuzzy open sets of $X$
$\text{IFSO}(X)$	- The family of all intuitionistic fuzzy semi open sets of $X$
$\text{IFPO}(X)$	- The family of all intuitionistic fuzzy pre open sets of $X$
$\text{IF}\alpha\text{O}(X)$	- The family of all intuitionistic fuzzy $\alpha$ open sets of $X$
$\text{IF}\beta\text{O}(X)$	- The family of all intuitionistic fuzzy $\beta$ open sets of $X$
$\text{IFGO}(X)$	- The family of all intuitionistic fuzzy generalized open sets of $X$
$\text{IF}\beta^{**}\text{GCSs}$	- Intuitionistic fuzzy $\beta^{**}$ generalized closed sets