Chapter 1

Preliminaries

Basic definitions, properties and results which were functional and constructive in the formulation of the thesis are collected and arranged in this Chapter.

1.1 Closed and Open Sets in Topological Spaces

Definition 1.1.1

For a subset S of a topological space (M, μ),

(i) The intersection of all closed sets containing S is called the closure of S and is denoted by $cl(S)$

(ii) The union of all open sets contained in S is called the interior of S and is denoted by $int(S)$

Definition 1.1.2 [Njastad, 1965] A subset S of a topological space (M, μ) is called an α-open set if $S \subseteq int(cl(int(S)))$. The complement of an α-open set is called α-closed.

Definition 1.1.3 [Mashhour et al., 1983] The α-closure of a subset S of a topological space (M, μ) is the intersection of all α-closed sets containing S and is denoted by $cl_\alpha(S)$.

Definition 1.1.4 [Maki, 1986] A subset S of a topological space (M, μ) is a Λ-set if it is equal to its kernel i.e., it is equal to the intersection of all open supersets of S.

Definition 1.1.5 [Francisco G. Arenas et al., 1997] A subset S of a topological space (M, μ) is called a λ-closed set if $S = L \cap D$, where L is a Λ-set and D is a closed set. The complement of λ-closed set is called λ-open.

Definition 1.1.6 [Caldas et al., 2007 b] The λ-closure of a subset S of a topological space (M, μ) is the intersection of all λ-closed sets containing S and is denoted by $cl_\lambda(S)$.

Definition 1.1.7 A subset S of a topological space (M, μ) is called

(i) **generalized closed (briefly g-closed) [Levine, 1970]** if $cl(S) \subseteq U$ whenever $S \subseteq U$ and U is open in (M, μ).

(ii) **generalized α-closed (briefly $g\alpha$-closed) [Maki et al., 1993]** if $cl_\alpha(S) \subseteq U$ whenever $S \subseteq U$ and U is α-open in (M, μ).

(iii) **α-generalized closed (briefly αg-closed) [Maki et al., 1994]** if $cl_\alpha(S) \subseteq U$ whenever $S \subseteq U$ and U is open in (M, μ).

(iv) **g^*-closed [Veera Kumar, 2000]** if $cl(S) \subseteq U$ whenever $S \subseteq U$ and U is g-open in (M, μ).

(v) **Λ-g-closed [Caldas et al., 2007 b]** if $cl_\lambda(S) \subseteq U$ whenever $S \subseteq U$ and U is λ-open in (M, μ).

(vi) **Λ-generalized closed (briefly Λ_g-closed) [Caldas et al., 2008 a]** if $cl(S) \subseteq U$ whenever $S \subseteq U$ and U is λ-open in (M, μ).

(vii) **$g\Lambda$-closed [Caldas et al., 2008 a]** if $cl_\lambda(S) \subseteq U$ whenever $S \subseteq U$ and U is open in (M, μ).

(viii) **$\Lambda_{g\alpha}$-closed [Ochanathevar Ravi et al., 2015]** if $cl_\alpha(S) \subseteq U$ whenever $S \subseteq U$ and U is λ-open in (M, μ).

(ix) **$g^{**}\Lambda$-closed [Balamani, 2020 b]** if $cl_\lambda(S) \subseteq U$ whenever $S \subseteq U$ and U is g^*-open in (M, μ).

The complements of the above-mentioned sets are called their respective open sets.

Lemma 1.1.8 [Francisco G. Arenas et al., 1997]

(i) Every Λ-set is λ-closed.

(ii) Every closed set and open set is λ-closed.

Lemma 1.1.9 [Caldas et al., 2007 b]

The following properties hold good for the subsets S and T of a topological space (M, μ).

(i) If S_i is λ-closed for each $i \in I$, then $\bigcap_{i \in I} S_i$ is also λ-closed.

(ii) If S_i is λ-open for each $i \in I$, then $\bigcup_{i \in I} S_i$ is also λ-open.

(iii) S is λ-closed if and only if $S = cl_\lambda(S)$.

(iv) Finite union of λ-closed sets need not be λ-closed.

(v) $cl_\lambda(S) = \bigcap\{F \subseteq M | S \subseteq F$ and F is λ-closed in $(M, \mu)\}$.

(vi) $S \subseteq cl_\lambda(S) \subseteq cl(S)$.

(vii) If $S \subseteq T$, then $cl_\lambda(S) \subseteq cl_\lambda(T)$.

1.2 Definition and Results Related to Spaces

Definition 1.2.1

A topological space (M, μ) is called a/an

(i) **α-space [Njastad, 1965]** if every α-closed subset of (M, μ) is closed in (M, μ).

(ii) **λ-space [Francisco G. Arenas et al., 1997]** if every λ-closed subset of (M, μ) is closed in (M, μ).

(iii) **$_{\alpha}T_b$-space [Devi et al., 1998]** if every αg-closed subset of (M, μ) is closed in (M, μ).

(iv) **$T^*_{1/2}$-space [Veera Kumar, 2000]** if every g^*-closed subset of (M, μ) is closed in (M, μ).

(v) **Door space [Dontchev, 1995]** if every subset of (M, μ) is either open or closed in (M, μ).

(vi) **Partition space [Neiminen, 1977]** if every open set of (M, μ) is closed in (M, μ).

Definition 1.2.2 A topological space (M, μ) is called a **$T_{1/2}$-space**

(i) **[Levine, 1970]** if every g-closed subset of (M, μ) is closed in (M, μ) (or)

(ii) **[Dunham, 1977]** if every singleton set in (M, μ) is open or closed in (M, μ) (or)

(iii) **[Francisco G. Arenas et. al., 1997]** if every subset of (M, μ) is λ-closed in (M, μ).

Result 1.2.3 Let (M, μ) be a topological space.

(i) In $\boldsymbol{T_1}$**-space,** every singleton set in (M, μ) is closed in (M, μ).

(ii) In $\boldsymbol{T_1}$**-space [Caldas et al., 2008 a]**, every Λ_g-closed subset of (M, μ) is closed in (M, μ).

1.3 Mappings in Topological Spaces

Results 1.3.1

Let (M, μ) and (N, ν) be the topological spaces. Let $u: (M, \mu) \to (N, \nu)$ be a map. Let S and T be the subsets of (M, μ) and (N, ν) respectively, then the following properties hold good

(i) If $S \subseteq T$, then $u(S) \subseteq u(T)$

(ii) If $S \subseteq T$, then $u^{-1}(S) \subseteq u^{-1}(T)$

(iii) Generally, $T \subseteq u^{-1}(u(T))$ but when u is injective, $T = u^{-1}(u(T))$

(iv) Generally, $u(u^{-1}(T)) \subseteq T$ but when u is surjective, $u\left(u^{-1}(T)\right) = T$

(v) If u is surjective then $[u(T)]^c \subseteq u(T^c)$

(vi) If u is bijective then $[u(T)]^c = u(T^c)$

Definition 1.3.2 Let (M, μ) and (N, ν) be the topological spaces. A map $u: (M, \mu) \to (N, \nu)$ is called

(i) **continuous [Munkres, Topology, A First Course]** if $u^{-1}(T)$ is closed in (M, μ) for every closed set T of (N, ν).

(ii) $\boldsymbol{\alpha}$**-continuous [Mashhour et al., 1983]** if $u^{-1}(T)$ is α-closed in (M, μ) for every closed set T of (N, ν).

(iii) **g-continuous [Balachandran et al., 1991]** if $u^{-1}(T)$ is g-closed in (M, μ) for every closed set T of (N, ν).

(iv) **αg-continuous [Devi et al., 1997]** if $u^{-1}(T)$ is αg-closed in (M, μ) for every closed set T of (N, ν).

(v) **$g\alpha$-continuous [Devi et al., 1997]** if $u^{-1}(T)$ is $g\alpha$-closed in (M, μ) for every closed set T of (N, ν).

(vi) **λ-continuous [Francisco G. Arenas et al., 1997]** if $u^{-1}(T)$ is λ-closed in (M, μ) for every closed set T of (N, ν).

(vii) **g^*-continuous [Veera Kumar, 2000]** if $u^{-1}(T)$ is g^*-closed in (M, μ) for every closed set T of (N, ν).

Definition 1.3.3 Let (M, μ) and (N, ν) be the topological spaces. A map $u: (M, \mu) \to (N, \nu)$ is called

(i) **Strongly continuous [Levine, 1960]** if $u^{-1}(T)$ is both open and closed in (M, μ) for every subset T of (N, ν).

(ii) **Totally continuous [Jain, 1980]** if $u^{-1}(T)$ is clopen in (M, μ) for every open subset T of (N, ν).

(iii) **Perfectly continuous [Noiri, 1984]** if $u^{-1}(T)$ is clopen in (M, μ) for every closed subset T of (N, ν).

(iv) **Contra continuous [Dontchev, 1996]** if $u^{-1}(T)$ is closed in (M, μ) for every open set T of (N, ν).

(v) **Contra λ-continuous [Caldas et al., 2006 a]** if $u^{-1}(T)$ is λ-open in (M, μ) for every closed set T of (N, ν).

Definition 1.3.4 Let (M, μ) and (N, ν) be the topological spaces. A map $u: (M, \mu) \to (N, \nu)$ is called

(i) **α-irresolute [Maheshwari and Thakur, 1980]** if $u^{-1}(T)$ is α-closed in (M, μ) for every α-closed set T of (N, ν).

(ii) **αg-irresolute [Devi et al. 1997]** if $u^{-1}(T)$ is αg-closed in (M, μ) for every αg-closed set T of (N, ν).

(iii) **$g\alpha$-irresolute [Devi et al. 1997]** if $u^{-1}(T)$ is $g\alpha$-closed in (M, μ) for every $g\alpha$-closed set T of (N, ν).

(iv) **g^*-irresolute [Veera Kumar, 2000]** if $u^{-1}(T)$ is g^*-closed in (M, μ) for every g^*-closed set T of (N, ν).

(v) **λ-irresolute [Caldas et al., 2007 b]** if $u^{-1}(T)$ is λ-closed in (M, μ) for every λ-closed set T of (N, ν).

1.4 Closed and Open Maps in Topological Spaces

Definition 1.4.1 Let (M, μ) and (N, ν) be the topological spaces. A map $u: (M, \mu) \to (N, \nu)$ is called

(i) **closed** if $u(T)$ is closed in (N, ν) for every closed set T of (M, μ).

(ii) **g-closed [Malghan, 1982]** if $u(T)$ is g-closed in (N, ν) for every closed set T of (M, μ).

(iii) **α-closed [Mashhour et al., 1983]** if $u(T)$ is α-closed in (N, ν) for every closed set T of (M, μ).

(iv) **$g\alpha$-closed [Devi et al., 1998]** if $u(T)$ is $g\alpha$-closed in (N, ν) for every closed set T of (M, μ).

(v) **αg-closed [Devi et al., 1998]** if $u(T)$ is αg-closed in (N, ν) for every closed set T of (M, μ).

(vi) **g^*-closed [Ratnesh Kumar Saraf and Miguel Caldas, 2007]** if $u(T)$ is g^*-closed in (N, ν) for every closed set T of (M, μ).

(vii) **λ-closed [Caldas et al., 2008 a]** if $u(T)$ is λ-closed in (N, ν) for every λ-closed set T of (M, μ).

(viii) **Contra open [Baker, 1997]** if $u(T)$ is closed in (N, ν) for every open set T in (M, μ).

Definition 1.4.2 [Caldas et al., 2008 b]

A surjective map $u:(M,\mu)\to(N,\nu)$ is called a **bi-contra continuous map** if u is contra continuous and $u^{-1}(T)$ is open in (M,μ) implies T is closed in (N,ν).

Definition 1.4.3 [Caldas et al., 2008 b]

A surjective map $u:(M,\mu)\to(N,\nu)$ is called a **bi-contra-α-continuous map** if u is contra α-continuous and $u^{-1}(T)$ is open in (M,μ) implies T is α-closed in (N,ν).

Definition 1.4.4 A map $u:(M,\mu)\to(N,\nu)$ is called a/an

(i) **quotient map [Munkres, Topology, A First Course]** if u is continuous and $u^{-1}(T)$ is open in (M,μ) implies T is an open set in (N,ν).

(ii) **α-quotient map [Lellis Thivagar, 1991]** if u is α-continuous and $u^{-1}(T)$ open in (M,μ) implies T is an α-open set in (N,ν).

Definition 1.4.5 A bijective map $u:(M,\mu)\to(N,\nu)$ is called a

(i) **homeomorphism [Munkres, Topology, A First Course]** if u is both open and continuous.

(ii) **g-homeomorphism [Maki, 1991]** if u is both g-open and g-continuous. i.e., u and u^{-1} are both g-continuous.

(iii) **gc-homeomorphism [Maki, 1991]** if both u and u^{-1} are g-irresolute.