Introduction

The study of topology emerged with the rudimentary knowledge of Set Theory, Algebra, Geometry and Analysis. General Topology has exhibited its elegance in both pure and applied aspects. In the fields of Computer aided geometric design and engineering (briefly, CAGD), Information systems, Artificial Intelligence and Image processing, one can visualize the existence of topological structures and the usage of topological properties. The concept of a topological space is concerned with generalizing the structure of sets in Euclidean spaces. Having Open sets as a powerful tool for defining topological spaces, Stone (1937) defined the notion of Regular open sets in his novel paper which related the theory of Boolean Rings to General Topology. Velicko (1968) introduced the concept of δ-open sets as a generalization of the regular open sets. This set was introduced to investigate the characterization of H-closed spaces. He observed that δ-open sets were stronger than open sets and the collection of all δ-open sets, denoted by τ_δ, is a topology on X. The semi-regularization of τ is a coarser topology τ_s, generated by the collection of all regular open sets as a base. In a semi-regular space, $\tau = \tau_s = \tau_\delta$.

Levine (1963) studied the notion of semi-open sets as a weaker form of the open sets. He also initiated the study of generalized closed sets (briefly, g-closed sets) in 1970. The idea of generalized closed sets is an efficient tool in characterizing topological spaces which satisfy weak separation axioms. Immediately after Levine's work on g-closed sets, several authors started generalizing various stronger and weaker forms of closed sets. Through the semi-regularization of the given topology τ and the related δ-closure operator (denoted by cl_δ), Donchev (1996) investigated the notion of δg-closed sets. This is a stronger form of g-closed sets and is properly placed between δ-closed and g-closed sets. He studied the concept of $T_{3/4}$-space, which reversed the relationship between δ-closed and δg-closed sets. In 2000, he developed the notions of gδ-closed and δg^*-closed sets to characterize almost weakly Hausdorff spaces. He also obtained the characterizations of almost weakly Hausdorff, T_δ and semi-regular spaces.

Georgiou (2004) developed the theory on generalization of δ-closed sets named as (Λ, δ)-closed sets using Λ-operator in terms of δ. The concept of δgs-closed sets was introduced by Park (2007), as a weaker form of Velicko's δ-closed sets. Eventually, Benchalli (2012) gave another generalization of δ-closed sets namely gδs-closed set. Among the various generalizations of δ-closed sets discussed so far, gδs-closed sets are weakest of all.

The classical paper of Zadeh (1965) comprises of the concepts of fuzzy sets and fuzzy set operations. Thereafter the paper of Chang (1968) paved way for tremendous growth of the numerous fuzzy topological concepts. Azad (1981) introduced the notion of fuzzy regular open sets in fuzzy topological spaces. Petricevic (1991) studied the concept of fuzzy δ-open sets in fuzzy topological spaces.

With these works as the motivation, yet another generalization of δ-closed sets namely λ_g^δ-closed sets is presented in this research work. This new notion is properly placed between δ-closed sets and δg^*-closed sets. Thereby, it is the nearest weaker form of δ-closed sets i.e., λ_g^δ-closed sets are much stronger than many existing generalizations of δ-closed sets. The association of λ_g^δ-closed sets with various other closed sets is presented in Figure 1 and Figure 2.

Notations:

- In all the diagrams $A \to B$ represents A implies B but not conversely and $A \not\leftrightarrow B$ represents A and B are independent of each other.
- In this Introduction part, Definitions are indicated by ■ and Results are indicated by •.

Figure 1: **Dependance Relationship**

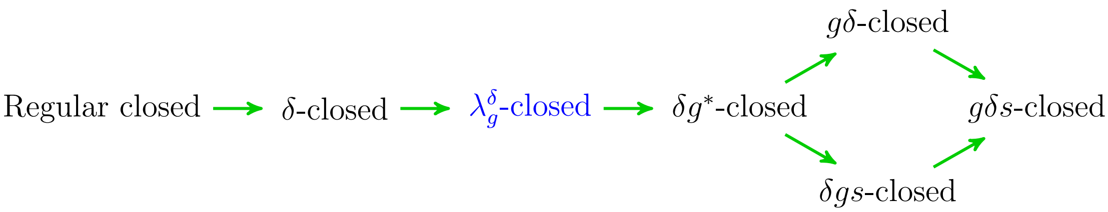
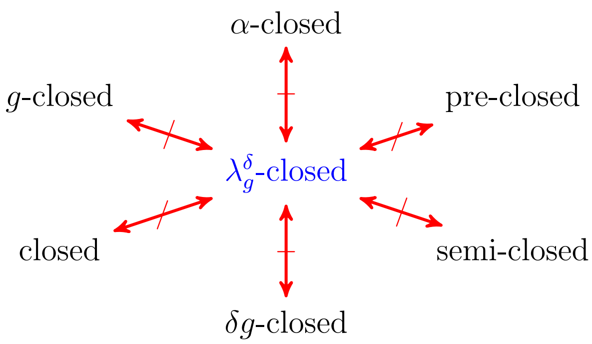


Figure 2: **Independence Relationship**



Methodology :

The research has been done by the following methods.

- Analytical method of comparing λ_g^δ-closed sets with existing closed sets.
- Producing Counter examples wherever necessary to substantiate the result.
- Analysis of preservation of topological properties by λ_g^δ-closed sets.
- Obtaining Characterization theorems.
- Apply the developed theory to suitable real life scenario and test its efficiency.

Chapter 1, deals with preliminary definitions and related concepts on various existing closed sets which are used in the remaining part of the study.

In **Chapter 2**, the notion of λ_g^δ-closed set is defined and its properties are studied. With the Real line and standard topology under consideration, the sets of the form

It is observed that λ_g^δ -closed sets are weaker than regular closed and δ-closed sets but stronger than δg^*-closed, gδ-closed, δgs-closed and gδs-closed sets.

Meanwhile, λ_g^δ-closed sets stand isolated from closed, g-closed, α-closed, pre-closed, semi-closed and δg-closed sets. Although λ_g^δ-closed sets and δg-closed sets are independent of each other, it is inciting to observe that they share a relationship in $T_{3/4}$-space.Arbitrary union of λ_g^δ-closed sets is λ_g^δ-closed, whereas finite intersection fails. This is supported by a suitable counter example.

The notions of λ_g^δ-open set, λ_g^δ-closure operator, λ_g^δ-interior operator are developed and their properties are analyzed. The family of all λ_g^δ-open sets happen to form an Alexandrov space. Subsequently, λ_g^δ-closure operator satisfies the Kuratowski closure axioms.

Important definitions and results:

■ A subset A of a topological space (X, τ) is called ***λ_g^δ-closed set*** if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is (Λ, δ)-open in X.

- Let A be a λ_g^δ-closed set in (X, τ). Then $cl_\delta(A) \setminus A$ does not contain any non-empty (Λ, δ)-closed set.
- If A is a (Λ, δ)-open set and a λ_g^δ-closed set of X then A is a δ-closed set of X.
- If every subset of X is λ_g^δ-closed then every δ-open set of X is δ-closed.
- If A is a λ_g^δ-closed and (Λ, δ)-open and F is δ-closed in X, then $A \cap F$ is δ-closed.
- If A is a λ_g^δ-closed set in X and $A \subseteq B \subseteq cl_\delta(A)$, then B is also a λ_g^δ-closed set.
- Let A be a λ_g^δ-closed set of X. Then A is δ-closed iff $cl_\delta(A) \setminus A$ is (Λ, δ)-closed.
- If every λ_g^δ-closed set is δ-closed then every singleton is (Λ, δ)-closed or δ-open.
- If A is λ_g^δ-closed then $(\Lambda, \delta)cl\{x\} \cap A \neq \phi$, for all $x \in cl_\delta\{A\}$.

- Let A be λ_g^δ-closed. If A is regular open then $pInt(A)$ and $sCl(A)$ are λ_g^δ-closed.

■ A subset A of a topological space (X, τ) is called ***λ_g^δ-open*** if its complement A^c is λ_g^δ-closed in (X, τ).

- If A is λ_g^δ-open in X then $G = X$ whenever G is (Λ, δ)-open and $int_\delta(A) \cup A^c \subseteq G$

- For each $x \in X$, either $\{x\}$ is (Λ, δ)-closed or $\{x\}$ is λ_g^δ-open in X. That is, for any topological space X, $X = (\Lambda, \delta)C(X, \tau) \cup \lambda_g^\delta O(X, \tau)$.

■ The ***λ_g^δ-closure of A*** (briefly $\lambda_g^\delta cl(A)$) in a topological space (X, τ) is defined to be the intersection of all λ_g^δ-closed sets containing A.

- If a subset A of (X, τ) is λ_g^δ-closed in X then $\lambda_g^\delta cl(A) = A$ but not conversely.

- For a topological space X with subsets A and B, the following conditions are valid:

  (i) $\lambda_g^\delta cl(\phi) = \phi$ and $\lambda_g^\delta cl(X) = X$.

  (ii) If $A \subseteq B$, then $\lambda_g^\delta cl(A) \subseteq \lambda_g^\delta cl(B)$.

  (iii) $A \subseteq \lambda_g^\delta cl(A)$.

  (iv) $\lambda_g^\delta cl(A \cup B) = \lambda_g^\delta cl(A) \cup \lambda_g^\delta cl(B)$.

  (v) $\lambda_g^\delta cl(A \cap B) \subseteq \lambda_g^\delta cl(A) \cap \lambda_g^\delta cl(B)$.

  (vi) $\lambda_g^\delta cl(\lambda_g^\delta cl(A)) = \lambda_g^\delta cl(A)$.

  (vii) For $A \subseteq X$, $\lambda_g^\delta cl(A) \subseteq cl_\delta(A)$.

- Let $A \subseteq X$. Then $x \in \lambda_g^\delta cl(A)$ iff $U \cap A \neq \phi$, for every λ_g^δ-open set U containing x.

■ The ***λ_g^δ- interior of A*** (briefly $\lambda_g^\delta int(A)$) in a topological space (X, τ) is defined to be the union of all λ_g^δ-open sets contained in A.

- If a subset A of (X, τ) is λ_g^δ-open then $\lambda_g^\delta int(A) = A$ but not conversely.

- For a topological space X with subsets A and B, the following conditions are valid:

  (i) $\lambda_g^\delta int(\phi) = \phi$ and $\lambda_g^\delta int(X) = X$.

(ii) If $A \subseteq B$, then $\lambda_g^\delta int(A) \subseteq \lambda_g^\delta int(B)$.

(iii) $\lambda_g^\delta int(A) \subseteq A$.

(iv) $\lambda_g^\delta int(A \cup B) \supseteq \lambda_g^\delta int(A) \cup \lambda_g^\delta int(B)$.

(v) $\lambda_g^\delta int(A \cap B) = \lambda_g^\delta int(A) \cap \lambda_g^\delta int(B)$.

(vi) $\lambda_g^\delta int(\lambda_g^\delta int(A)) = \lambda_g^\delta int(A)$.

(vii) For $A \subseteq X$, $\lambda_g^\delta int(A) \subseteq int_\delta(A)$.

Chapter 3 deals with various notions related to λ_g^δ-closed sets namely λ_g^δ-neighborhood, λ_g^δ-limit point, λ_g^δ-derived set, λ_g^δ-frontier, λ_g^δ-boundary, λ_g^δ-exterior and λ_g^δ-saturated set. Properties related to these concepts are studied. Additionally, λ_g^δ-open sets are characterized using the concept of grill in topological spaces.

Important definitions and results:

■ A subset N of a topological space (X, τ) is called ***λ_g^δ-neighborhood*** of $x \in X$ if there exists a λ_g^δ-open set Q such that $x \in Q \subseteq N$.

■ A subset N of $A \subseteq X$ is called a ***λ_g^δ-neighborhood of A*** if there exists a λ_g^δ-open set Q such that $A \subseteq Q \subseteq N$.

- If a subset N of a topological space X is λ_g^δ-open then N is a λ_g^δ-neighborhood of each of its points.

- If F is a λ_g^δ-closed subset of a topological space and $x \in F^c$ then there exists a λ_g^δ-neighborhood N of x such that $N \cap F = \phi$.

- In a topological space (X, τ) with $x \in X$, the following results are true.

  (i) $\lambda_g^\delta N(x) \neq \phi$.

  (ii) If $N \in \lambda_g^\delta N(x)$ then $x \in N$.

  (iii) If $N \in \lambda_g^\delta N(x)$ and $N \subseteq M$ then $M \in \lambda_g^\delta N(x)$.

  (iv) If $N \in \lambda_g^\delta N(x)$ and $M \in \lambda_g^\delta N(x)$ then $N \cap M \in \lambda_g^\delta N(x)$.

(v)If $N \in \lambda_g^\delta N(x)$ then there exists $M \in \lambda_g^\delta N(x)$ such that $M \subseteq N$ and $M \in \lambda_g^\delta N(y)$, for all $y \in M$.

- Let (X, τ) be a topological space and $x \in X$. Suppose that a collection $\mathscr{A}_x$ satisfies

  (i) $N \in \mathscr{A}_x$ such that $x \in N$

  (ii) $N, M \in \mathscr{A}_x \Rightarrow N \cap M \in \mathscr{A}_x$

  then $\mathscr{B}$ forms a basis for a topology where $\mathscr{B} = \{\phi\} \cup \{G \subseteq X | x \in G \Rightarrow$ there exists $N \in \mathscr{A}_x$ such that $x \in N \subseteq G\}$.

■ Let $x \in A \subseteq X$. Then x is a ***λ_g^δ-limit point*** of A if every λ_g^δ-neighborhood of x contains at least one point of A other than x. That is, $N \cap (A \setminus \{x\}) \neq \phi$, for all $N \in \lambda_g^\delta N(x)$.

- Let $A, B \subseteq X$. Then the following statements are valid.

  (i) $\lambda_g^\delta D(\phi) = \phi$.

  (ii) $\lambda_g^\delta D(A) \subseteq D_\delta(A)$.

  (iii)If $A \subseteq B$ then $\lambda_g^\delta D(A) \subseteq \lambda_g^\delta D(B)$.

  (iv) $\lambda_g^\delta D(A \cup B) = \lambda_g^\delta D(A) \cup \lambda_g^\delta D(B)$.

  (v) $\lambda_g^\delta D(A \cap B) \subseteq \lambda_g^\delta D(A) \cap \lambda_g^\delta D(B)$.

  (vi) $\lambda_g^\delta D(\lambda_g^\delta D(A)) \setminus A \subseteq \lambda_g^\delta D(A)$.

  (vii) $\lambda_g^\delta D(A \cup \lambda_g^\delta D(A)) \subseteq A \cup \lambda_g^\delta D(A)$.

- Let $A \subseteq X$. If A is λ_g^δ-closed then $\lambda_g^\delta D(A) \subseteq A$.

■ For a subset A of a topological space (X, τ), ***λ_g^δ-frontier*** of A is denoted by $\lambda_g^\delta F(A)$ and defined as $\lambda_g^\delta F(A) = \lambda_g^\delta cl(A) \setminus \lambda_g^\delta int(A)$.

- For a subset A of a topological space (X, τ), the following results are true.

  (i) $\lambda_g^\delta cl(A) = \lambda_g^\delta int(A) \cup \lambda_g^\delta F(A)$.

(ii) $\lambda_g^\delta int(A) \cap \lambda_g^\delta F(A) = \phi$.

(iii) $\lambda_g^\delta F(A) = \lambda_g^\delta cl(A) \cap \lambda_g^\delta cl(X \setminus A)$.

(iv) $\lambda_g^\delta F(A)$ is δ-closed.

(v) $\lambda_g^\delta F(A) = \lambda_g^\delta F(X \setminus A)$.

(vi) $Fr_\delta(\lambda_g^\delta F(A)) \subseteq \lambda_g^\delta F(A)$.

- Let $A \subseteq B$ and $\lambda_g^\delta int(B) = \phi$ then $\lambda_g^\delta F(A) \subseteq \lambda_g^\delta F(B)$.

■ For a subset A of a topological space (X, τ), $\boldsymbol{\lambda_g^\delta}$***-boundary*** of A is denoted by $\lambda_g^\delta B(A)$ and defined as $\lambda_g^\delta B(A) = A \setminus \lambda_g^\delta int(A)$.

- For a subset A of a topological space (X, τ), the following results are true.

(i) $\lambda_g^\delta B(\phi) = \phi$.

(ii) $\lambda_g^\delta B(X) = \phi$.

(iii) $A = \lambda_g^\delta int(A) \cup \lambda_g^\delta B(A)$.

(iv) If A is $\lambda_g^\delta - open$ then $\lambda_g^\delta B(A) = \phi$.

(v) $\lambda_g^\delta int(A) \cap \lambda_g^\delta B(A) = \phi$.

(vi) $\lambda_g^\delta B(\lambda_g^\delta int(A)) = \phi$.

■ For a subset A of a topological space (X, τ), $\boldsymbol{\lambda_g^\delta}$***-exterior*** of A is denoted by $\lambda_g^\delta E(A)$ and defined as $\lambda_g^\delta E(A) = X \setminus \lambda_g^\delta cl(A)$.

- For a subset A of a topological space (X, τ), the following results are true.

(i) $Ext_\delta(A) \subseteq \lambda_g^\delta E(A)$.

(ii) $\lambda_g^\delta E(X) = \phi$.

(iii) $\lambda_g^\delta E(\phi) = X$.

(iv) $\lambda_g^\delta E(A) = \lambda_g^\delta int(X \setminus A)$.

(v) If $A \subseteq B$ then $\lambda_g^\delta E(A) \supseteq \lambda_g^\delta E(B)$.

(vi) $\lambda_g^\delta E(A \cup B) \subseteq \lambda_g^\delta E(A) \cup \lambda_g^\delta E(B)$.

(vii) $\lambda_g^\delta E(A \cap B) \supseteq \lambda_g^\delta E(A) \cap \lambda_g^\delta E(B)$.

(viii) $\lambda_g^\delta E(\lambda_g^\delta E(A)) = \lambda_g^\delta int(\lambda_g^\delta cl(A))$.

(ix) $\lambda_g^\delta E(A) = \lambda_g^\delta E(X \setminus \lambda_g^\delta E(A))$.

(x) $X = \lambda_g^\delta int(A) \cup \lambda_g^\delta E(A) \cup \lambda_g^\delta F(A)$.

(xi) $\lambda_g^\delta int(A) \subseteq \lambda_g^\delta E(\lambda_g^\delta E(A))$.

- $Ext_\delta(A) \cup Ext_\delta(B) \subseteq \lambda_g^\delta E(A \cap B)$.

■ A subset A of a topological space (X, τ) is said to be ***λ_g^δ-saturated*** if $\lambda_g^\delta cl(\{x\}) \subseteq A$ for every $x \in A$.

- Every λ_g^δ-closed set is a λ_g^δ-saturated set but not conversely.

■ Let $(X, \tau^{\lambda_g^\delta}, \mathcal{G})$ be a grill λ_g^δ-space. We define a function $\psi_{\lambda_g^\delta \mathcal{G}} : P(X) \to P(X)$, denoted by $\psi_{\lambda_g^\delta \mathcal{G}}(A, \tau^{\lambda_g^\delta})$ or $\psi_{\lambda_g^\delta \mathcal{G}}(A)$ (for $A \in P(X)$), called the ***λ_g^δ-operator*** associated with the grill $\mathcal{G}$ and $\tau^{\lambda_g^\delta}$, and is defined by $\psi_{\lambda_g^\delta \mathcal{G}}(A) = \{x \in X \mid U \cap A \in \mathcal{G}$, for all λ_g^δ-open set containing $x\}$.

- Let $(X, \tau^{\lambda_g^\delta})$ be a λ_g^δ-space and $\mathcal{G}$, $\mathcal{H}$ be two grills on X. Then for a subset A of X, the following conditions are valid:

(i) $\mathcal{G} \subseteq \mathcal{H} \Rightarrow \psi_{\lambda_g^\delta \mathcal{G}}(A) \subseteq \psi_{\lambda_g^\delta \mathcal{H}}(A)$.

(ii) $\psi_{\lambda_g^\delta (\mathcal{G} \cup \mathcal{H})}(A) = \psi_{\lambda_g^\delta \mathcal{G}}(A) \cup \psi_{\lambda_g^\delta \mathcal{H}}(A)$.

- Let $(X, \tau^{\lambda_g^\delta}, \mathcal{G})$ be a grill λ_g^δ-space. Then for any two subsets A and B of X, the following conditions hold:

(i) $\psi_{\lambda_g^\delta \mathcal{G}}(A) \subseteq \psi_{\lambda_g^\delta \mathcal{G}}(B)$, if $A \subseteq B$.

(ii) $\psi_{\lambda_g^\delta \mathcal{G}}(A \cup B) = \psi_{\lambda_g^\delta \mathcal{G}}(A) \cup \psi_{\lambda_g^\delta \mathcal{G}}(B)$.

(iii) $\psi_{\lambda_g^\delta \mathcal{G}}(A \cap B) \subseteq \psi_{\lambda_g^\delta \mathcal{G}}(A) \cap \psi_{\lambda_g^\delta \mathcal{G}}(B)$.

(iv) $\psi_{\lambda_g^\delta \mathcal{G}}(A) = \phi$, if $A \notin \mathcal{G}$

(v) $\psi_{\lambda_g^\delta \mathcal{G}}(A) \subseteq \psi_{\delta \mathcal{G}}(A)$.

(vi) $\psi_{\lambda_g^\delta \mathcal{G}}(A) \subseteq \lambda_g^\delta cl(A) \subseteq cl_\delta(A)$.

- Let $(X, \tau^{\lambda_g^\delta}, \mathcal{G})$ be a grill λ_g^δ-space. Then for any two subsets A and B of X, the following conditions hold:

  (i) $\psi_{\lambda_g^\delta \mathcal{G}}(A) \setminus \psi_{\lambda_g^\delta \mathcal{G}}(B) \subseteq \psi_{\lambda_g^\delta \mathcal{G}}(A \setminus B)$.

  (ii) If $B \notin \mathcal{G}, \psi_{\lambda_g^\delta \mathcal{G}}(A \cup B) = \psi_{\lambda_g^\delta \mathcal{G}}(A) = \psi_{\lambda_g^\delta \mathcal{G}}(A \setminus B)$.

  (iii) If $(A \setminus B) \cup (B \setminus A) \notin \mathcal{G}$ then $\psi_{\lambda_g^\delta \mathcal{G}}(A) = \psi_{\lambda_g^\delta \mathcal{G}}(B)$.

- Let $(X, \tau^{\lambda_g^\delta}, \mathcal{G})$ be a grill λ_g^δ-space and $A \subseteq X$. Then

  (i) $\phi \notin \mathcal{G} \Rightarrow \psi_{\lambda_g^\delta \mathcal{G}}(\phi) = \phi$.

  (ii) $\psi_{\lambda_g^\delta \mathcal{G}}(A) \setminus \psi_{\lambda_g^\delta \mathcal{G}}(\psi_{\lambda_g^\delta \mathcal{G}}(A)) \subseteq \psi_{\lambda_g^\delta \mathcal{G}}(A \setminus \psi_{\lambda_g^\delta \mathcal{G}}(A))$.

  (iii) $\psi_{\lambda_g^\delta \mathcal{G}}(A) \setminus \psi_{\lambda_g^\delta \mathcal{G}}(B) = \psi_{\lambda_g^\delta \mathcal{G}}(A \setminus B) \setminus \psi_{\lambda_g^\delta \mathcal{G}}(B)$.

  (iv) $A \cap \psi_{\lambda_g^\delta \mathcal{G}}(X) = \phi$, for every λ_g^δ-open set A with $A \notin \mathcal{G}$.

- Let $(X, \tau^{\lambda_g^\delta}, \mathcal{G})$ be a grill λ_g^δ-space and $A \subseteq X$ be λ_g^δ-closed. Then $\psi_{\lambda_g^\delta \mathcal{G}}(A) \subseteq A$.

- Let $(X, \tau^{\lambda_g^\delta}, \mathcal{G})$ be a grill λ_g^δ-space and $A \subseteq X$ be λ_g^δ-closed. Then $\psi_{\lambda_g^\delta \mathcal{G}}(\psi_{\lambda_g^\delta \mathcal{G}}(A)) \subseteq \psi_{\lambda_g^\delta \mathcal{G}}(A)$.

- Let $(X, \tau^{\lambda_g^\delta}, \mathcal{G})$ be a grill λ_g^δ-space and $A \subseteq X$. If V is λ_g^δ-open set containing x then $\psi_{\lambda_g^\delta \mathcal{G}}(A) = \psi_{\lambda_g^\delta \mathcal{G}}(V \cap A)$.

- Let $(X, \tau^{\lambda_g^\delta}, \mathcal{G})$ be a grill λ_g^δ-space. Then a function $\zeta_{\lambda_g^\delta \mathcal{G}} : P(X) \to P(X)$ defined by $\zeta_{\lambda_g^\delta \mathcal{G}}(A) = A \cup \psi_{\lambda_g^\delta \mathcal{G}}(A)$, satisfies Kuratowski closure axioms, for every λ_g^δ-open set A.

In **Chapter 4**, five new types of separation spaces are introduced and their properties are analyzed. The newly introduced spaces are as follows.
A topological space (X, τ) is said to be a

(i) $_{\lambda_g^\delta}\boldsymbol{T_\delta}$ ***-space*** if every λ_g^δ-closed subset of (X, τ) is δ-closed in (X, τ).

(ii) $_{\delta g^*}\boldsymbol{T_{\lambda_g^\delta}}$ ***-space*** if every δg^*-closed subset of (X, τ) is λ_g^δ-closed in (X, τ).

(iii) $_{g\delta}\boldsymbol{T_{\lambda_g^\delta}}$ ***-space*** if every $g\delta$-closed subset of (X, τ) is λ_g^δ-closed in (X, τ).

(iv) $_{\delta gs}T_{\lambda_g^\delta}$ ***-space*** if every δgs-closed subset of (X, τ) is λ_g^δ-closed in (X, τ).

(v) $_{g\delta s}T_{\lambda_g^\delta}$ ***-space*** if every $g\delta s$-closed subset of (X, τ) is λ_g^δ-closed in (X, τ).

These spaces are useful in reversing the dependence relationship between λ_g^δ-closed set and various other types of closed sets discussed in Chapter 1. Under the respective separation space, the dependency turns into equality and thereby they coincide. The following figures represent the relationship of these five spaces with few already existing spaces and also the interrelationship among these spaces.

Figure 3: **Relationship of the Separation Spaces with Already Existing Spaces**

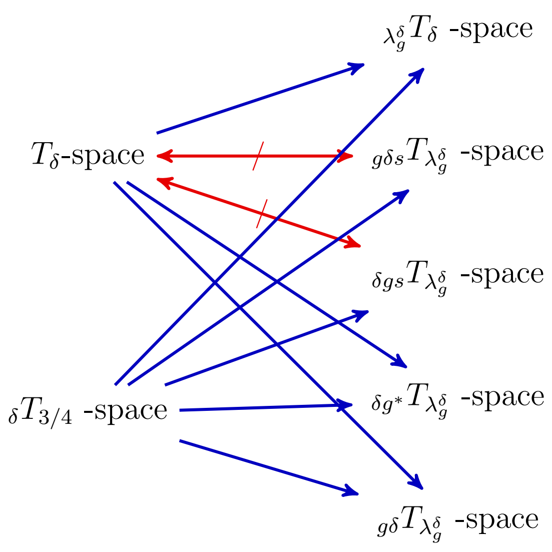
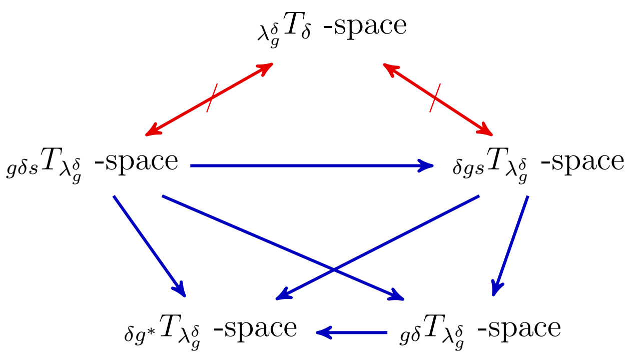


Figure 4: **Interrelationship between the Separation Spaces**



Important definitions and results:

- Every δg-closed set is a λ_g^δ-closed set in the following spaces:
  - (i) $T_{3/4}$ -space,
  - (ii) ${}_{\delta gs}T_{\lambda_g^\delta}$ -space,
  - (iii) ${}_{\delta g^*}T_{\lambda_g^\delta}$ -space,
  - (iv) ${}_{g\delta s}T_{\lambda_g^\delta}$ -space.
- If a topological space (X,τ) is ${}_{g\delta}T_{\lambda_g^\delta}$ as well as ${}_{\lambda_g^\delta}T_\delta$ then it is a T_δ -space.
- If a topological space (X,τ) is ${}_{\delta gs}T_{\lambda_g^\delta}$ as well as ${}_{\lambda_g^\delta}T_\delta$ then it is a $T_{3/4}$ -space.
- For a topological space (X,τ) which is ${}_{\lambda_g^\delta}T_\delta$, the following conditions are equivalent:
  - (i) X is almost weakly Hausdorff;
  - (ii) Every singleton of (X,τ) is δ-closed or δ-open;
  - (iii) Every singleton of (X,τ) is λ_g^δ-closed or λ_g^δ-open.

Chapter 5 is on various types of continuities using λ_g^δ-closed sets. Here, λ_g^δ-continuity, quasi λ_g^δ-continuity, perfectly λ_g^δ-continuity, totally λ_g^δ-continuity, strongly λ_g^δ-continuity and contra λ_g^δ-continuity are defined with their properties being discussed. The association between these continuities is studied with the support of counter examples and the association is presented below.

Figure 5: **Dependence Relationship of λ_g^δ-continuity**

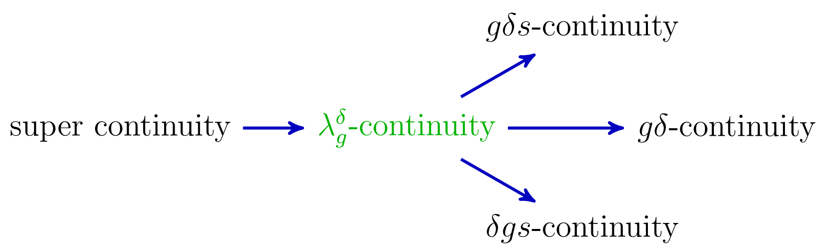
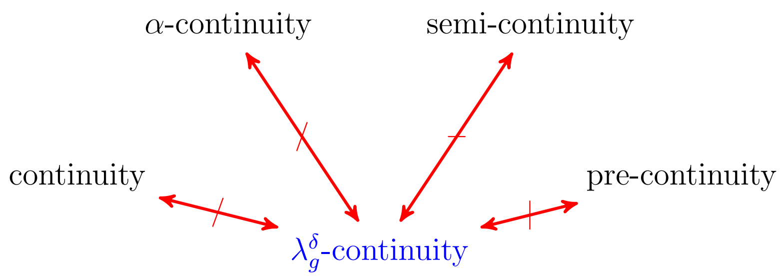


Figure 6: **Isolation of λ_g^δ-continuity**



Important definitions and results:

■ A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $\boldsymbol{\lambda_g^\delta}$***-continuous*** if the inverse image of every open set in (Y, σ) is λ_g^δ-open in (X, τ).

- If a function $f : X \to Y$ is λ_g^δ-continuous then for each $x \in X$ and each open V of $f(x)$, there exists a λ_g^δ-set U containing x such that $f(U) \subseteq V$.

- If for any function $f : X \to Y$ and $A \subseteq X$ such that $f(\lambda_g^\delta cl(A)) \subseteq \lambda_g^\delta cl(f(A))$ then the inverse image of every δ-closed set in Y is λ_g^δ-closed in X.

- For a function $f : X \to Y$, the following conditions are true:

  (i) f is λ_g^δ-continuous. $\Rightarrow$

  (ii) $\lambda_g^\delta cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, for each $B \subseteq Y$. $\Rightarrow$

  (iii) f$(\lambda_g^\delta cl(A)) \subseteq cl(f(A))$, for each $A \subseteq X$. $\Rightarrow$

  (iv) $f^{-1}(int(B)) \subseteq \lambda_g^\delta int(f^{-1}(B))$, for each $B \subseteq Y$.

- The composition of two λ_g^δ-continuous functions need not be a λ_g^δ-continuous function.

- For functions $f : (X, \tau) \to (Y, \sigma)$, $g : (Y, \sigma) \to (Z, \eta)$ and $g \circ f : (X, \tau) \to (Z, \eta)$, we have the following:

(i) If f is λ_g^δ-continuous and g is super continuous then $g \circ f$ is λ_g^δ-continuous,

(ii) If f is super continuous and g is super continuous then $g \circ f$ is λ_g^δ-continuous,

(iii) If f is λ_g^δ-continuous and g is continuous then $g \circ f$ is λ_g^δ-continuous(resp. $g\delta$-continuous, $g\delta s$-continuous and δgs-continuous).

- If $f : (X, \tau) \to (Y, \sigma)$ is λ_g^δ-continuous then for each $x \in X$ and each open set V containing $f(x)$, there is a λ_g^δ-open(resp. $g\delta$-open, $g\delta s$-open and δgs-open) set U containing x such that $f(U) \subseteq V$.

■ A function $f : (X, \tau) \to (Y, \sigma)$ is called ***quasi λ_g^δ-continuous*** if the inverse image of every λ_g^δ-open set in (Y, σ) is open in (X, τ).

- For a function $f : X \to Y$, the following are equivalent:

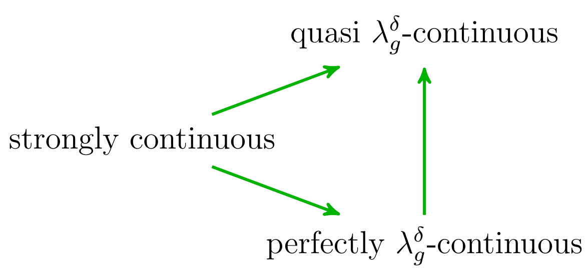
(i) quasi λ_g^δ-continuous;

(ii) $f^{-1}(B)$ is closed in X, for every λ_g^δ-closed B in Y.

(iii) For each $x \in X$ and each λ_g^δ-open set B containing $f(x)$, there exists an open set A containing x such that $f(A) \subseteq B$.

■ A function $f : (X, \tau) \to (Y, \sigma)$ is called ***perfectly λ_g^δ-continuous*** if the inverse image of every λ_g^δ-open set in (Y, σ) is clopen in (X, τ).

Figure 7: **Relationship between strongly continuous, quasi λ_g^δ-continuous and perfectly λ_g^δ-continuous functions**



- Let (X, τ) be a partition space, (Y, σ) be a topological space and $f : (X, \tau) \to (Y, \sigma)$ be any function. Then the following are equivalent.

(i) f is perfectly λ_g^δ-continuous;

(ii) f is quasi λ_g^δ-continuous.

- Let $f : X \to Y$ and $g : Y \to Z$ be any two functions. Then their composition $g \circ f : X \to Z$ is

  (i) perfectly λ_g^δ-continuous if g is perfectly λ_g^δ-continuous and f is continuous,

  (ii) continuous if g is λ_g^δ-continuous and f is quasi λ_g^δ-continuous.

  (iii) continuous if g is λ_g^δ-continuous and f is perfectly λ_g^δ-continuous.

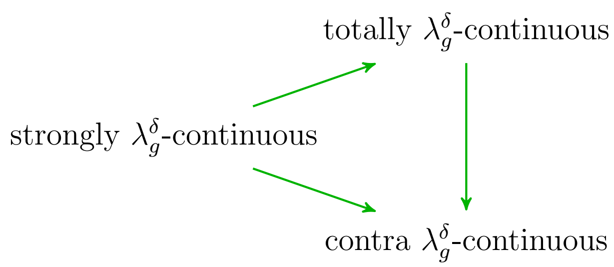
■ A function $f : (X, \tau) \to (Y, \sigma)$ is called

(i) ***totally λ_g^δ-continuous*** if the inverse image of every open subset of (Y, σ) is λ_g^δ-clopen in (X, τ).

(ii) ***strongly λ_g^δ-continuous*** if the inverse image of every subset of (Y, σ) is λ_g^δ-clopen in (X, τ).

(iii) ***contra λ_g^δ-continuous*** if the inverse image of every closed set of (Y, σ) is λ_g^δ-open in (X, τ).

Figure 8: **Relationship between strongly λ_g^δ-continuous, totally λ_g^δ-continuous and contra λ_g^δ-continuous functions**



- The composition of two contra λ_g^δ-continuous functions need not be a contra λ_g^δ-continuous function.

- For topological spaces $(X,\tau),(Y,\sigma),(Z,\eta), f:(X,\tau)\to(Y,\sigma), g:(Y,\sigma)\to(Z,\eta)$ and $g\circ f:(X,\tau)\to(Z,\eta)$ the following results are true.

  (i) If f is quasi λ_g^δ-continuous and g is super continuous then $g\circ f$ is continuous.

  (ii) If f is a quasi λ_g^δ-continuous and g is a totally λ_g^δ-continuous then $g\circ f$ is a perfectly continuous.

  (iii) If f is perfectly λ_g^δ-continuous and g is super continuous then $g\circ f$ is perfectly continuous.

  (iv) If f is continuous and g is perfectly λ_g^δ-continuous (resp. quasi λ_g^δ-continuous) then $g\circ f$ is perfectly λ_g^δ-continuous (resp. quasi λ_g^δ-continuous).

  (v) If f is super continuous and g is perfectly λ_g^δ-continuous (resp. quasi λ_g^δ-continuous) then $g\circ f$ is perfectly λ_g^δ-continuous (resp. quasi λ_g^δ-continuous)

  (vi) If f is a perfectly λ_g^δ-continuous and g is a strongly λ_g^δ-continuous then $g\circ f$ is a strongly continuous.

  (vii) If f is a strongly λ_g^δ-continuous and g is any function then $g\circ f$ is a strongly λ_g^δ-continuous.

  (viii) If f is a quasi λ_g^δ-continuous and g is a contra λ_g^δ-continuous then $g\circ f$ is a contra continuous function.

  (ix) If f is a contra λ_g^δ-continuous and g is a continuous (resp. super continuous) then $g\circ f$ is a contra λ_g^δ-continuous.

  (x) If f is a totally λ_g^δ-continuous and g is a continuous (resp. super continuous) then $g\circ f$ is a totally λ_g^δ-continuous.

λ_g^δ-irresoluteness and contra λ_g^δ-irresoluteness are introduced and their properties relating to composition are analyzed. Characteristics of these functions by inducing surjection, bijection on various types of continuity are presented. The inheritance of the nature of the domain is analyzed under these functions.

■ A function $f:(X,\tau)\to(Y,\sigma)$ is called

(i) ***λ_g^δ-irresolute*** if $f^{-1}(B)$ is λ_g^δ-closed in (X,τ) for every λ_g^δ-closed set B in (Y,σ).

(ii) ***contra λ_g^δ-irresolute*** if $f^{-1}(B)$ is λ_g^δ-closed in (X,τ) for every λ_g^δ-open set B in (Y,σ).

- For a function $f:(X,\tau)\to(Y,\sigma)$ with (Y,σ) being an almost weakly Hausdorff space, every λ_g^δ-continuous function is λ_g^δ-irresolute.

- Composition of two λ_g^δ-irresolute functions is a λ_g^δ-irresolute function.

- For topological spaces $(X,\tau),(Y,\sigma),(Z,\eta), f:(X,\tau)\to(Y,\sigma)$, $g:(Y,\sigma)\to(Z,\eta)$ and $g\circ f:(X,\tau)\to(Z,\eta)$ the following results are true.

  (i) If f is a λ_g^δ-irresolute function and g is a λ_g^δ-continuous (resp. totally λ_g^δ-continuous, strongly λ_g^δ-continuous, contra λ_g^δ-continuous, contra λ_g^δ-irresolute) function then $g\circ f$ is a λ_g^δ-continuous (resp. totally λ_g^δ-continuous, strongly λ_g^δ-continuous, contra λ_g^δ-continuous, contra λ_g^δ-irresolute) function.

  (ii) If f is a contra λ_g^δ-irresolute (resp. strongly continuous, quasi λ_g^δ-continuous, perfectly λ_g^δ-continuous) function and g is a λ_g^δ-irresolute function then $g\circ f$ is a contra λ_g^δ-irresolute (resp. strongly continuous, quasi λ_g^δ-continuous, perfectly λ_g^δ-continuous) function.

- If $f:(X,\tau)\to(Y,\sigma)$ is λ_g^δ-irresolute then

  (i) For $A\subseteq X, f(\lambda_g^\delta cl(A))\subseteq cl_\delta(f(A))$.

  (ii) For $B\subseteq Y, \lambda_g^\delta cl(f^{-1}(B))\subseteq f^{-1}(cl_\delta(A))$.

- If $f:(X,\tau)\to(Y,\sigma)$ is surjective, λ_g^δ-irresolute and closed with (X,τ) being an almost weakly Hausdorff space then every λ_g^δ-closed set is closed in (Y,σ).

In **Chapter 6**, notions of λ_g^δ-open and λ_g^δ-closed functions are taken for study and their behaviours are characterized in almost weakly Hausdorff space. Two types of homeomorphisms namely λ_g^δ-homeomorphism and $\lambda_g^{\delta*}$-homeomorphism are developed and their properties are obtained. It is observed that the set of all $\lambda_g^{\delta*}$-homeomorphisms form a group under composition of functions.

Important definitions and results:

■ A function $f : (X, \tau) \to (Y, \sigma)$ is

(i) $\boldsymbol{\lambda_g^\delta}$***-open*** if $f(A)$ is λ_g^δ-open for every open set A in X.

(ii) $\boldsymbol{\lambda_g^\delta}$***-closed*** if $f(A)$ is λ_g^δ-closed for every closed set A in X.

- Let $f : (X, \tau) \to (Y, \sigma)$ be a map with $f(int(A)) \subseteq \lambda_g^\delta int(f(A))$, for every $A \subseteq X$. Then f is λ_g^δ-open.

- Let $f : (X, \tau) \to (Y, \sigma)$ be λ_g^δ-closed then $\lambda_g^\delta cl(f(A)) \subseteq f(cl(A))$, for each $A \subseteq X$.

- For topological spaces (X, τ), (Y, σ), (Z, η), $f : (X, \tau) \to (Y, \sigma)$, $g : (Y, \sigma) \to (Z, \eta)$ and $g \circ f : (X, \tau) \to (Z, \eta)$ the following results are true.

  (i) If $g \circ f$ is λ_g^δ-open and f is continuous, surjective then g is λ_g^δ-open,

  (ii) If $g \circ f$ is open and g is λ_g^δ-continuous, injective then f is λ_g^δ-open.

- For a bijective function $f : (X, \tau) \to (Y, \sigma)$, the following are equivalent.

  (i) f is λ_g^δ-open;

  (ii) f is λ_g^δ-closed;

  (iii) f^{-1} is λ_g^δ-continuous.

- A bijective function $f : (X, \tau) \to (Y, \sigma)$ is λ_g^δ-closed iff for each subset B in Y and each open set U in X containing $f^{-1}(B)$, there exists a λ_g^δ-open set V in Y such that

  (i) $B \subseteq V$ and

  (ii) $f^{-1}(V) \subseteq U$.

- Let $f : (X, \tau) \to (Y, \sigma)$ be any function with (Y, σ) being an almost weakly Hausdorff space. Then every λ_g^δ-closed function is a closed function.

- For topological spaces $(X,\tau),(Y,\sigma),(Z,\eta)$, $f:(X,\tau)\to(Y,\sigma)$, $g:(Y,\sigma)\to(Z,\eta)$ and $g\circ f:(X,\tau)\to(Z,\eta)$ the following are true.

  (i) If f and g are both λ_g^δ-closed with (Y,σ) being an almost weakly Hausdorff space then $g\circ f$ is λ_g^δ-closed.

  (ii) If f is closed and g is λ_g^δ-closed then $g\circ f$ is λ_g^δ-closed.

  (iii) If f is closed and g is λ_g^δ-closed with (Y,σ) being an almost weakly Hausdorff space then $g\circ f$ is closed.

- Let $f:X\to Y$ and $g:Y\to Z$ such that $g\circ f:X\to Z$ is λ_g^δ-closed. Then the following are true.

  (i) If f is continuous, surjective then g is λ_g^δ-closed,

  (ii) If g is λ_g^δ-irresolute, injective then f is λ_g^δ-closed,

  (iii) If f is λ_g^δ-continuous, surjective and X is an almost weakly Hausdorff space then g is λ_g^δ-closed,

  (iv) If f is $g\delta$-continuous, surjective and X is an almost weakly Hausdorff space then g is λ_g^δ-closed,

  (v) If g is strongly λ_g^δ-continuous, injective then f is closed.

- If a function $f:X\to Y$ is λ_g^δ-open then $f^{-1}(\lambda_g^\delta cl(B))\subseteq cl(f^{-1}(B))$, for every $B\subseteq Y$.

- If a function $f:X\to Y$ is λ_g^δ-open then for each $x\in X$ and for each neighborhood U of $x\in X$, there exists a λ_g^δ-neighborhood W of $f(x)$ such that $W\subseteq f(U)$.

■ A bijective function $f:(X,\tau)\to(Y,\sigma)$ is called $\boldsymbol{\lambda_g^\delta}$***-homeomorphism*** if f is both λ_g^δ-continuous and λ_g^δ-open.

- If a bijective function $f:X\to Y$ is λ_g^δ-continuous then the following statements are equivalent.

  (i) f is λ_g^δ-open;

(ii) f is λ_g^δ-homeomorphism;

(iii) f is λ_g^δ-closed.

■ A bijective function $f : (X,\tau) \to (Y,\sigma)$ is called ***$\lambda_g^{\delta*}$-homeomorphism*** if both f and f^{-1} are λ_g^δ-irresolute. The set of all $\lambda_g^{\delta*}$-homeomorphisms of (X,τ) onto itself is denoted by $\lambda_g^{\delta*}h(X,\tau)$.

- Composition of two $\lambda_g^{\delta*}$-homeomorphisms is a $\lambda_g^{\delta*}$-homeomorphism.

- The set of all $\lambda_g^{\delta*}$-homeomorphisms of (X,τ) denoted by $\lambda_g^{\delta*}h(X,\tau)$ forms a group under composition of functions.

- If $f : X \to Y$ is a $\lambda_g^{\delta*}$-homeomorphism then

(i) For $A \subseteq X$, $f(\lambda_g^\delta cl(A)) \subseteq cl_\delta(f(A))$,

(ii) For $B \subseteq Y$, $\lambda_g^\delta cl(f^{-1}(B)) \subseteq f^{-1}(cl_\delta(B))$,

(iii) For $B \subseteq Y$, $f^{-1}(\lambda_g^\delta cl(B)) \subseteq cl_\delta(f^{-1}(B))$,

(iv) For $A \subseteq X$, $\lambda_g^\delta cl(f(A)) \subseteq f(cl_\delta(A))$.

In **Chapter 7**, various other interesting separation axioms namely λ_g^δ-compactness, λ_g^δ-connectedness, λ_g^δ-Lindelof and λ_g^δ-G_i, $i = 1,2$ axioms are exhibited. λ_g^δ-compactness is characterized using the finite intersection property and λ_g^δ-connectedness is characterized using λ_g^δ-Frontier.

■ A collection $\mathcal{A}$ of a topological space (X,τ) is said to cover X (or) to be a covering of X if the union of elements of $\mathcal{A}$ is equal to X. It is said to be a ***λ_g^δ-open covering of X*** if its elements are λ_g^δ-open subsets of (X,τ).

■ A non-empty collection $\{A_i | i \in I\}$ of λ_g^δ-open sets in a topological space (X,τ) is called a ***λ_g^δ-open cover*** of a subset B of (X,τ) if $B \subseteq \cup\{A_i | i \in I\}$.

■ A subset B of a topological space (X,τ) is called ***λ_g^δ-compact*** if every λ_g^δ-open cover of X has a finite subcover.

■ A subset B of a topological space (X, τ) is called ***λ_g^δ-compact relative to X*** if for every collection $\{A_i | i \in I\}$ of λ_g^δ-open subsets of (X, τ) such that $B \subseteq \cup\{A_i | i \in I\}$ there exists a finite subset I_0 of I such that $B \subseteq \cup\{A_i | i \in I_0\}$.

- A λ_g^δ-closed subset of a λ_g^δ-compact, λ_g^δ-topological space X is λ_g^δ-compact relative to X.
- Let $f : X \to Y$ be a surjective, λ_g^δ-continuous function. If X is λ_g^δ-compact then Y is compact.
- Let $f : X \to Y$ be a surjective, quasi λ_g^δ-continuous function. If X is compact then Y is λ_g^δ-compact.
- Let $f : X \to Y$ be a surjective, perfectly λ_g^δ-continuous function. If X is compact then Y is λ_g^δ-compact.
- If $f : X \to Y$ is λ_g^δ-irresolute and $B \subseteq X$ is λ_g^δ-compact relative to X then $f(B)$ is λ_g^δ-compact relative to Y.
- A topological space X is λ_g^δ-compact iff each family of λ_g^δ-closed subsets of X with the finite intersection property has a non-empty intersection.

■ A topological space (X, τ) is called ***λ_g^δ-connected*** if X cannot be expressed as a union of two disjoint, non-empty, λ_g^δ-open sets.

- For a topological space X, the following are equivalent.
  - (i) X is λ_g^δ-connected;
  - (ii) X and ϕ are the only subsets of X which are both λ_g^δ-open and λ_g^δ-closed;
  - (iii) Each λ_g^δ-continuous function of X into a discrete space Y with atleast two points is a constant function.
- Let $f : X \to Y$ be a surjective, λ_g^δ-continuous function. If X is λ_g^δ-connected then Y is connected.

- Let $f : X \rightarrow Y$ be a surjective, λ_g^δ-irresolute function. If X is λ_g^δ-connected then Y is λ_g^δ-connected.

- Every topological space which is both ${}_{\lambda_g^\delta}T_\delta$ and connected is λ_g^δ-connected.

■ A subset A of a topological space (X, τ) is called

(i) ***λ_g^δ-regular closed*** if $A = \lambda_g^\delta cl(\lambda_g^\delta int(A))$.

(ii) ***λ_g^δ-regular open*** if $A = \lambda_g^\delta int(\lambda_g^\delta cl(A))$.

(iii) ***λ_g^δ-regular*** if it is both λ_g^δ-regular closed and λ_g^δ-regular open.

- A subset A of a topological space (X, τ) is λ_g^δ-regular iff $\lambda_g^\delta Fr(A) = \phi$.

- For a topological space (X, τ), the following are equivalent:

  (i) X is λ_g^δ-connected.

  (ii) X and ϕ are the only λ_g^δ-regular subsets of X.

  (iii) Each λ_g^δ-continuous function of X into a discrete space Y with atleast two points is a constant function.

  (iv) Every non-empty proper subset has a non-empty λ_g^δ-Frontier.

- Let $f : X \rightarrow Y$ be a λ_g^δ-open, λ_g^δ-closed(resp. δ-open, δ-closed) injection. If Y is λ_g^δ-connected then X is also λ_g^δ-connected.

- If $f : X \rightarrow Y$ is a totally λ_g^δ-continuous function from a λ_g^δ-connected space X to Y then Y has the indiscrete topology.

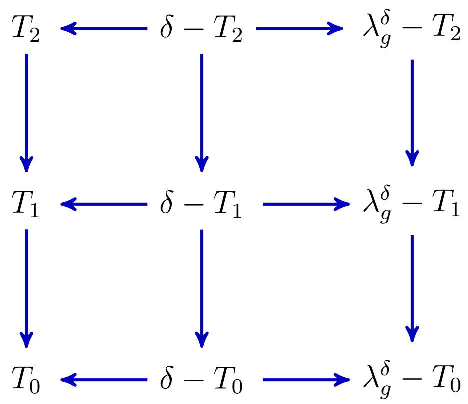
- If $f : X \rightarrow Y$ is a strongly λ_g^δ-continuous bijective function and Y is a topological space with atleast two points then X is not λ_g^δ-connected.

- If a topological space X is almost weakly Hausdorff and connected then it is λ_g^δ-connected.

■ A topological space (X, τ) is ***λ_g^δ-Lindelof*** if every λ_g^δ-open cover of X contains a countable subcover.

- A surjective, λ_g^δ-irresolute image of a λ_g^δ-Lindelof space is λ_g^δ-Lindelof.
- A surjective, λ_g^δ-continuous image of a λ_g^δ-Lindelof is Lindelof.
- A surjective, quasi λ_g^δ-continuous image of a Lindelof space is λ_g^δ-Lindelof. Separation axioms using λ_g^δ-closed sets namely λ_g^δ-T_i, $i = 0, 1, 2$ are introduced. The properties and characterizations of λ_g^δ-closed sets are studied under δ-T_i, $i = 0, 1, 2$ and λ_g^δ-T_i, $i = 0, 1, 2$ axioms. Further the relationships of λ_g^δ-T_i, δ-T_i and T_i, $i = 0, 1, 2$ axioms are also found and presented in the Figure below.

Figure 9: **Relation between the Separation Axioms**



- If (X, τ) is δ-T_0, then λ_g^δ-closures of distinct points are distinct.
- If (X, τ) is δ-T_1 space, every singleton is λ_g^δ-closed.
- If (X, τ) is δ-T_2 space, for each $x \in X$, $\{x\} = \cap\{\lambda_g^\delta cl(U) | U$ is λ_g^δ-open in X and $x \in U\}$.

■ A topological space (X, τ) is said to be $\boldsymbol{\lambda_g^\delta}$-$\boldsymbol{T_0}$ **space** if for each pair of distinct points $x, y \in X$, there exists a λ_g^δ-open set containing one point but not the other.

- A topological space (X, τ) is λ_g^δ-T_0 iff λ_g^δ-closures of distinct points are distinct.

■ A topological space (X, τ) is said to be $\boldsymbol{\lambda_g^\delta}$-$\boldsymbol{T_1}$ **space** if for each pair of distinct points $x, y \in X$, there exist λ_g^δ-open sets U and V such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.

- If singletons are λ_g^δ-closed then (X,τ) is λ_g^δ-T_1 but not conversely.

■ A topological space is said to be **λ_g^δ-T_2 space** if for any pair of distinct points x and y in X, there exist disjoint λ_g^δ-open sets U and V such that $x \in U$ and $y \in V$.

- In a topological space (X,τ), the following are equivalent.

  (i) (X,τ) is λ_g^δ-T_2.

  (ii) For each $x \neq y$ in X, there exists a λ_g^δ-open set U such that $x \in U$ and $y \notin \lambda_g^\delta cl\{U\}$.

  (iii) For each $x \in X$, $\{x\} = \cap\{\lambda_g^\delta cl(U)|$ U is λ_g^δ-open set in X and $x \in U\}$.

■ A topological space (X,τ) is called a ***$\lambda_g^\delta G_1$-space*** if for any point $p \in X$ and any connected subset M of X with $p \notin M$, there exist λ_g^δ-open sets U and V such that $p \in U, M \subseteq V, U \cap M = \phi$ and $\{p\} \cap V = \phi$.

- If every connected subset of X is λ_g^δ-closed then for any two disjoint connected subsets M and N of X, there exist λ_g^δ-open sets U and V such that $M \subseteq U$, $N \subseteq V$, $U \cap N = \phi$ and $M \cap V = \phi$.

- If for any two disjoint connected subsets M and N of X, there exist λ_g^δ-open sets U and V such that $M \subseteq U$, $N \subseteq V$, $U \cap N = \phi$ and $V \cap M = \phi$ then X is a $\lambda_g^\delta G_1$-space.

- Let X be a topological space and Q be its subspace. Then a subset A of Q is λ_g^δ-open in Q if A can be written as $A = Q \cap K$ where K is λ_g^δ-open in X.

- Every δ-open subspace Q of a $\lambda_g^\delta G_1$-space X is $\lambda_g^\delta G_1$.

- A bijective, continuous and λ_g^δ-irresolute image of a $\lambda_g^\delta G_1$-space is a $\lambda_g^\delta G_1$-space.

■ A topological space (X,τ) is called ***$\lambda_g^\delta G_2$-space*** if for every connected set F and a point $p \notin F$, there exist λ_g^δ-open sets U and V such that $p \in U$, $F \subseteq V$ and $U \cap V = \phi$.

- Every $\lambda_g^\delta G_2$-space is a $\lambda_g^\delta T_2$-space.

- A δ-open subspace of a $\lambda_g^\delta G_2$-space is $\lambda_g^\delta G_2$.
- If a topological space X is $\lambda_g^\delta G_2$ then for any point $p \in X$ and any connected subset M not containing p, $\lambda_g^\delta cl(U) \cap M = \phi$, where U is a λ_g^δ-open neighborhood of p.

Chapter 8 deals with an application of λ_g^δ-closed sets in biometric analysis - face recognition. A face recognition system is a computer application capable of identifying or verifying a person from a digital image or a video frame from a video source. In biometrics, face recognition plays a phenomenal role. Face recognition is broadly classified into face identification and face verification. Many researches have been conducted on both face identification and face verification, with greater focus on the latter. In this work, three type of closed sets for assessing the similarity in a facial feature under protocols namely Regular closed, δ-closed and λ_g^δ-closed are compared. The protocols are tested against accuracy and time consumption for attaining the results using MATLAB. Among these protocols, it is proved that λ_g^δ-closed based protocol gives better results in accuracy aspect than the other two.

The study of fuzzy sets was initiated by Zadeh. Since the generalization of the usual notion of a set into a fuzzy set by Zadeh in his classic paper of 1965, many abstract structures were generalized using fuzzy sets. Fuzzy topological spaces were introduced by Chang in 1968. Azad introduced the concept of fuzzy regular open sets and fuzzy regular closed sets in fuzzy topological spaces. Petricevic (1991) introduced the concept of fuzzy δ-open sets and fuzzy δ-closed sets in fuzzy topological spaces. In 2004, Georgiou presented the notion of (Λ, δ)-closed sets in general topology. Thereafter this notion grasped higher significance due its nature of being partially δ-open and partially δ-closed. The closure and interior operators related to these sets are analyzed. Theory on somewhat fuzzy λ_g^δ-continuous function is developed with the properties being analyzed.

Important definitions and results:

- A fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$ is called a ***fuzzy*** $\boldsymbol{(\Lambda, \delta)}$***-closed*** (briefly $F(\Lambda, \delta)$-closed) set if $A = K \wedge L$, where K is a $F\Lambda_\delta$-set and L is a fuzzy

δ-closed set.

- The following are equivalent for a fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$.

  (i) A is $F(\Lambda, \delta)$-closed;

  (ii) $A = K \wedge cl_\delta(A)$, where K is a $F\Lambda_\delta$-set;

  (iii) $A = F\Lambda_\delta(A) \wedge cl_\delta(A)$;

  (iv) $A = F\Lambda_\delta(A) \wedge L$, where L is a fuzzy δ-closed set.

- Let $(X, \mathcal{F})$ be a fuzzy topological space. Then

  (i)Arbitrary intersection of $F(\Lambda, \delta)$-closed sets is $F(\Lambda, \delta)$-closed in $(X, \mathcal{F})$.

  (ii)Arbitrary union of $F(\Lambda, \delta)$-open sets is $F(\Lambda, \delta)$-open in $(X, \mathcal{F})$.

- The following are equivalent for a fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$.

  (i) A is $F(\Lambda, \delta)$-open;

  (ii) $A = K \vee int_\delta(A)$, where K is a FV_δ-set;

  (iii) $A = FV_\delta(A) \vee int_\delta(A)$;

  (iv) $A = FV_\delta(A) \vee L$, where L is a fuzzy δ-open set.

■ A fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$ is called a ***fuzzy $\boldsymbol{\lambda_g^\delta}$-closed set*** (briefly $F\lambda_g^\delta$-closed set) if $cl_\delta(A) \leq U$, whenever $A \leq U$ and U is a fuzzy (Λ, δ)-open set.

- A fuzzy subset A is $F\lambda_g^\delta$-closed iff $A\bar{q}B \Rightarrow cl_\delta(A)\bar{q}B$, for every fuzzy (Λ, δ)-closed set B of $(X, \mathcal{F})$.

- If A is $F\lambda_g^\delta$-closed in $(X, \mathcal{F})$ and x_r is a fuzzy point of $(X, \mathcal{F})$ such that $x_r q cl_\delta(A)$ then $cl_\delta(x_r) q A$.

■ A fuzzy subset A of a fuzzy topological space $(X, \mathcal{F})$ is called a ***fuzzy $\boldsymbol{\lambda_g^\delta}$-open set*** (briefly $F\lambda_g^\delta$-open set) if $U \leq int_\delta(A)$, whenever $U \leq A$ and U is a fuzzy (Λ, δ)-closed set.

■ The ***Fuzzy*** $\boldsymbol{\lambda_g^\delta}$***-closure*** (briefly $F\lambda_g^\delta$-closure) of a fuzzy subset A in a fuzzy topological space $(X, \mathcal{F})$ is defined as

$$F\lambda_g^\delta cl(A) = \wedge\{P : A \leq P \text{ and } P \in F\lambda_g^\delta C(X, \mathcal{F})\}$$

and denoted by $F\lambda_g^\delta cl(A)$.

- The ***Fuzzy*** $\boldsymbol{\lambda_g^\delta}$***-interior*** (briefly $F\lambda_g^\delta$-interior) of a fuzzy subset A in a fuzzy topological space $(X, \mathcal{F})$ is defined as

$$F\lambda_g^\delta int(A) = \vee\{Q : Q \leq A \text{ and } Q \in F\lambda_g^\delta O(X, \mathcal{F})\}$$

and denoted by $F\lambda_g^\delta int(A)$.

■ A fuzzy function $f : (X, \mathcal{F}) \to (Y, \mathcal{G})$ is called ***somewhat fuzzy*** $\boldsymbol{\lambda_g^\delta}$***-continuous*** if for $B \in \mathcal{G}$ and $f^{-1}(B) \neq 0$, there exists a $F\lambda_g^\delta$-open set in $(X, \mathcal{F})$ such that $A \neq 0$ and $A \leq f^{-1}(B)$. That is, $F\lambda_g^\delta int(f^{-1}(B)) \neq 0$.

- Composition of a somewhat fuzzy λ_g^δ-continuous function and a fuzzy continuous function (resp. fuzzy δ-continuous function) is a somewhat fuzzy λ_g^δ-continuous function.

- If f is a somewhat fuzzy λ_g^δ-continuous function then for a fuzzy closed set B of $(Y, \mathcal{G})$, such that $f^{-1}(B) \neq 1$, there exists a fuzzy λ_g^δ-closed set $A \neq 1$ of X such that $f^{-1}(B) \leq A$.

- Let $f : (X, \mathcal{F}) \to (Y, \mathcal{G})$ be a one-one and onto fuzzy function. Then the following are equivalent.

  (i) f is somewhat fuzzy λ_g^δ-continuous;

  (ii) If A is a fuzzy closed set in $(Y, \mathcal{G})$ such that $f^{-1}(A) \neq 1$, then there exists a fuzzy λ_g^δ-closed set $B \neq 1$ in $(X, \mathcal{F})$ such that $B \geq f^{-1}(A)$;

  (iii) If A is a fuzzy λ_g^δ-dense set in $(X, \mathcal{F})$ then $f(A)$ is a fuzzy λ_g^δ-dense set in $(Y, \mathcal{G})$.