

**Certain Studies Relating to Truncated Life Tests  
Through Acceptance Sampling Procedures**

By

**S. Sutharani**

Supervisor

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**A Thesis Submitted to**

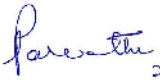
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**In Partial Fulfilment of the Requirements for the Degree of  
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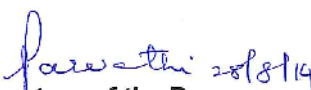
## Certificate

This is certify that the thesis entitled "**Certain Studies Relating to Truncated Life Tests Through Acceptance Sampling Procedures**" submitted to the Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for award of the degree **Doctor of Philosophy in Mathematics**, is a record of original work done by **Mrs. S. Sutharani** during the period of her study in the Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, under my supervision and guidance and the thesis has not formed the basis for the award of any Degree / Diploma / Associate ship / Fellowship or similar title of any candidate of any University.

  
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## Declaration

I hereby declare that the thesis entitled "Certain Studies Relating to Truncated Life Tests Through Acceptance Sampling Procedures" submitted to the Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, in partial fulfillment of the requirements for the award of the degree of **Doctor of Philosophy in Mathematics** is a record of original work done by me under the supervision and guidance of **Dr. (Tmt). A.R. Sudamani Ramaswamy**, M.Sc., M.Phil., Ph.D., Associate Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and it has not formed the basis for the award of any Degree / Diploma / Associate ship / Fellowship or similar title of any candidate of any University.

  
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Signature of the Candidate

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