Chapter I

CHAPTER- I

Supra Soft Sets and Supra Soft Topological Spaces

Section 1.1

Soft Sets and Soft Topological Spaces

Definition: 1.1.1

Let X be an initial universe and E be a set of parameters. Let P(X) denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a **soft set over** , where F is mapping given by $F: A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e -approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \emptyset$ i.e.

$F_A = \{F(e): e \in A \subseteq E, F: A \rightarrow P(X)\}$. The family of all these soft sets denoted by $SS(X)_A$.

Definition: 1.1.2

Let F_A, $G_B \in SS(X)_E$. Then F_A is **soft subset** of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if $A \subseteq B$, and $F(e) \subseteq G(e)$, $\forall e \in A$.

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A, $G_B \tilde{\supseteq} F_A$.

Definition: 1.1.3

Two soft subset F_A and G_B over a common universe set X are said to be **soft equal** if F_A is soft subset of G_B and G_B is soft subset of F_A.

Definition: 1.1.4

The **complement of a soft set** (F, A), denoted by $(F, A)'$, is defined by $(F, A)' = (F', A)$, $F': A \rightarrow P(X)$ is mapping given by $F'(e) = X - F(e), \forall e \in A$ and F' is called soft complement of the function F.

Clearly $(F')'$ is the same as F and $((F, A)')' = (F, A)$.

Definition: 1.1.5

The **difference of two soft sets** (F, E) and (G, E) over the common universe X, denoted by $(F, E) - (G, E)$ is the soft set (H, E) where for all $e \in E$, $H(e) = F(e) - G(e)$.

Definition: 1.1.6

Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as **x belongs to the soft set (F, E)** whenever $x \in F(e)$ for all $e \in E$.

Definition: 1.1.7

A soft set (F, A) over X is said to be a **NULL soft set** denoted by $\tilde{\phi}$ or ϕ_A if for all $e \in A, F(e) = \emptyset$ (null set).

Definition: 1.1.8

A soft set (F, A) over X is said to be an **absolute soft set** denoted by $\tilde{A}$ or X_A if for all $e \in A, F(e) = X$. Clearly we have $X'_A = \emptyset_A$ and $\emptyset'_A = X_A$.

Definition: 1.1.9

The **Union of two soft sets** (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e), e \in A - B \\ G(e), e \in B - A \\ F(e) \cup G(e), e \in A \cap B. \end{cases}$$

Definition: 1.1.10

The **intersection of two soft sets** (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.

Note : We consider only soft sets (F, E) over a universe X in which all the parameter set E are same. We denote the family of these soft sets by $SS(X)_E$.

Definition: 1.1.11

Let I be an arbitrary indexed set and $L = \{(F_i, E), i \in I\}$ be a **subfamily of** $SS(X)_E$.

The **union of L** is the soft set (H, E), where $H(e) = \cup_{i\in I} F_i(e)$ for each e ∈ E. we write $\cup_{i\in I} (F_i, E) = (H, E)$.

The **intersection of L** is the soft set (M, E), where $M(e) = \cap_{i\in I} F_i(e)$ for each e ∈ E. we write $\cap_{i\in I} (F_i, E) = (M, E)$.

Definition: 1.1.12

Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then τ ⊆ $SS(X)_E$ is called a **soft topology on X** if

1. $\widetilde{X}, \widetilde{\phi} \in \tau$, where $\widetilde{\phi}(e) = \phi$ and $\widetilde{X}(e) = X, \forall e \in E$,
2. the union of any number of soft sets in τ belongs to τ,
3. the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a **soft topological space over X .**

Definition: 1.1.13

Let (X, τ, E) be a soft topological space. A soft set (F, A) over X is said to be **closed soft set in X**, if it relative complement (F, A)′ is open soft set.

Definition: 1.1.14

Let (X, τ, E) be a soft topological space. The members of τ are said to be **open soft sets in X.** We denote the set of all open soft sets over X by OS(X, τ, E), or when there can be no confusion by OS(X) and the set of all closed soft sets by CS(X, τ, E), or CS(X).

Definition: 1.1.15

Let (X, τ, E) be a soft topological space and (F, A) ∈ $SS(X)_E$.The **soft closure of (F, A),** denoted by cl(F, A) is the intersection of all closed soft super sets of

(F, A).Clearly $cl(F, A)$ is th smallest closed soft set over X which contains (F, A) i.e $cl(F,A) = \tilde{\cap}\{(H,C):(H,C) \text{is closed soft set and } (F,A) \tilde{\subseteq} (H,C)\}$.

Definition: 1.1.16

Let (X, τ, E) be a soft topological space and $(F, A) \in SS(X)_E$.The **soft interior of (G, B)**, denoted by $int(G, B)$ is the union of all open soft subsets of (G, B).Clearly $int(G, B)$ is the largest open soft set over X which contained in (G, B) i.e $int(G,B) = \tilde{\cup}\{(H,C):(H,C) \text{is an open soft set and } (H,C) \tilde{\subseteq} (G,B)\}$.

Definition: 1.1.17

The soft set $(F, E) \in SS(X)_E$ is called a **soft point in X_E** if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \emptyset$ for each $e' \in E - \{e\}$, and the soft point (F, E) is denoted by x_e.

Theorem: 1.1.18

The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it.

Definition: 1.1.19

The soft point x_e **is said to be belonging to the soft set (G, A)**, denoted by $x_e \tilde{\in} (G, A)$, if for the element $e \in A$, $F(e) \subseteq G(e)$.

Definition: 1.1.20

A soft set (G, B) in a soft topological space (X, τ, E) is called a **soft neighbourhood of the soft point** $x_e \in X_E$ if there exists an open soft set (H, C) such that $x_e \tilde{\in} (H, C) \tilde{\subseteq} (G, B)$.

A soft set (G, B) in a soft topological space (X, τ, E) is called a **soft neighbourhood of the soft(F, A)** if there exist an open soft set(H, C) such that $(F, A) \tilde{\in} (H, C) \tilde{\subseteq} (G, B)$. The neighbourhood system of a soft point x_e denoted by $N_\tau(x_e)$, is the family of all its neighbourhoods.

Theorem: 1.1.21

Let (X, τ, E) be a soft topological space. A soft point $e_x \tilde{\in} cl(F, A)$ if and only if each soft neighbourhood of e_x intersects (F, A).

Definition: 1.1.22

Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. Define $\tau_{(F,E)} = \{(G, E) \tilde{\cap} (F, E): (G, E) \in \tau\}$, which is soft topology on (F, E). This soft topology is called a **soft relative topology** of τ on (F, E), and $[(F, E), \tau_{(F,E)}]$ is called a **soft subspace** of (X, τ, E).

Definition: 1.1.23

Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets, $u: X \rightarrow Y$ and $p: A \rightarrow B$ be mappings. Let $f_{pu}: SS(X)_A \rightarrow SS(Y)_B$ be a mapping. Then;

If $(F, A) \in SS(X)_A$. Then the **image of (F, A) under f_{pu}**, written as $f_{pu} = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that

$$f_{pu}(F)(b) = \begin{cases} \bigcup_{x \in p^{-1}(b) \cap A} u(F(a)), & p^{-1}(b) \cap A \neq \phi, \\ \phi, & otherwise. \end{cases}$$

for all $b \in B$.

If $(G, B) \in SS(Y)_B$. Then the **inverse image of (G, B) under f_{pu}**, written as $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that

$$f_{pu}^{-1}(G)(a) = \begin{cases} \cup^{-1}(G(p(a))), & p(a) \in B, \\ \phi, & otherwise. \end{cases}$$

for all $a \in A$.

The soft function f_{pu} is called surjective if p and u are surjective, also is said to be injective if p and u are injective.

Definition: 1.1.24

Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces and $f_{pu}: SS(X)_A \to SS(Y)_B$ be a function. Then

The function f_{pu} is called **continuous soft** (cts-soft) if $f_{pu}^{-1}(G, B) \in \tau_1$ $\forall (G, B) \in \tau_2$.

The function f_{pu} is called **open soft** if $f_{pu}(G, A) \in \tau_2$ $\forall (G, A) \in \tau_1$.

Definition: 1.1.25

Let (X, τ, E) be a soft topological space and $x, y \in X$ such that $x \neq y$. Then (X, τ, E) is called a **soft Hausdorff space** or **T_2 space** if there exist open soft sets (F, E) and (G, E) such that $x \in (F, E)$, $y \in (G, E)$ and $(F, E) \tilde{\cap} (G, E) = \emptyset$.

Section 1.2

Supra soft topological spaces

Definition: 1.2.1

Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\mu \subseteq SS(X)_E$ is called **supra soft topology** on X with a fixed set E if

1. $\widetilde{X}, \widetilde{\phi} \in \mu$,
2. the union of any number of soft sets in μ belongs to μ.

The triplet (X, μ, E) is called a **supra soft topological space** or (**supra soft spaces**) over X .

Remark: 1.2.2

Every soft topological space is supra soft topological space, but the converse is not true in general as shown in the following example.

Example: 1.2.3

Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1, e_2\}$ and $\mu = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E)\}$ where $(F_1, E), (F_2, E)$ are soft sets over X defined as follows:

$F_1(e_1) = \{h_1, h_2\}$, $F_1(e_2) = \{h_2, h_3\}$,

$F_2(e_1) = \{h_1, h_3\}$, $F_2(e_2) = \{h_1, h_2\}$.

Then (X, μ, E) is supra soft topology, but it is not soft topology.

Definition: 1.2.4

Let (X, τ, E) be a soft topological space and (X, μ, E) be a supra soft topological space. We say that **μ is a supra soft topology associated with τ** if $\tau \subset \mu$.

Definition: 1.2.5

Let (X, μ, E) be a supra soft topological space over X, then the members of μ are said to be **supra open soft sets** in X. We denote the set of all supra open soft sets over X by $SOS(X, \mu, E)$, or when there can be no confusion by $SOS(X)$ and the set of all supra closed soft sets by $SCS(X, \mu, E)$ or $SCS(X)$.

Definition: 1.2.6

Let (X, μ, E) be a supra soft topological space. A soft set (F, A) over X is said to be **supra closed soft set** in X, if its relative complement $(F, A)'$ is supra open soft set.

Definition: 1.2.7

The soft set $(F, E) \in SS(X)_E$ is called **supra soft point** in X_E, denoted by x_e, if there exist $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \emptyset$ for each $e' \in E - \{e\}$.

Definition: 1.2.8

The supra soft point x_e is said to be **belonging to the soft set** (G, A), denoted by $x_e \tilde{\in} (G, A)$, if for the element $e \in A, F(e) \subseteq G(e)$.

Definition: 1.2.9

A soft set (G, E) in a supra soft topological space (X, μ, E) is called **supra soft neighbourhood** (briefly: **supra nbd**) of the supra soft point $x_e \tilde{\in} X_E$ if there exist a supra open soft set (H, A) such that $x_e \tilde{\in} (H, E) \tilde{\subseteq} (G, E)$. The supra soft neighbourhood system of a supra soft point x_e, denoted by supra $N_\mu(x_e)$, is the family of all its supra soft neighbourhoods.

Definition: 1.2.10

Let (X, μ, E) be a supra soft topological space over and $(F, E) \in SS(X)_E$. Then the **supra soft interior** of (G, E), denoted by $int^s(G, E)$ is the soft union of all supra open soft subsets of (G, E). Clearly $int^s(G, E)$ is the largest supra open soft set over X which contained in (G, E) i.e

$$int^s(G, E) = \tilde{\cup} \{(H, E): (H, E) \text{ is the supra open soft set and } (H, E) \tilde{\subseteq} (G, E)\}.$$

Definition: 1.2.11

Let (X, μ, E) be a supra soft topological space over and $(F, E) \in SS(X)_E$. Then the **supra soft closure** of (F, E), denoted by $cl^s(F, E)$ is the soft intersection of all supra closed super soft sets of (F, E). Clearly $cl^s(F, E)$ is the smallest supra closed soft set over X which contains (F, E) i.e

$$cl^s(F, E) = \tilde{\cap} \{(H, E): (H, E) \text{ is the supra closed soft set and } (F, E) \tilde{\subseteq} (H, E)\}.$$

Definition: 1.2.12

Let (X, μ, E) be a supra soft topological space over and $(G, E) \in SS(X)_E$. Then $x_e \in SS(X)_E$ is called **supra limit soft point** of (G, E) if $((G, E) - x_e) \tilde{\cap} (H, E) \neq \emptyset$ $\forall (H, E) \in SOS(X)$. The set of all supra limit soft points of (F, E) is called the supra soft derived of (F, E) and denoted by $d^s(F, E)$ or F_E^{sd}.

Theorem: 1.2.13

Let (X, μ, E) be a supra soft topological space over and $(F, E), (G, E) \in SS(X)_E$. Then

1. $cl^s(F_E) \tilde{\cup} cl^s(G_E) \tilde{\subseteq} cl^s((F_E) \tilde{\cup} (G_E))$.

2. $d^s(F_E) \tilde{\cup} d^s(G_E) \tilde{\subseteq} d^s((F_E) \tilde{\cup} (G_E))$.

3. $int^s((F_E) \tilde{\cap} (G_E)) \tilde{\subseteq} int^s(F_E) \tilde{\cap} int^s(G_E)$.

Remark: 1.2.14

The equality of Theorem 1.2.13 is not true in general as shown in the following examples.

Example: 1.2.15

1. Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$ and $\mu = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined as follows:

$F_1(e) = \{h_1, h_3\}$, $F_2(e) = \{h_2, h_4\}$, $F_3(e) = \{h_1, h_3, h_4\}$.

Then μ defines a supra soft topology on X . Let (G, E) and (H, E) be two soft sets over X defined by

$G(e) = \{h_1, h_3\}$ and $H(e) = \{h_2\}$.

Then $d^s((G_E)\ \widetilde{\cup}\ (H_E))\ \widetilde{\cup}\ d^s(G_E)\ \widetilde{\cup}\ d^s(H_E)$.

2. Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$ and $\mu = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined as follows:

$F_1(e) = \{h_1, h_3\}$, $F_2(e) = \{h_2, h_4\}$, $F_3(e) = \{h_1, h_3, h_4\}$.

Then μ defines a supra soft topology on X . Let (G, E) and (H, E) be two soft sets over X defined by

$G(e) = \{h_1, h_3\}$ and $H(e) = \{h_2\}$.

Then $d^s((G_E)\ \widetilde{\cup}\ (H_E))\ \widetilde{\cup}\ d^s(G_E)\ \widetilde{\cup}\ d^s(H_E)$.

3. Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e\}$ and $\mu = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined as follows:

$F_1(e) = \{h_1, h_3\}$, $F_2(e) = \{h_2, h_4\}$, $F_3(e) = \{h_1, h_3, h_4\}$.

Then μ defines a supra soft topology on X . Let (G, E) and (H, E) be two soft sets over X defined by

$G(e) = \{h_1, h_3, h_4\}$ and $H(e) = \{h_2, h_3, h_4\}$.

Then $int^s(G_E)\ \widetilde{\cap}\ int^s(H_E) int^s\ \widetilde{\cup}\ ((G_E)\ \widetilde{\cap}\ (H_E))$.

Section 1.3

Different kinds of subsets of Supra Soft topological spaces

Definition: 1.3.1

Let (X, μ, E) be a supra soft topological space. A mapping $\gamma: SS(X)_E \to SS(X)_E$ is said to be an operation on $SOS(X)$ if $F_E \widetilde{\subseteq} \gamma(F_E)$ $\forall$ $F_E \in SOS(X)$.The collection of all **supra γ-open soft sets** is denoted by

$SOS(\gamma) = \{F_E : F_E \widetilde{\subseteq} \gamma(F_E),\ F_E \in SS(X)_E\}$. Also, the complement of supra γ- open soft set is called **supra γ- closed soft set**, i. e

$SCS(\gamma) = \{F_E : F_E \text{ is a supra } \gamma - \text{open soft},\ F_E \in SS(X)_E\}$ is the family of all supra γ- closed soft sets.

Definition: 1.3.2

Let (X, μ, E) be a supra soft topological space. Different cases of γ-operation on $SS(X)_E$ are as follows:

1. If $\gamma = int^s(cl^s)$, then γ is called **supra pre-open soft operator**. We denote the set of all supra pre-open sets by $SPOS(X, \mu, E)$, or when there can be no confusion by $SPOS(X)$ and the set of all supra pre-closed soft sets by $SPCS(X, \mu, E)$, or $SPCS(X)$.

2. If $\gamma = int^s(cl^s(int^s))$, then γ is called **supra α-open soft operator**. We denote the set of all supra α -open sets by $S\alpha OS(X, \mu, E)$, or $S\alpha OS(X)$ and the set of all supra α -closed soft sets by $S\alpha CS(X, \mu, E)$, or $S\alpha CS(X)$.

3. If $\gamma = cl^s(int^s)$, then γ is called **supra semi-open soft operator**. We denote the set of all supra semi-open sets by $SSOS(X, \mu, E)$, or $SSOS(X)$ and the set of all supra semi-closed soft sets by $SSCS(X, \mu, E)$, or $SSCS(X)$.

4. If $\gamma = cl^s(int^s(cl^s))$, then γ is called **supra β-open soft operator**. We denote the set of all supra β-open sets by $S\beta OS(X, \mu, E)$, or $S\beta OS(X)$ and the set of all supra β -closed soft sets by $S\beta CS(X, \mu, E)$, or $S\beta CS(X)$.

Theorem: 1.3.3

Let (X, μ, E) be a supra soft topological space and $\gamma: SS(X)_E \to SS(X)_E$ is said to be an operation on $SOS(X)$.

If $\gamma \in \{int^s(cl^s), int^s(cl^s(int^s)), cl^s(int^s), cl^s(int^s(cl^s))\}$. Then

1. Arbitrary soft union of supra γ-open soft sets is supra γ-open soft.

2. Arbitrary soft intersection of supra γ-closed soft sets is supra γ-closed soft.

Proof:

1. We give the proof for the case of supra pre-open soft operator i.e $\gamma = int^s(cl^s)$. Let $\{F_{jE} : j \in J\} \subseteq SPOS(X)$. Then $\forall j \in J$, $F_{jE} \tilde{\subseteq} int^s\left(cl^s(F_{jE})\right)$. It follows that $\bigcup_j F_{jE} \tilde{\subseteq} \bigcup_j int^s(cl^s(F_{jE})) \tilde{\subseteq} int^s(\bigcup_j cl^s(F_{jE})) \tilde{\subseteq} int^s(cl^s(\bigcup_j F_{jE}))$.

Hence $\bigcup_j F_{jE} \in SPOS(X)$ $\forall j \in J$. The rest of the proof is similar.

Remark: 1.3.4

The soft intersection of two supra pre-open (resp. supra β-open, supra α-open, supra semi-open) soft sets need not to be supra pre-open (supra β-open, supra α-open, supra semi-open) as shown in the following examples.

Example: 1.3.5

1. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined as follows:

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_1\}$,

$F_2(e_1) = \{h_1, h_2\}$, $F_2(e_2) = \{h_2, h_3\}$,

$F_3(e_1) = \{h_2, h_3\}$, $F_3(e_2) = \{h_2, h_3\}$.

Then μ defines a supra soft topology on X. Hence the soft sets (G, E) and (H, E) which defines as follows:

$G(e_1) = \{h_1, h_3\}$, $G(e_2) = \{h_1, h_3\}$,

$H(e_1) = \{h_2, h_3\}$, $H(e_2) = \{h_2, h_3\}$, are supra pre-open soft sets of (X, μ, E), but their soft intersection $(G, E) \tilde{\cap} (H, E) = (M, E)$, where $M(e_1) = \{h_3\}$, $M(e_2) = \{h_3\}$, is not pre-open soft set.

2. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined as follows:

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_1\}$,

$F_2(e_1) = \{h_1, h_2\}$, $F_2(e_2) = \{h_2, h_3\}$,

$F_3(e_1) = \{h_2, h_3\}$, $F_3(e_2) = \{h_2, h_3\}$.

Then μ defines a supra soft topology on X. Hence the soft sets (G, E) and (H, E) which defines as follows:

$G(e_1) = \{h_1, h_3\}$, $G(e_2) = \{h_1, h_3\}$,

$H(e_1) = \{h_2, h_3\}$, $H(e_2) = \{h_2, h_3\}$,

are supra β-open soft sets of (X, μ, E), but their soft intersection $(G, E) \tilde{\cap} (H, E) = (M, E)$, where $M(e_1) = \{h_3\}$, $M(e_2) = \{h_3\}$, is not β -open soft set.

3. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$ where $(F_1, E), (F_2, E)$ are soft sets over X defined as follows:

$F_1(e_1) = \{h_1, h_2\}$, $F_1(e_2) = \{h_2\}$,

$F_1(e_1) = \{h_1, h_3\}$, $F_2(e_2) = \{h_2, h_3\}$.

Then μ defines a supra soft topology on X. Hence the soft sets $(F_1, E), (F_2, E)$ are supra α-open soft sets of (X, μ, E), but their soft intersection $(F_1, E) \tilde{\cap} (F_2, E) = (M, E)$ where $M(e_1) = \{h_1\}$, $M(e_2) = \{h_2\}$, is not α-open soft set.

4. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E)\}$ where $(F_1, E), (F_2, E)$ are soft sets over X defined as follows:

$F_1(e_1) = \{h_1, h_2\}$, $F_1(e_2) = \{h_2\}$,

$F_1(e_1) = \{h_1, h_3\}$, $F_2(e_2) = \{h_2, h_3\}$.

Then μ defines a supra soft topology on X. Hence the soft sets (G, E) and (H, E) which defines as follows:

$G(e_1) = \{h_1, h_2\}$, $G(e_2) = \{h_2, h_3\}$,

$H(e_1) = \{h_1, h_3\}$, $H(e_2) = \{h_2, h_3\}$,

are supra semi-open soft sets of (X, μ, E), but their soft intersection $(G, E) \tilde{\cap} (H, E) = (M, E)$, where $M(e_1) = \{h_1\}$, $M(e_2) = \{h_2\}$, is not semi-open soft set.

Theorem: 1.3.6

In a supra soft topological space (X, μ, E) the following statements hold,

1. every supra open (resp. closed) soft set is supra pre-open (resp. pre-closed) soft.

2. every supra open (resp. closed) soft set is supra semi-open (resp. semi-closed) soft.

3. every supra open (resp. closed) soft set is supra α-open (resp. α-closed) soft.

4. every supra open (resp. closed) soft set is supra β-open (resp. β-closed) soft.

Proof:

The assertion is proved in the case of supra open soft sets and the other cases are similar.

1. Let $(F, E) \in SOS(X)$. Then $int^s(F, E) = (F, E)$. Since $(F, E) \tilde{\subseteq} cl^s(F, E)$, then $(F, E) \tilde{\subseteq} int^s(cl^s(F, E))$. Therefore, $(F, E) \in SPOS(X)$.

2. Let $(F, E) \in SOS(X)$. Then $int^s(F, E) = (F, E)$. Since $(F, E) \tilde{\subseteq} cl^s(F, E)$, then $(F, E) \tilde{\subseteq} cl^s(int^s(F, E))$. Thus, $(F, E) \in SSOS(X)$.

3. Let $(F, E) \in SOS(X)$. Then $int^s(F, E) = (F, E)$. Since $(F, E) \tilde{\subseteq} cl^s(F, E)$, then $(F, E) \tilde{\subseteq} int^s(cl^s(F, E)) = int^s(cl^s(int^s(F, E)))$. Hence $(F, E) \in S\alpha OS(X)$.

4. Let $(F,E) \in SOS(X)$. Then $int^s(F,E) = (F,E)$. Since $(F,E) \tilde{\subseteq} cl^s(F,E)$, then $(F,E) \tilde{\subseteq} int^s(cl^s(F,E))$. Hence $(F,E) \tilde{\subseteq} cl^s(F,E) \tilde{\subseteq} cl^s(int^s(cl^s(F,E)))$. Therefore, $(F,E) \in S\beta OS(X)$.

Remark: 1.3.7

The converse of the theorem 1.3.6 is not true in general as shown in the following example.

Example: 1.3.8

1. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined as follows:

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_1\}$,

$F_2(e_1) = \{h_1, h_2\}$, $F_2(e_2) = \{h_2, h_3\}$,

$F_3(e_1) = \{h_2, h_3\}$, $F_3(e_2) = \{h_2, h_3\}$.

Then μ defines a supra soft topology on X. Hence the soft set (G, E), which defined by

$G(e_1) = \{h_1, h_3\}$, $G(e_2) = \{h_1, h_3\}$, is a supra pre-open soft set of (X, μ, E), but it is not supra open soft.

2. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{\tilde{X}, \tilde{\emptyset}, (F_1, E), (F_2, E)\}$ where $(F_1, E), (F_2, E)$ are soft sets over X defined as follows:

$F_1(e_1) = \{h_1, h_2\}$, $F_1(e_2) = \{h_2\}$,

$F_1(e_1) = \{h_1, h_3\}$, $F_2(e_2) = \{h_2, h_3\}$.

Then μ defines a supra soft topology on X. Hence the soft set (G, E) where

$G(e_1) = \{h_1, h_2\}$, $G(e_2) = \{h_2, h_3\}$, is a supra semi-open soft set of (X, μ, E), but it is not supra open soft.

3. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{\tilde{X}, \tilde{\emptyset}, (F_1, E), (F_2, E)\}$ where $(F_1, E), (F_2, E)$ are soft sets over X defined as follows:

$F_1(e_1) = \{h_1, h_2\}$, $F_1(e_2) = \{h_2\}$,

$F_1(e_1) = \{h_1, h_3\}$, $F_2(e_2) = \{h_2, h_3\}$.

Then μ defines a supra soft topology on X. Hence the soft set (G, E) where

$G(e_1) = \{h_1, h_2\}$, $G(e_2) = \{h_2, h_3\}$, is a supra α-open soft set of(X, μ, E), but it is not supra open soft.

4. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E).(F_3, E)$ are soft sets over X defined as follows:

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_1\}$,

$F_2(e_1) = \{h_1, h_2\}$, $F_2(e_2) = \{h_2, h_3\}$,

$F_3(e_1) = \{h_2, h_3\}$, $F_3(e_2) = \{h_2, h_3\}$.

Then μ defines a supra soft topology on X. Hence the soft set (G, E), which defined by

$G(e_1) = \{h_1, h_3\}$, $G(e_2) = \{h_1, h_3\}$, is a supra β-open soft set of (X, μ, E), but it is not supra open soft.

Theorem: 1.3.9

Let (X, μ, E) be a supra soft topological space, then the following statements hold,

1. every supra α-open (resp. α-closed) soft set is supra semi-open (resp. semi-closed) soft.

2. every supra semi-open (resp. semi-closed) soft set is supra β-open (resp. β-closed) soft.

3. every supra pre-open (resp. pre-closed) soft set is supra β-open (resp. β-closed) soft.

4. every supra α-open (resp. α-closed) soft set is supra pre-open (resp. pre-closed) soft.

Proof:

The assertion is proved in the case of supra open soft sets and the other cases are similar.

1. Let $(F, E) \in S\alpha OS(X)$.

Then $(F,E) \tilde{\subseteq} int^s(cl^s(int^s(F,E))) \tilde{\subseteq} int^s(cl^s(F,E))$. Hence $(F,E) \in SSOS(X)$.

2. Let $(F,E) \in SSOS(X)$. Then $(F,E) \tilde{\subseteq} cl^s(int^s(F,E))$. Since $(F,E) \tilde{\subseteq} cl^s(F,E)$, then $(F,E) \tilde{\subseteq} cl^s(int^s(F,E)) \tilde{\subseteq} cl^s(int^s(cl^s(F,E)))$. Thus $(F,E) \in S\beta OS(X)$.

3. Let $(F,E) \in SPOS(X)$.

Then $(F,E) \tilde{\subseteq} int^s(cl^s(F,E) \tilde{\subseteq} cl^s(int^s(cl^s(F,E)))$. Hence $(F,E) \in S\beta OS(X)$.

4. Let$(F,E) \in S\alpha OS(X)$. Since $int^s(F,E) \tilde{\subseteq} cl^s(F,E)$.

Then $cl^s(int^s(F,E)) \tilde{\subseteq} cl^s(F,E))$.

Hence $(F,E) \tilde{\subseteq} int^s(cl^s(int^s(F,E))) \tilde{\subseteq} int^s(cl^s(F,E))$.

Thus $(F,E) \tilde{\subseteq} int^s(cl^s(F,E))$. It follows that $(F,E) \in SPOS(X)$.

Remark: 1.3.10

The converse of the theorem is not true in general and is shown in the following example.

Example: 1.3.11

1. Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1, e_2\}$ and

$\mu = \{\tilde{X}, \tilde{\phi}, (F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E)\}$

where $(F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E)$ are supra soft sets over X defined as follows:

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_1\}$

$F_2(e_1) = \{h_2\}$, $F_2(e_2) = \{h_2\}$

$F_3(e_1) = \{h_1, h_2\}$, $F_3(e_2) = \{h_1, h_2\}$,

$F_4(e_1) = \{h_1, h_4\}$, $F_4(e_2) = \{h_1, h_4\}$,

$F_5(e_1) = \{h_1, h_2, h_4\}$, $F_5(e_2) = \{h_1, h_2, h_4\}$.

Then μ defines a supra soft topology on X. Hence the soft sets (G,E), which defined as follows:

$G(e_1) = \{h_1, h_3\}$, $G(e_2) = \{h_1, h_3\}$.

is a supra semi-open oft set of (X, μ, E), but it is not supra α-open soft.

2. Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1, e_2\}$ and

$\mu = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$

where $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$ are supra soft sets over X defined as follows:

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_1\}$

$F_2(e_1) = \{h_1, h_2\}$, $F_2(e_2) = \{h_1, h_2\}$

$F_3(e_1) = \{h_2, h_3\}$, $F_3(e_2) = \{h_2, h_3\}$,

$F_4(e_1) = \{h_1, h_2, h_3\}$, $F_4(e_2) = \{h_1, h_2, h_3\}$,

Then μ defines a supra soft topology on X. Hence the soft sets (G, E), which defined as follows:

$G(e_1) = \{h_2\}$, $G(e_2) = \{h_2\}$.

is a supra β-open oft set of (X, μ, E), but it is not supra semi-open soft.

3. Let $X = \{h_1, h_2, h_3, h_4\}$, $E = \{e_1, e_2\}$ and

$\mu = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$

where $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)$ are supra soft sets over X defined as follows:

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_1\}$

$F_2(e_1) = \{h_2\}$, $F_2(e_2) = \{h_2\}$

$F_3(e_1) = \{h_1, h_2\}$, $F_3(e_2) = \{h_1, h_2\}$,

$F_4(e_1) = \{h_1, h_4\}$, $F_4(e_2) = \{h_1, h_4\}$,

$F_5(e_1) = \{h_1, h_2, h_4\}$, $F_5(e_2) = \{h_1, h_2, h_4\}$.

Then μ defines a supra soft topology on X. Hence the soft sets (G, E), which defined as follows:

$G(e_1) = \{h_1, h_3\}$, $\qquad$ $G(e_2) = \{h_1, h_3\}$.

is a supra β-open oft set of (X, μ, E), but it is not supra pre-open soft.

4. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\mu = \{\widetilde{X}, \widetilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined as follows:

$F_1(e_1) = \{h_1\}$, $\qquad$ $F_1(e_2) = \{h_1\}$,

$F_2(e_1) = \{h_1, h_2\}$, $\qquad$ $F_2(e_2) = \{h_1, h_2\}$,

$F_3(e_1) = \{h_2, h_3\}$, $\qquad$ $F_3(e_2) = \{h_2, h_3\}$.

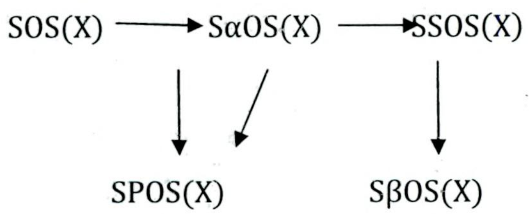
Then μ defines a supra soft topology on X. Hence the soft set (G, E), which defined by

$G(e_1) = \{h_1, h_3\}$, $\qquad$ $G(e_2) = \{h_1, h_3\}$.

is a supra pre-open soft set of (X, μ, E), but it is not supra α-open soft.

Remark: 1.3.12

The following implications hold from Theorem 1.3.6 and Theorem 1.3.9 for a soft topological space (X, μ, E). These implications are not reversible.



Theorem: 1.3.13

Let (X, τ, E) be a soft topological space, $\gamma: SS(X)_E \to SS(X)_E$ be one of the operations defined in Definition 1.3.2 and $F_E \in SS(X)_E$. Then the results hold:

1. $\gamma(\text{int}^s(F'_E)) = \widetilde{X} - \gamma(\text{cl}^s(F_E))$.

2. $\gamma(\text{cl}^s(F'_E)) = \widetilde{X} - \gamma(\text{int}^s(F_E))$.

Proof:

We give the proof for the case of supra pre-open soft operator i.e $\gamma = (\text{int}^s(\text{cl}^s))$, the other cases is similar.

1. Let $x_e \notin Pcl^s(F_E)$. Then $\exists\ G_E \in SPO(\widetilde{X}, x_e)$ such that $G_E \tilde{\cap} F_E = \emptyset$, hence $x_e \in G_E \tilde{\subseteq} F'_E$. Thus, $x_e \in Pint^s(F'_E)$. This means that, $\widetilde{X} - Pcl^s(F_E) \tilde{\subseteq} Pint^s(F'_E)$.

2. Let $x_e \in Pint^s(F'_E)$. Since $Pint^s(F'_E) \tilde{\cap} F_E = \emptyset$, so $x_e \notin Pcl^s(F_E)$. It follows that $x_e \in \widetilde{X} - Pcl^s(F_E)$. Therefore $Pint^s(F'_E) \tilde{\subseteq} \widetilde{X} - Pcl^s(F_E)$.

Let $x_e \in Pint^s(F_E)$. Then $\forall\, G_E \in SPO(\widetilde{X}, x_e)$, $x_e \in G_E \tilde{\subseteq} F_E$, hence $G_E \tilde{\cap} F'_E = \emptyset$. Thus, $x_e \notin Pint^s(F'_E)$. This means that, $\widetilde{X} - Pint^s(F_E) \tilde{\subseteq} Pcl^s(F'_E)$.

Let $x_e \notin Pcl^s(F'_E)$. Then $\exists\ G_E \in SPO(\widetilde{X}, x_e)$ such that $G_E \tilde{\cap} F'_E = \emptyset$, hence $x_e \in G_E \tilde{\subseteq} F_E$. It follows that $x_e \in Pint^s(F_E)$. This means that, $Pcl^s(F'_E) \tilde{\subseteq} \widetilde{X} - Pint^s(F_E)$. This completes the proof.

Theorem: 1.3.14

Let (X, μ, E) be a supra soft topological space and $F_E \in SS(X)_E$. Then

1. $F_E \in SSOS(X)$ if and only if $cl^s(F_E) = cl^s(int^s(F_E))$.

2. $G_E \in SSOS(X)$, then $G_E \tilde{\cap} cl^s(F_E) \tilde{\subseteq} cl^s(G_E \tilde{\cap} F_E)$.

Theorem: 1.3.15

Let (X, τ, E) be a soft topological space and $F_{E,}, G_E, \in SS(X)_E$. Then

1. $F_E \in S\alpha SOS(X)$ if and only if $\exists\ H_{E,} \in SOS(X)$ such that $H_E \tilde{\subseteq} F_E \tilde{\subseteq} int^s(cl^s(H_E))$.

2. If $F_E \in S\alpha OS(X)$ and $F_E \tilde{\subseteq} G_E \tilde{\subseteq} int^s(cl^s(F_E))$, Then $G_E \in S\alpha OS(X)$.

Proof:

1. Necessity :Suppose that $int^s(F_E) = H_{E,} \in SOS(X)$. Then $H_E \tilde{\subseteq} F_E \tilde{\subseteq} int^s(cl^s(H_E))$.

z Let $H_E \tilde{\subseteq} F_E \tilde{\subseteq} int^s(cl^s(H_E))$, $H_E \in SOS(X)$. Then

$int^s(H_E) = (H_E) \tilde{\subseteq} int^s(H_E)$.

It follows that $F_E \tilde{\subseteq} int^s(cl^s(int^s(H_E))) \tilde{\subseteq} int^s(cl^s(int^s(F_E)))$.

Thus, $F_E \in S\alpha SOS(X)$.

2. Let $F_E \in S\alpha SOS(X)$, then $F_E \tilde{\subseteq} int^s(cl^s(int^s(F_E)))$. Hence

$F_E \tilde{\subseteq} G_E \tilde{\subseteq} int^s(cl^s(int^s(cl^s(int^s(H_E))))) \tilde{\subseteq} int^s(cl^s(int^s(F_E))) \tilde{\subseteq} int^s(cl^s(int^s(G_E)))$

Thus, $G_E \in S\alpha SOS(X)$.

Theorem: 1.3.16

Let (X, μ, E) be a supra soft topological space and $F_E \in SS(X)_E$. Then

1. $F_E \in S\alpha OS(X)$ if and only if $F_E \in SPOS(X) \tilde{\cap} SSOS(X)$.

2. $F_E \in S\alpha CS(X)$ if and only if $F_E \in SPCS(X) \tilde{\cap} SSCS(X)$.

Proof:

1. Necessity: Let $F_E \in S\alpha OS(X)$, Then $F_E \tilde{\subseteq} int^s(cl^s(int^s(F_E)))$. Hence $F_E \tilde{\subseteq} cl^s(int^s(F_E))$ and $F_E \tilde{\subseteq} int^s(cl^s(F_E))$. Thus $F_E \in SPOS(X) \tilde{\cap} SSOS(X)$.

Sufficiency: Let $F_E \in SPOS(X) \tilde{\cap} SSOS(X)$. Then $F_E \tilde{\subseteq} cl^s(int^s(F_E))$ and $F_E \tilde{\subseteq} int^s(cl^s(F_E))$. Thus, $F_E \tilde{\subseteq} int^s(cl^s(cl^s(int^s(F_E)))) = int^s(cl^s(int^s(F_E)))$. It follows that $F_E \in S\alpha OS(X)$.

2. The proof is similar to the above way.

Theorem: 1.3.17

Let (X, μ, E) be a supra soft topological space and $F_E \in SS(X)_E$. Then $F_E \in SPCS(X)$ if and only if $cl^s(int^s(F_E)) \tilde{\subseteq} F_E$.

Proof:

Let $F_E \in SPCS(X)$, then F'_E is a supra pre-open soft set, This means that, $F'_E \tilde{\subseteq} int^s(cl^s(\tilde{X} - F_E)) = \tilde{X} - cl^s(int^s(F_E))$. Therefore, $cl^s(int^s(F_E) \tilde{\subseteq} F_E$. Conversely, let $cl^s(int^s(F_E)) \tilde{\subseteq} F_E$. Then $(\tilde{X} - F_E) \tilde{\subseteq} int^s(cl^s(\tilde{X} - F_E))$, hence $\tilde{X} - F_E$ is a supra pre-open soft set. Therefore, F_E is a supra pre-closed soft set.

Theorem: 1.3.18

Let (X, μ, E) be a supra soft topological space. If $F_E \in S\alpha OS(X)$ and $F'_E \in SPOS(X)$. Then $F_E \in SOS(X)$.

Proof:

Le$F_E \in S\alpha OS(X)$ and $F'_E \in SPOS(X)$. Then $F_E \in SPCS(X)$. Hence $cl^s(int^s(F_E) \tilde{\subseteq} F_E \tilde{\subseteq} int^s(cl^s(int^s(F_E)) \tilde{\subseteq} cl^s(int^s(F_E))$. This means that, $cl^s(int^s(F_E) = F_E$. Thus, $F_E \tilde{\subseteq} int^s(cl^s(int^s(F_E)) = int^s(F_E)$. Therefore $F_E \in SOS(X)$.

Theorem: 1.3.19

Let (X, μ, E) be a supra soft topological space and $F_E \in SS(X)_E$. Then $F_E \in S\alpha CS(X)$ if and only if $cl^s(int^s(cl^s(F_E))) \tilde{\subseteq} F_E$.

Proof:

Let $F_E \in S\alpha CS(X)$, then F'_E is a supra α-open soft set. This means that, $F'_E \tilde{\subseteq} int^s\left(cl^s(int^s(\tilde{X} - F_E))\right) = \tilde{X} - (cl^s(int^s(cl^s(F_E))))$. Therefore, $cl^s(int^s(cl^s(F_E))) \tilde{\subseteq} F_E$. Conversely, let $cl^s(int^s(cl^s(F_E))) \tilde{\subseteq} F_E$. Then $(\tilde{X} - F_E) \tilde{\subseteq} int^s\left(cl^s(int^s(\tilde{X} - F_E))\right)$, hence $\tilde{X} - F_E$ is a supra α-open soft set. Therefore, F_E is a supra α -closed soft set.

Theorem: 1.3.20

Let (X, μ, E) be a supra soft topological space and $F_E \in SS(X)_E$. Then $F_E \in SSCS(X)$ if and only if $int^s(cl^s(F_E)) \tilde{\subseteq} F_E$.

Proof:

Let $F_E \in SSCS(X)$, then F'_E is a supra semi-open soft set, This means that, $F'_E \tilde{\subseteq} cl^s(int^s(\tilde{X} - F_E)) = \tilde{X} - (int^s(cl^s(F_E)))$. Therefore, $int^s(cl^s(F_E)) \tilde{\subseteq} F_E$. Conversely, let $int^s(cl^s(F_E)) \tilde{\subseteq} F_E$. Then $(\tilde{X} - F_E) \tilde{\subseteq} cl^s(int^s(\tilde{X} - F_E))$, hence $\tilde{X} - F_E$ is a supra semi-open soft set. Therefore, F_E is a supra semi-closed soft set.

Corollary: 1.3.21

Let (X, μ, E) be a supra soft topological space and $F_E \in SS(X)_E$. Then $F_E \in SSCS(X)$ if and only if $F_E = F_E \tilde{\cup} int^s(cl^s(F_E))$.

Theorem: 1.3.22

Let (X, μ, E) be a supra soft topological space and $F_E \in SS(X)_E$. Then $F_E \in S\beta CS(X)$ if and only if $int^s(cl^s(int^s(F_E))) \tilde{\subseteq} F_E$.

Proof:

Let $F_E \in S\beta CS(X)$, then F'_E is a supra β-open soft set. This means that. $F'_E \tilde{\subseteq} cl^s(int^s(cl^s(\tilde{X} - F_E))) = \tilde{X} - int^s(cl^s(int^s(F_E)))$. Therefore. $int^s(cl^s(int^s(F_E))) \tilde{\subseteq} F_E$. Conversely, let $int^s(cl^s(int^s(F_E))) \tilde{\subseteq} F_E$. Then $(\tilde{X} - F_E) \tilde{\subseteq} cl^s(int^s(cl^s(\tilde{X} - F_E)))$, hence $\tilde{X} - F_E$ is a supra β-open soft set. Therefore, F_E is a supra β-closed soft set.

Section 1.4

Decomposition of some forms of supra soft continuity

Definition 1.4.1

Let (X, τ_1, A) and (X, τ_2, B) be soft topological spaces. Let μ_1 be an associated supra soft topology with τ_1. Let $u: X \to Y$ and $p: A \to B$ e a mappings. Let $f_{pu}: SS(X)_A \to SS(Y)_B$ be a function. Then, the function

1. f_{pu} is called **supra continuous soft function(supra cts soft)** if $f_{pu}^{-1}(G, B) \in SOS(X, \mu_1, E)\ \forall (G, B) \in OS(Y)$.

2. f_{pu} is called **supra pre-continuous soft function(supra pre-cts soft)** if $f_{pu}^{-1}(G, B) \in SPOS(X, \mu_1, E)\ \forall (G, B) \in OS(Y)$.

3. f_{pu} is called **supra semi-continuous soft function(supra semi-cts soft)** if $f_{pu}^{-1}(G, B) \in SSOS(X, \mu_1, E)\ \forall (G, B) \in OS(Y)$.

4. f_{pu} is called **supra α-continuous soft function(supra α-cts soft)** if $f_{pu}^{-1}(G, B) \in S\alpha OS(X, \mu_1, E)\ \forall (G, B) \in OS(Y)$.

5. f_{pu} is called supra **β-continuous soft function(supra β-cts soft)** if $f_{pu}^{-1}(G, B) \in S\beta OS(X, \mu_1, E)\ \forall (G, B) \in OS(Y)$.

Theorem: 1.4.2

Let (X, τ_1, A) and (X, τ_2, B) be soft topological spaces. Let μ_1 be a an associated supra soft topology with τ_1. Let $u: X \to Y$ and $p: A \to B$ be a mappings. Let $f_{pu}: SS(X)_A \to SS(Y)_B$ be a function. Then for the classes, supra pre-continuous (resp. supraα-continuous soft, supra semi-continuous soft, supra β-continuous soft) functions the following are equivalent (we give an example for the classes of supra pre-continuous soft functions).

1. f_{pu} is a supra pre-continuous soft function.

2. $f_{pu}^{-1}(H, B) \in SPCS(X, \mu_1, E)\ \forall (H, B) \in CS(Y)$.

3. $f_{pu}\left(Pcl^s(G, A)\right) \subseteq cl_{\tau_2}\left(f_{pu}(G, A)\right) \forall (G, A) \in SS(X)_A$.

4. $Pcl^s\ (f_{pu}^{-1}(H, B)) \subseteq\ f_{pu}^{-1}(cl_{\tau_2}(H, B))\ \forall (H, B) \in SS(Y)_B$.

5. $f_{pu}^{-1}\left(int_{\tau_2}(H, B)\right) \subseteq Pint^s(f_{pu}^{-1}(H, B))\ \forall (H, B) \in SS(Y)_B$.

Proof:

(1→2)Let (H, B) be a closed soft set over Y. Then $(H, B)' \in OS(Y)$ and $f_{pu}^{-1}\ (H, B)' \in SPOS(X, \mu_1, E)$ from definition 1.4.1. Since

$f_{pu}^{-1}\ (H, B)' = (f_{pu}^{-1}\ (H, B))'$. Thus, we get $f_{pu}^{-1}(H, B) \in SPCS(X, \mu_1, E)$.

(2→3) Let $(G, A) \in SS(X)_A$.
Since $(G, A) \tilde{\subseteq} f_{pu}^{-1}\left(f_{pu}(G, A)\right) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}\left(f_{pu}(G, A))\right) \in SPCS(X, \mu_1, E)$. from(2)
Then $(G, A) \tilde{\subseteq} Pcl^s(G, A) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}\left(f_{pu}(G, A))\right)$.
Hence $f_{pu}\left(Pcl^s(G, A)\right) \tilde{\subseteq} f_{pu}(f_{pu}^{-1}(cl_{\tau_2}\left(f_{pu}(G, A)))\right) \tilde{\subseteq} cl_{\tau_2}\left(f_{pu}(G, A)\right)$.

Thus, $f_{pu}\left(Pcl^s(G, A)\right) \tilde{\subseteq} cl_{\tau_2}\left(f_{pu}(G, A)\right)$.

(3→4)Let $(H, B) \in SS(Y)_B$ and $(G, A) = f_{pu}^{-1}(H, B)$.
Then $f_{pu}\left(Pcl^s f_{pu}^{-1}(H, B)\right) \tilde{\subseteq} cl_{\tau_2}\left(f_{pu}(f_{pu}^{-1}(H, B))\right)$ from(3).Hence
$Pcl^s(f_{pu}^{-1}(H, B)) \tilde{\subseteq} f_{pu}^{-1}\left(f_{pu}(Pcl^s(f_{pu}^{-1}(H, B)))\right) \tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(f_{pu}(f_{pu}^{-1}(H, B))))$
$\tilde{\subseteq} f_{pu}^{-1}(cl_{\tau_2}(H, B))$. Thus, $Pcl^s\ (f_{pu}^{-1}(H, B) \tilde{\subseteq}\ f_{pu}^{-1}(cl_{\tau_2}(H, B))$.

(4→2) Let (H, B) be a closed soft set over Y.

Then $Pcl^s\left(f_{pu}^{-1}(H,B)\right) \tilde{\subseteq} f_{pu}^{-1}\left(cl_{\tau_2}(H,B)\right) = f_{pu}^{-1}(H,B)\ \forall (H,B) \in SS(Y)_B$ from(4).

but clearly $f_{pu}^{-1}(H,B) \tilde{\subseteq}\ Pcl^s(f_{pu}^{-1}(H,B))$.

This means that, $f_{pu}^{-1}(H,B) = \ Pcl^s(f_{pu}^{-1}(H,B)) \in SPCS(X, \mu_1, E)$.

(1→5)Let $(H,B) \in SS(Y)_B$. Then $f_{pu}^{-1}(int_{\tau_2}(H,B)) \in\ SPOS(X, \mu_1, E)$ from(1). Hence $f_{pu}^{-1}(int_{\tau_2}(H,B)) = \ Pint^s(f_{pu}^{-1} int_{\tau_2}(H,B)) \ \tilde{\subseteq}\ Pint^s(f_{pu}^{-1}(H,B))$.Thus.

$f_{pu}^{-1}\left(int_{\tau_2}(H,B)\right) \subseteq Pint^s(f_{pu}^{-1}(H,B))$.

(5→1)Let (H,B) be an open soft set over Y. Then $int_{\tau_2}(H,B) = (H,B)$ and $f_{pu}^{-1}(int_{\tau_2}(H,B)) = \ f_{pu}^{-1}(H,B) \tilde{\subseteq}\ Pint^s(f_{pu}^{-1}(H,B))$ from(5). But we have $Pint^s(f_{pu}^{-1}(H,B)) \tilde{\subseteq}\ f_{pu}^{-1}(H,B)$. This means that,

$Pint^s(f_{pu}^{-1}(H,B) = \ f_{pu}^{-1}(H,B) \in\ SPOS(X, \mu_1, E)$. Thus f_{pu} is a supra pre-continuous soft function.

Theorem: 1.4.3

Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let μ_1 be an associated supra soft topology with τ_1. Let $u: X \to Y$ and $p: A \to B$ be a mappings. Let $f_{pu}: SS(X)_A \to SS(Y)_B$ be a function. Then

1. every supra continuous soft function is supra pre-continuous soft function.

2. every supra continuous soft function is supra semi-continuous soft function.

3. every supra continuous soft function is supra α-continuous soft function.

4. every supra continuous soft function is supra β-continuous soft function.

Theorem: 1.4.4

Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let μ_1 be an associated supra soft topology with τ_1. Let $u: X \to Y$ and $p: A \to B$ be a mappings. Let $f_{pu}: SS(X)_A \to SS(Y)_B$ be a function. Then

1. every supra α-continuous soft function is supra semi-continuous soft function.

2. every supra semi-continuous soft function is supra β-continuous soft function.

3. every supra pre-continuous soft function is supra β-continuous soft function.

4. every supra α-continuous soft function is supra pre-continuous soft function.

Theorem: 1.4.5

Let (X, τ_1, A) and (Y, τ_2, B) be soft topological spaces. Let μ_1 be an associated supra soft topology with τ_1. Let $u: X \rightarrow Y$ and $p: A \rightarrow B$ be a mappings. Let $f_{pu}: SS(X)_A \rightarrow SS(Y)_B$ be a function. Then is a supra α-continuous soft function if and only if it is a supra pre-continuous and supra semi-continuous soft function.

Corollary: 1.4.6

For a soft topological space (X, τ, E) and its associated supra soft topology μ we have the following implications.