Introduction

Topology is a branch of mathematics which is good at extracting global qualitative features from complicated geometric structures. General topology known as point set topology is the branch of topology dealing with the basic set theoretic definitions and constructions used in topology. It is the foundation of most other branches of topology, including differential topology, geometric topology, algebraic topology and network topology. The fundamental concepts in point set topology are continuity, compactness and connectedness. The topological structures are modeled suitably in the fields of computer graphics, pattern recognition, artificial intelligence, data mining, information systems, rough set theory, quantum physics etc.

The concepts of closed and open sets play an important role in the study of topological spaces. Stone (1937) introduced the regular open sets in topological spaces, as a stronger form of open sets and Levine (1960) initiated and studied the concept of semi open sets in topological spaces, as a weaker form of open sets.

Several authors introduced and studied different kinds of open and closed sets in topological spaces. Njastad (1965) introduced the concept of α-open sets which are weaker than open sets and proved that the collection of all α-open sets forms a finer topology. Further Levine (1970) introduced the notion of generalized closed (g-closed) sets in topological spaces.

Mashhour et al.(1982), Andrijevic (1986), Bhattacharya and Lahiri (1987) and Arya and Nour (1990) introduced and studied the weaker forms of open sets respectively pre open sets, semi pre open sets, semi generalized open sets and generalized semi open sets. Veera Kumar (2000a) introduced and investigated ψ-closed sets and Ramya and Parvathi (2013) defined ψg-closed sets in topological spaces.

Levine (1970) introduced the concept of continuous maps in topology. Following Levine several researchers have shown interest in the generalizations of continuous maps. Mashhour et al. (1983) introduced and studied α-continuous maps in topological spaces. The generalized continuous (g-continuous) maps were introduced and studied by Balachandran et al. (1991). Crossley and Hildebrand (1972) introduced the concept of irresolute maps.

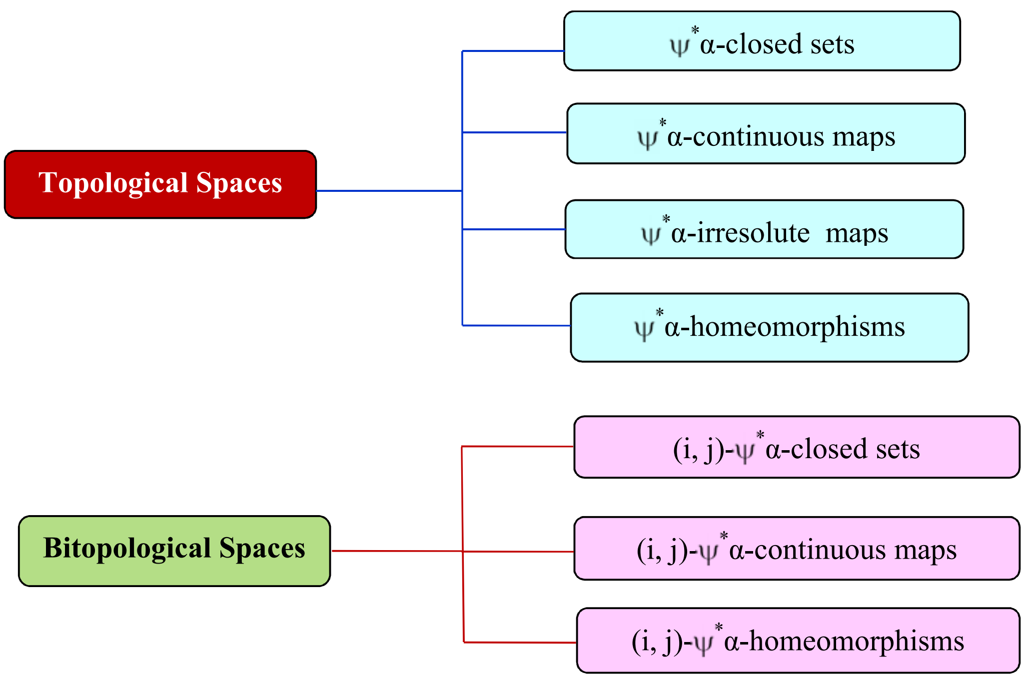
Open maps and closed maps are very useful in topological spaces. The concepts of homeomorphism play an important role in topological spaces. Malghan (1982) introduced the concept of generalized closed maps in topological spaces. Maki et al. (1991a) introduced generalized homeomorphisms and gc-homeomorphisms and studied their properties. The researchers have extended the notion of generalized closed sets to open maps, closed maps and homeomorphisms.

Following the introduction of bitopological spaces by Kelly (1963) research work has been carried out to generalize the topological concepts to bitopological spaces. Fukutake (1985) initiated the concept of g-closed sets in bitopological spaces. Sheik John and Sundram (2004) introduced g^*-closed sets in bitopological spaces. Using the concepts of g-closed sets El-Tantawy and Abu-Donia (2005) introduced and studied generalization of separation axioms in bitopological spaces.

PROFILE OF THE PRESENT WORK

In the present work, a new class of generalized closed sets called $\psi^*\alpha$-closed sets is defined and their properties are analyzed in topological and bitopological spaces using α-closed sets and ψg-open sets.

Concepts considered in this thesis are



Basic definitions and results relevant to various generalized closed sets in topological and bitopological spaces are presented in the first chapter.

In chapter 2, a new class of generalized closed sets namely $\psi^*\alpha$-closed sets in topological spaces is introduced with the definition, **A subset A of a topological space (X, τ) is said to be a $\psi^*\alpha$-closed set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ψg-open in (X, τ).** A comparison of $\psi^*\alpha$-closed sets with other existing closed sets, properties of $\psi^*\alpha$-closed sets and characterizations of $\psi^*\alpha$-closed sets are studied. A similar study is also carried out for $\psi^*\alpha$-open sets. As an application of $\psi^*\alpha$-closed sets five new spaces namely, ${}_{\psi^*\alpha}T_c$-space, ${}_{\psi^*\alpha}T_\alpha$-space, ${}_{g\alpha}T_{\psi^*\alpha}$-space, ${}_{\alpha g}T_{\psi^*\alpha}$-space and ${}_{\psi g}T_{\psi^*\alpha}$-space are introduced and their properties are analyzed.

In chapter 3, using $\psi^*\alpha$-closed sets the concept of $\psi^*\alpha$-continuity is defined and various types of continuous maps namely quasi $\psi^*\alpha$-continuous maps, perfectly $\psi^*\alpha$-continuous maps, totally $\psi^*\alpha$-continuous maps, strongly $\psi^*\alpha$-continuous maps and contra $\psi^*\alpha$-continuous maps are established. Properties and interrelations among these maps are discussed. The impact of newly defined spaces with respect to continuous mappings and composition of mappings are investigated.

Two new classes of irresolute maps called $\psi^*\alpha$-irresolute maps and contra $\psi^*\alpha$-irresolute maps are introduced in chapter 4. Properties and the association of these maps with other existing irresolute maps are analyzed.

In chapter 5, $\psi^*\alpha$-closed maps, $\psi^*\alpha$-open maps, quasi $\psi^*\alpha$-closed maps, quasi $\psi^*\alpha$-open maps, $\psi^*\alpha$-homeomorphisms and $\psi^*\alpha$-quotient maps are defined and their properties and characterizations are discussed. Further the study of $\psi^*\alpha$-compact spaces and $\psi^*\alpha$-connected spaces are carried out and the standard results in topological spaces are extended to these spaces.

In chapter 6, (i, j)-$\psi^*\alpha$-closed sets in bitopological space are introduced with the following definition. **A subset A of a bitopological space (X, τ_1, τ_2) is called (i, j)-$\psi^*\alpha$-closed if τ_j-$\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_i-ψg-open in (X, τ_i), where i, j = 1, 2 and $i \neq j$.** A comparative study between (i, j)-$\psi^*\alpha$-closed sets and the existing generalized (i, j)-closed sets is carried out. As an application of (i, j)-$\psi^*\alpha$-closed sets, four new spaces namely, (i, j)-$_{\psi*\alpha}T_c$-space, (i, j)-$_{\psi*\alpha}T_\alpha$-space, (i, j)-$_{g\alpha}T_{\psi*\alpha}$-space and (i, j)-$_{\alpha g}T_{\psi*\alpha}$-space are introduced and their interrelations among them are discussed.

The concepts of (i, j)-$\psi^*\alpha$-σ_k-continuous maps, quasi (i, j)-$\psi^*\alpha$-continuous maps, perfectly (i, j)-$\psi^*\alpha$-continuous maps, totally (i, j)-$\psi^*\alpha$-σ_k-continuous maps, strongly (i, j)-$\psi^*\alpha$-σ_k-continuous maps, contra (i, j)-$\psi^*\alpha$-σ_k-continuous maps, (i, j)-$\psi^*\alpha$-irresolute maps and contra (i, j)-$\psi^*\alpha$-irresolute maps are introduced in chapter 7. Properties and their interdependency are analyzed.

In chapter 8, (i, j)-$\psi^*\alpha$-closed maps, (i, j)-$\psi^*\alpha$-open maps, quasi (i, j)-$\psi^*\alpha$-closed maps, quasi (i, j)-$\psi^*\alpha$-open maps and (i, j)-$\psi^*\alpha$-homeomorphisms are defined and their properties are studied.

- ✶ For the topological spaces of three elements or four elements various open and closed sets are found out and the collections are tabulated in **Appendix I** and **Appendix II** for verification of examples.