On Regular Generalized Semipreclosed sets in Intuitionistic fuzzy topological spaces

Vaishnavy, V

(13PMA014)

Thesis submitted to

Avinashilingam Institute for Home Science and Higher Education for Women,

Coimbatore - 641 043

In Partial Fulfilment of the Requirements for the

Degree of Master of Science in Mathematics

March, 2015

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Parvathi 31/3/15
Signature of the Head of the Department

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Signature of the Supervisor

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CONTENT

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TITLE

INTRODUCTION

INTRODUCTION

Intuitionistic fuzzy topology is considered to be an interesting as well as rapidly growing field in Mathematics. Before a decade only a handful number of research works were being undertaken in this concept but now intuitionistic fuzzy topology has become one among the most famous research areas in Mathematics. The origin of this particular concept is fuzzy sets. Ever since the establishment of fuzzy sets by Zadeh[1965], fuzzy has invaded almost all branches of Mathematics. Later the introduction of fuzzy topology by Chang[1967] was an annexation towards the hike of fuzzy sets and fuzzy topology. The perception of intuitionistic fuzzy sets by Atanassov[1986] was a breakthrough towards the evolution of intuitionistic fuzzy topology. Using the notion of intuitionistic fuzzy sets, Coker[1997] has constructed the basic concepts of intuitionistic fuzzy topological spaces. We are well aware of the existence of sets and fuzzy sets. Now let us proceed to know the necessity and importance of introducing intuitionistic fuzzy sets in the field of mathematics.

"Intuitionistic fuzzy sets" occupies our daily lives in the following way.

Consider a garment manufacturing company which manufactures about 2000 garments per day. Out of the 2000 manufactured garments 1750 are being distributed to the dealers for sale. Then what about the remaining 250 garments?

Have they not been sold out?

Sets and fuzzy sets say "YES" they have not been sold. But this need not be case every time, because there may be customers who come directly to the garment company to make their purchase. Here comes "INTUITIONISTIC FUZZY SETS". Intuitionistic fuzzy sets contributes to the fact that the statement, "the remaining count of garments were not sold out" is not true. Rather it reveals that this count may also include the garments which were directly purchased by the customers at the company, the garments which were circulated to the external designer agencies for additional designing, the garments which were defective and hence forwarded for rework and finally the garments which were not sold out. For this accuracy of statistical values we have intuitionistic fuzzy sets.

In this thesis work a new class of intuitionistic fuzzy set namely "Intuitionistic fuzzy regular generalized semipreclosed set" is being introduced. Further its corresponding open set, continuous mapping, contra continuous mapping and almost continuous mapping are being introduced and their respective properties are discussed.

In Chapter I, the recent developments in intuitionistic fuzzy topology contributed by various authors are presented. This forms the basement for the remaining chapters of this thesis.

In Chapter II, Intuitionistic fuzzy regular generalized semipreclosed sets are introduced. The relationship between this newly introduced set and few of the already existing intuitionistic fuzzy sets is being discussed. Further some of the characterizations of this newly introduced set are discussed.

In Chapter III Regular generalized semipreopen sets in intuitionistic fuzzy topological spaces are introduced and the relationship between this newly introduced set and some of the previously existing intuitionistic fuzzy sets are discussed. Additionally, the theoretical applications of intuitionistic fuzzy regular generalized semipre open sets are presented.

In Chapter IV, Intuitionistic fuzzy regular generalized semipre continuous mappings are introduced and the liaison of it with some of the previously existing continuous mappings in intuitionistic fuzzy topological spaces are investigated. Further some fascinating theorems concerning intuitionistic fuzzy regular generalized semipre continuous mappings are discussed.

In Chapter V intuitionistic fuzzy contra regular generalized semipre continuous mappings are introduced which is followed by the relationship of it with some of the already existing contra continuous mappings in intuitionistic fuzzy topological spaces. Further few interesting theorems concerning it are discussed.

In Chapter VI, intuitionistic fuzzy almost regular generalized semipre continuous mappings are introduced and some interesting theorems are discussed. Moreover its liaison with few of the already existing intuitionistic fuzzy continuous mappings are presented. Throughout this thesis (X, τ). (Y, σ) and (Z, γ) denote intuitionistic fuzzy topological spaces on which no separation axioms are assumed unless otherwise explicitly mentioned.

REVIEW OF LITERATURE

REVIEW OF LITERATURE

The notion of fuzzy sets was given by Zadeh[1965]. Later this concept was extended to intuitionistic fuzzy sets by Atanassov[1986]. Using the notion of intuitionistic fuzzy sets, Coker[1997] introduced the concept of intuitionistic fuzzy topological spaces. This was eventually followed by the introduction of three new classes of sets namely intuitionistic fuzzy semi closed sets, intuitionistic fuzzy pre closed sets, intuitionistic fuzzy α - closed sets by Gurcay, Coker and Haydar[1997]. In 1986 Andrijevic[1986] introduced a new class of sets in general topology namely semi-preopen sets. Later this was succeeded by the introduction of intuitionistic fuzzy semi-pre open sets and intuitionistic fuzzy semi-pre closed sets by Young Bae Jun and Seok - Zun Song[2005]. These sets were then generalized by Santhi, R and Jayanthi, D[2009]. Further intuitionistic fuzzy semi-pre continuous mappings were introduced by the Santhi, R and Jayanthi, D[2010]. Biljana Krsteska and Erdal Ekici[2007] introduced intuitionistic fuzzy contra pre continuous mappings and intuitionistic fuzzy contra strongly pre continuous mappings.

1. FUZZY SETS

[Lotfi A. Zadeh., 1965]

In this article, the author has introduced a new class of sets namely fuzzy sets which are characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. Further the author has provided the notions of inclusion, union, intersection, complement, etc with respect to the fuzzy sets.

2. INTUITIONISTIC FUZZY SETS

[Krassimir T. Atanassov., 1986]

In this article, the author has provided the notion of intuitionistic fuzzy sets. This is considered to be the generalization on fuzzy sets. The highlight of this particular article is that some relations and operations concerning classical sets are extended to intuitionistic fuzzy sets.

3. FUZZY TOPOLOGICAL SPACES

[C. L. Chang., 1968]

In this article, the author has introduced fuzzy topological spaces. This concept is considered to be the generalization of general topological spaces. In brief, the basic concepts such as fuzzy open set, fuzzy closed set, fuzzy neighbourhood, fuzzy continuity etc., are discussed in depth.

4. INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Dogan Coker., 1997]

In this article, the author has introduced intuitionistic fuzzy topological space. The notions of intuitionistic fuzzy interior and intuitionistic fuzzy closure are being provided and this is followed by the discussion of some important properties concerning them. Furthermore, the notion of intuitionistic fuzzy continuity is provided.

5. ON FUZZY CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[H. Gurcay., D. Coker., and Haydar Es. A., 1997]

This article consists of the notions of intuitionistic fuzzy semiopen, ituitionistic fuzzy preopen and intuitionistic fuzzy α-open sets and their corresponding closed sets. Further the relationship between these sets are established.

6. INTUITIONISTIC FUZZY ALPHA - CONTINUITY AND INTUITIONISTIC FUZZY PRECONTINUITY

[Joung Kon Jeon., Young Bae Jun., and Jin Han Park., 2005]

In this article, the notion of intuitionistic fuzzy point is provided. This is followed by the discussion of the relationship between some of the previously defined intuitionistic fuzzy sets.

7. INTUITIONISTIC FUZZY SEMI-PRE OPEN SETS AND INTUITIONISTIC FUZZY SEMI-PRE CONTINUOUS MAPPINGS

[Young Bae Jun., and Seok - Zun Song., 2005]

This article consists of the notion of intuitionistic fuzzy semi-preopen sets and its corresponding closed sets. The properties regarding the union and intersection of these sets are portrayed. Also, the relationship between this new class of sets and some of the previously existing sets are discussed.

8. RELATION BETWEEN SEMIPRECLOSED SETS AND BETA CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Jayanthi, D., 2013]

This article is considered to be a significant one as it clearly distinguishes intuitionistic fuzzy semipreclosed sets and beta closed sets. The author has additionally provided the relationship between these two sets.

9. INTUITIONISTIC FUZZY GENERALIZED SEMI-PRE CLOSED SETS

[Santhi, R., and Jayanthi, D., 2009]

In this article the concept of intuitionistic fuzzy semi-pre closed was generalized. One of the interesting aspect of this article is that it consists of the relation between the particular set under consideration and various other previously existing intuitionistic fuzzy sets.

10. INTUITIONISTIC FUZZY CONTRA STRONG PRECONTINUITY

[Biljana Krsteska., and Erdal Ekici., 2007]

This article consists of the notion of intuitionistic fuzzy contra strong pre continuity. Additionally the authors have presented the notions of intuitionistic fuzzy contra continuous mapping, intuitionistic fuzzy contra pre continuous mapping, intuitionistic fuzzy contra alpha continuous mapping.

CHAPTER I

CHAPTER I

PRELIMINARIES

Definition 1.1 [2] : An ***intuitionistic fuzzy set*** (IFS in short) A is an object having the form

$$A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the function $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A=\langle x, \mu_A, \nu_A\rangle$ instead of denoting $A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 1.2 [2] : Let A and B be two IFSs of the form

$$A=\{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

and

$$B=\{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}.$$

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$

(d) $A\cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle: x \in X\}$

(e) $A\cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle: x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 1.3 [4] : An ***intuitionistic fuzzy topology*** (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim , 1_\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called the ***intuitionistic fuzzy topological space*** (IFTS in short) and any IFS in τ is known as an ***intuitionistic fuzzy open set*** (IFOS in short) in X. The compliment A^c of an IFOS A in IFTS (X,τ) is called an ***intuitionistic fuzzy closed set*** (IFCS in short) in X.

Definition 1.4 [4] : Let (X, τ) be an IFTS and $A= \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the ***intuitionistic fuzzy interior*** and ***intuitionistic fuzzy closure*** are defined by

$int(A) = \cup\{ G/ G$ is an IFOS in X and $G \subseteq A \}$

$cl(A) = \cap\{K/ K$ is an IFCS in X and $A \subseteq K\}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 1.5 [6] : An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X,τ) is said to be an

(i) ***intuitionistic fuzzy regular closed set*** (IFRCS in short) if $A = cl(int(A))$

(ii) ***intuitionistic fuzzy regular open set*** (IFROS in short) if $A= int(cl(A))$

Definition 1.6 [6] : An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X,τ) is said to be an

(i) ***intuitionistic fuzzy semi closed set*** (IFSCS in short) if $int(cl(A)) \subseteq A$

(ii) ***intuitionistic fuzzy pre closed set*** (IFPCS in short) if $cl(int(A)) \subseteq A$

(iii) ***intuitionistic fuzzy α closed set*** (IFαCS in short) if $cl(int(cl(A))) \subseteq A$

(iv) ***intuitionistic fuzzy β closed set*** (IFβCS in short) if $int(cl(int(A))) \subseteq A$

The respective complements of the above IFCSs are called their respective IFOSs. The family of all IFSCSs, IFPCSs, IFαCSs and IFβCSs (respectively IFSOSs, IFPOSs, IFαOSs and IFβOSs) of an IFTS (X, τ) are respectively denoted by IFSC(X), IFPC(X), IFαC(X), IFβC(X) (respectively IFSO(X), IFPO(X), IFαO(X), IFβO(X)).

Definition 1.7 [16] : An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X,τ) is said to be an

(i) ***intuitionistic fuzzy semi-pre closed set*** (IFSPCS in short) if there exists an IFPCS B such that $\text{int}(B) \subseteq A \subseteq B$

(ii) ***intuitionistic fuzzy semi-pre open set*** (IFSPOS in short) if there exists an IFPOS B such that $B \subseteq A \subseteq \text{cl}(B)$

The family of all IFSPCSs (respectively IFSPOSs) of an IFTS (X,τ) is denoted by IFSPC(X) (respectively IFSPO(X)).

Every IFSCS (respectively IFSOS) and every IFPCS (respectively IFPOS) is an IFSPCS (respectively IFSPOS). But the separate converses need not hold in general.

Definition 1.8 [12] : Let A be an IFS in an IFTS (X, τ). Then the ***semi-pre interior*** and the ***semi-pre closure*** of A are defined as

$\text{spint}\,(A) = \cup\,\{G \mid G$ is an IFSPOS in X and $G \subseteq A\}$
$\text{spcl}\,(A) = \cap\,\{K \mid K$ is an IFSPCS in X and $A \subseteq K\}$

Note that for any IFS A in (X, τ), we have $\text{spcl}(A^c) = (\text{spint}(A))^c$ and $\text{spint}(A^c) = (\text{spcl}(A))^c$.

Definition 1.9 [14] : An IFS A is an

(i) ***intuitionistic fuzzy generalized closed set*** (IFGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS

(ii) ***intuitionistic fuzzy regular generalized closed set*** (IFRGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is IFROS.

Definition 1.10 [10] : An ***intuitionistic fuzzy point*** (IFP in short), written as $p_{(\alpha,\beta)}$, is defined to be an intuitionistic fuzzy set of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 1.11 [14] : Two IFSs are said to be ***q-coincident*** ($A_q B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 1.12 [14] : Two IFSs are said to be ***not q-coincident*** ($A_q^c B$ in short) if and only if $A \subseteq B^c$.

Definition 1.13 [12] : An IFS A in an IFTS (X,τ) is said to be an ***intuitionistic fuzzy generalized semi-pre closed set*** (IFGSPCS for short) if $\mathrm{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Definition 1.14 [14] : An IFTS (X,τ) is said to be an ***intuitionistic fuzzy $T_{1/2}$ space*** (IF$T_{1/2}$ space in short) if every IFGCS in (X,τ) is an IFCS in (X,τ).

Definition 1.15 [13] : If every IFGSPCS in (X, τ) is an IFSPCS in (X, τ), then the space can be called as an ***intuitionistic fuzzy semi- pre $T_{1/2}$ space*** (IFSP$T_{1/2}$ space in short).

Definition 1.16 [6] : Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an ***intuitionistic fuzzy continuous*** (IF continuous in short) ***mapping*** if $f^{-1}(B) \in \mathrm{IFO}(X)$ for every $B \in \sigma$.

Definition 1.17 [9] : Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

(i) ***intuitionistic fuzzy semi continuous*** (IFS continuous in short) ***mapping*** if $f^{-1}(B) \in \mathrm{IFSO}(X)$ for every $B \in \sigma$

(ii) ***intuitionistic fuzzy α continuous*** (IFα-continuous in short) ***mapping*** if $f^{-1}(B) \in \mathrm{IF\alpha O}(X)$ for every $B \in \sigma$

(iii) ***intuitionistic fuzzy pre continuous*** (IFP continuous in short) ***mapping*** if $f^{-1}(B) \in \mathrm{IFPO}(X)$ for every $B \in \sigma$.

Every IF continuous mapping is an IFα-continuous mapping and every IFα-continuous mapping is an IFS continuous mapping as well as an IFP continuous mapping, but the separate converses may not be true in general.

Definition 1.18 [14] : Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an ***intuitionistic fuzzy generalized continuous*** (IFG continuous in short) ***mapping*** if $f^{-1}(B) \in$ IFGC(X) for every IFCS $B \subseteq Y$.

Every IF continuous mapping is an IFG continuous mapping but the converse may not be true in general.

Definition 1.19 [16] : Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an ***intuitionistic fuzzy semi-pre continuous*** (IFSP continuous in short) ***mapping*** if $f^{-1}(B) \in$ IFSPO(X) for every $B \in \sigma$.

Every IFS continuous mapping and IFP continuous mappings are IFSP continuous mapping but the converses may not be true in general[14].

Definition 1.20 [13] : A mapping f: (X, τ) → (Y, σ) is called an ***intuitionistic fuzzy generalized semi-pre continuous*** (IFGSP continuous for short) ***mapping*** if $f^{-1}(V)$ is an IFGSPCS in (X, τ) for every IFCS V of (Y, σ).

Definition 1.21 [10] : Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

(i) ***intuitionistic fuzzy contra continuous mapping*** (IFC continuous mapping in short) if $f^{-1}(B) \in$ IFO(X) for each IFCS B in Y

(ii) ***intuitionistic fuzzy contra α continuous mapping*** (IFCα continuous mapping in short) if $f^{-1}(B) \in$ IFαO(X) for each IFCS B in Y

(iii) ***intuitionistic fuzzy contra pre continuous mapping*** (IFCP continuous mapping in short) if $f^{-1}(B) \in$ IFPO(X) for each IFCS B in Y

Result 1.22 [6] : Let A be an IFS in (X, τ), then

(i) $scl(A) = A \cup int(cl(A))$

(ii) $sint(A) = A \cap cl(int(A))$

Result 1.23 [11] : Let A be an IFS in (X, τ), then

(i) $\alpha cl(A) = A \cup cl(int(cl(A)))$

(ii) $\alpha int(A) = A \cap int(cl(int(A)))$

Theorem 1.24 [5] : Let (X,τ) be an intuitionistic fuzzy topological space and A, B be intuitionistic fuzzy sets in X. Then the following properties hold

a) $\mathrm{int}(A) \square A$

b) $A \square \mathrm{cl}(A)$

c) $A \square B \square \mathrm{int}(A) \square \mathrm{int}(B)$

d) $A \square B \square \mathrm{cl}(A) \square \mathrm{cl}(B)$

e) $\mathrm{int}(\mathrm{int}(A)) = \mathrm{int}(A)$

f) $\mathrm{cl}(\mathrm{cl}(A)) = \mathrm{cl}(A)$

g) $\mathrm{int}(A \cap B) = \mathrm{int}(A) \cap \mathrm{int}(B)$

h) $\mathrm{cl}(A \square B) = \mathrm{cl}(A) \square \mathrm{cl}(B)$

i) $\mathrm{int}(1\sim) = 1\sim$

j) $\mathrm{cl}(0\sim) = 0\sim$

Corollary 1.25 [4] : Let A, B and C be intuitionistic fuzzy sets in X. Then,

a) $(A \square B)$ and $(C \square D) \square (A \square C) \square (B \square D)$ and $(A \cap C) \square (B \cap D)$

b) $A \square B$ and $A \square C \square A \square (B \cap C)$

c) $A \square C$ and $B \square C \square (A \square B) \square C$

d) $A \square B$ and $B \square C \square A \square C$

e) $(A \square B)^c = A^c \cap B^c$

f) $(A \cap B)^c = A^c \square B^c$

g) $A \square B \square B^c \square A^c$

h) $(A^c)^c = A$

i) $0\sim^c = 1\sim$

j) $1\sim^c = 0\sim$

Corollary 1.26 [4] : Let A, $A_i (i \in J)$ be intuitionistic fuzzy sets in X and B, $B_j (j \in K)$ be intuitionistic fuzzy sets in Y and f : X → Y be a function. Then,

a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

c) $A \subseteq f^{-1}(f(A))$ [If f is injective , then $A=f^{-1}(f(A))$]

d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective , then $B=f(f^{-1}(B))$]

e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

g) $f^{-1}(0\sim) = 0\sim$

h) $f^{-1}(1\sim) = 1\sim$

i) $f^{-1}(B^c) = (f^{-1}(B))^c$

CHAPTER II

CHAPTER II

INTUITIONISTIC FUZZY REGULAR GENERALIZED SEMIPRECLOSED SETS

In this chapter, we have introduced the notion of intuitionistic fuzzy regular generalized semipreclosed sets and studied some of their properties including the relation between intuitionistic fuzzy regular generalized semipreclosed sets and few of the other already existing intuitionistic fuzzy sets.

Definition 2.1 : An IFS A in an IFTS (X,τ) is said to be an ***intuitionistic fuzzy regular generalized semipreclosed set*** (IFRGSPCS in short) if spcl(A) □ U whenever A □ U and U is an IFROS in (X,τ).

The family of all IFRGSPCSs of an IFTS (X,τ) is denoted by IFRGSPC(X).

Example 2.2 : Let X = {a, b} and G = ⟨x, (0.5, 0.4), (0.5, 0.6)⟩ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.6$. Then τ = {0~, G, 1~} is an IFT on X. Let A = ⟨x, (0.4, 0.2), (0.6, 0.7)⟩ be an IFS in X. Then, IFPC(X) = {0~, 1~, μ_a □ [0,1], μ_b □ [0,1], ν_a □ [0,1], ν_b □ [0,1] / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Therefore, IFSPC(X) = {0~, 1~, μ_a □ [0,1], μ_b □ [0,1], ν_a □ [0,1], ν_b □ [0,1] / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}.

Now spcl(A) = A. We have A□ G. Hence spcl(A) □ G, where G is an IFROS in X. This implies that A is an IFRGSPCS in X.

Theorem 2.3 : Every IFCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof: Let A be an IFCS then cl(A) = A. Let A □ U and U is an IFROS. Then, spcl(A) □ cl(A) = A □ U, by hypothesis, A is an IFRGSPCS.

Theorem 2.4 : Every IFGCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof : Let A □ U and U be an IFROS. Since every IFROS is an IFOS, U is an IFOS in X. Then by hypothesis cl(A) □ U. As spcl(A) □ cl(A) we have spcl(A) □ U. Hence A is an IFRGSPCS.

Theorem 2.5 : Every IFSPCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof : Let A be an IFSPCS and A □ U, where U is an IFROS. Then since spcl(A) = A and A □ U, we have spcl(A) □ U. Hence A is an IFRGSPCS.

Theorem 2.6 : Every IFβCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof : Let A be an IFβCS and A □ U, U is an IFROS. Then since βcl(A) = A and A □ U, we have βcl(A) □ U. Hence A is an IFRGSPCS.

Theorem 2.7 : Every IFSCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof : Let A be an IFSCS. Since every IFSCS is an IFSPCS[16], by Theorem 2.5 we get A is an IFRGSPCS.

Theorem 2.8 : Every IFPCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof : Let A be an IFPCS. Since every IFPCS is an IFSPCS[16], by Theorem 2.5 we get A is an IFRGSPCS.

Theorem 2.9 : Every IFαCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

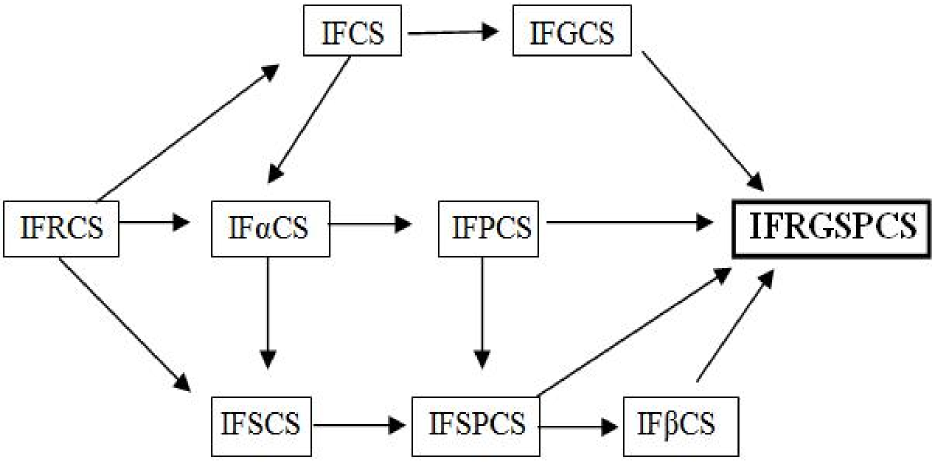
Proof : Let A be an IFαCS. Since every IFαCS is an IFSPCS [16] , by Theorem 2.5 we get A is an IFRGSPCS.

Theorem 2.10 : Every IFRCS in (X,τ) is an IFRGSPCS in (X,τ) but not conversely.

Proof : Let A be an IFRCS. Since every IFRCS is an IFCS [14], by Theorem 2.3 we get A is an IFRGSPCS.

Thus we proved that every IFCS, IFPCS, IFSCS, IFRCS, IFGCS, IFSPCS, IFαCS, IFβCS are IFRGSPCS.

The diagrammatic representation of the above theorems is as follows.



In the above diagram the reverse implications are not true in general. This can be easily seen from the following examples.

Example 2.11 : Let $X = \{a, b\}$ and $G = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.6$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let $A = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$ be an IFS in X. Then, IFPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, IFSPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. As spcl(A) = A, we have $A \subseteq G$ implies spcl(A) $\subseteq G$, where G is an IFROS in X. This implies that A is an IFRGSPCS in X. Now since cl(A) = $G^c \neq A$, A is not an IFCS in X. Also $A \subseteq G$ but cl(A) = $G^c \nsubseteq G$. Therefore A is not an IFGCS in X. Now cl(int(A)) = cl(0~) = $0\sim \neq A$. Therefore A is not an IFRCS in X. Hence A is an IFRGSPCS but not IFCS, IFGCS, IFRCS.

Example 2.12 : Let $X = \{a, b\}$ and $G = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.6$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.4$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let $A = \langle x, (0.5, 0.7), (0.5, 0.3)\rangle$ be an IFS in X. Then, IFPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, IFSPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. As spcl(A) = 1~, we have $A \subseteq 1\sim$ implies spcl(A) $\subseteq 1\sim$, where 1~ is an IFROS. This implies that A is an IFRGSPCS in X. Now since cl(int(A)

= cl(G) = 1~ ⊈ A we get A is not an IFPCS in X. Further int(cl(int(A))) = int(1~) = 1~ ⊈ A. Hence A is not an IFβCS in X. Also int(cl(A)) = int(1~) = 1~ ⊈ A. Thus A is not an IFSCS in X. Now since cl(int(cl(A))) = cl(1~) = 1~ ⊈ A, A is not an IFαCS in X. Further there exists no IFPCS B such that int(B) □ A □ B. Therefore A is not an IFSPCS in X. Hence A is an IFRGSPCS but not IFPCS, IFβCS, IFSCS, IFαCS, IFSPCS.

Theorem 2.13 : Let (X,τ) be an IFTS. Then for every A □ IFRGSPC(X) and for every B □ IFS(X), A □ B □ spcl(A) □ B □ IFRGSPC(X).

Proof : Let B □ U and U be an IFROS. Then since, A □ B, A □ U. By hypothesis, B □ spcl(A). Therefore spcl(B) □ spcl(spcl(A)) = spcl(A) □ U, since A is an IFRGSPCS. Hence B □ IFRGSPC(X).

Theorem 2.14 : An IFS A of an IFTS (X,τ) is an IFRGSPCS if and only if $A\ {}_q^c\ F$ □ spcl(A) ${}_q^c$ F for every IFRCS F of X.

Proof: Necessity : Let F be an IFRCS and $A\ {}_q^c\ F$, then by Definition 1.12, A □ F^c, where F^c is an IFROS. Then spcl(A) □ F^c, by hypothesis. Hence again by Definition 1.12, spcl(A) ${}_q^c$ F.

Sufficiency : Let U be an IFROS such that A □ U. Then U^c is an IFRCS and A □ $(U^c)^c$. By hypothesis, $A\ {}_q^c\ U^c$ □ spcl(A) ${}_q^c$ U^c. Hence by Definition 1.12, spcl(A) □ $(U^c)^c$ = U. Therefore spcl(A) □ U. Hence A is an IFRGSPCS.

Theorem 2.15 : Let (X,τ) be an IFTS. Then every IFS in (X,τ) is an IFRGSPCS if and only if IFSPO(X) = IFSPC(X).

Proof: Necessity : Suppose that every IFS in (X,τ) is an IFRGSPCS. Let U □ IFRO(X), then U □ IFSPO(X) and by hypothesis, spcl(U) □ U □ spcl(U). This implies spcl(U) = U. Therefore U □ IFSPC(X). Hence IFSPO(X) □ IFSPC(X). Let A □ IFSPC(X), then A^c □ IFSPO(X) □ IFSPC(X). That is, A^c □ IFSPC(X). Therefore A □ IFSPO(X). Hence IFSPC(X) □ IFSPO(X). Thus IFSPO(X) = IFSPC(X). **Sufficiency :** Suppose that IFSPO(X) = IFSPC(X). Let A □ U and U be an IFROS. Then U □ IFSPO(X) and spcl(A) □ spcl(U) = U, since U □ IFSPC(X), by hypothesis. Therefore A is an IFRGSPCS in X.

Theorem 2.16 : If A is an IFROS and an IFRGSPCS in (X,τ) then A is an IFSPCS in (X,τ).

Proof : Since A □ A and A is an IFROS, by hypothesis, spcl(A) □ A. But A □ spcl(A). Therefore spcl(A) = A. Hence A is an IFSPCS.

Theorem 2.17 : Let A be an IFRGSPCS in (X,τ) and p(α, β) be an IFP in X such that int(p(α, β)) $_q$ spcl(A), then cl(int(p(α, β))) $_q$ A.

Proof : Let A be an IFRGSPCS and let int(p(α, β)) $_q$ spcl(A). If cl(int(p(α, β))) $_q^c$ A then by Definition 1.12, A □ $[\mathrm{cl}(\mathrm{int}(p(\alpha, \beta)))]^c$ where $[\mathrm{cl}(\mathrm{int}(p(\alpha, \beta)))]^c$ is an IFROS. Then by hypothesis, spcl(A) □ $[\mathrm{cl}(\mathrm{int}(p(\alpha, \beta)))]^c = \mathrm{int}(\mathrm{cl}(p(\alpha, \beta)^c))$ □ $\mathrm{cl}(p(\alpha, \beta)^c) = (\mathrm{int}(p(\alpha, \beta)))^c$. This implies that spcl(A) □ $(\mathrm{int}(p(\alpha, \beta)))^c$. Therefore by Definition 1.12, int(p(α, β)) $_q^c$ spcl(A), which is a contradiction to the hypothesis. Hence cl(int(p(α, β))) $_q$ A.

Theorem 2.18 : Let F □ A □ X where A is an IFROS and an IFRGSPCS in X. Then F is an IFRGSPCS in A if and only if F is an IFRGSPCS in X.

Proof : Necessity : Let U be an IFROS in X and F □ U. Also let F be an IFRGSPCS in A. Then clearly F □ A ∩ U and A ∩ U is an IFROS in A. Hence the semipre closure of F in A, $\mathrm{spcl}_A(F)$ □ A ∩ U. By Theorem 2.16, A is an IFSPCS. Therefore spcl(A) = A and the semipre closure of F in X, spcl(F) □ spcl(F) ∩ spcl(A) = spcl(F) ∩ A = $\mathrm{spcl}_A(F)$ □ A ∩ U □ U. That is, spcl(F) □ U whenever F □ U. Hence F is an IFRGSPCS in A.

Sufficiency : Let V be an IFROS in A such that F □ V. Since A is an IFROS in X ,V is an IFROS in X. Therefore spcl(F) □ V, since F is an IFRGSPCS in X. Thus $\mathrm{spcl}_A(F)$ = spcl(F) ∩ A □ V ∩ A □ V. Hence F is an IFRGSPCS in A.

Theorem 2.19 : Let (X, τ) be an IFTS, then for every A □ IFSPC(X) and for every IFS B in X, int(A) □ B □ A □ B □ IFRGSPC(X).

Proof : Let A be an IFSPCS in X. Then by Definition 1.6, there exits an IFPCS, say C such that int(C) □ A □ C. By hypothesis, B □ A. Therefore B □ C. Since int(C) □ A, int(C) □ int(A) and int(C) □ B. Thus int(C) □ B □ C and by Definition 1.7, B □ IFSPC(X). Hence by Theorem 2.5, B □ IFRGSPC(X).

CHAPTER III

CHAPTER III

REGULAR GENERALIZED SEMIPREOPEN SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

In this chapter, we have introduced the notion of intuitionistic fuzzy regular generalized semipreopen sets and studied some of their properties.

Definition 3.1 : The complement A^c of an IFRGSPCS A in an IFTS (X,τ) is called an ***intuitionistic fuzzy regular generalized semipreopen set*** (IFRGSPOS in short) in X.

The family of all IFRGSPOSs of an IFTS (X,τ) is denoted by IFRGSPO(X).

Theorem 3.2 : Every IFOS, IFGOS, IFSOS, IFPOS, IFSPOS, IFαOS, IFβOS, IFROS is an IFRGSPOS but the converses are not true in general.

Proof: Straightforward.

Example 3.3 : Let $X = \{a, b\}$ and $G = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.6$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let $A = \langle x, (0.6, 0.7), (0.4, 0.2)\rangle$ be an IFS in X. Then, IFPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, IFSPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. As $spcl(A^c) = A^c$, we have $A^c \subseteq G^c$ implies $spcl(A^c) \subseteq G^c$, where G is an IFROS in X. This implies that A^c is an IFRGSPCS in X and hence A is an IFRGSPOS. Now since $int(A) = G \neq A$, A is not an IFOS in X. Also $G^c \subseteq A$ but $G^c \nsubseteq int(A)$. Therefore A is not an IFGOS in X. Now $int(cl(A)) = int(1\sim) = 1\sim \neq A$. Therefore A is not an IFROS in X. Hence A is an IFRGSPOS but not IFOS, IFGOS, IFROS.

Example 3.4 : Let $X = \{a, b\}$ and $G = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.6$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.4$. Then $\tau = \{0\sim, G, 1\sim\}$ is an IFT on X. Let $A = \langle x, (0.5, 0.3), (0.5, 0.7)\rangle$ be an IFS in X. Then, IFPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, IFSPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_a + \nu_a \leq 1$

and $\mu_b + \nu_b \leq 1\}$. As spcl(A^c) = 1~, we have A^c □ 1~ implies spcl(A^c) □ 1~, where 1~ is an IFRCS. This implies that A^c is an IFRGSPCS in X. Hence A is an IFRGSPOS in X. Now since A ⊈ int(cl(A)) = int(G^c) = 0~, we get A is not an IFPOS in X. Further A ⊈ cl(int(cl(A))) = cl(int(G^c)) = cl(0~) = 0~. Hence A is not an IFβOS in X. Also A ⊈ cl(int(A)) = cl(0~) = 0~. Thus A is not an IFSOS in X. Now since A ⊈ int(cl(int(A))) = int(cl(0~)) = int(0~) = 0~, A is not an IFαOS in X. Further there exists no IFPOS B such that A □ B □ cl(A). Therefore A is not an IFSPOS in X. Hence A is an IFRGSPOS but not IFPOS, IFβOS, IFSOS, IFαOS, IFSPOS.

Theorem 3.5 : Let (X, τ) be an IFTS. Then for every A □ IFRGSPO(X) and for every B □ IFS(X), spint(A) □ B □ A □ B □ IFRGSPO(X).

Proof: Let A be any IFRGSPOS of X and B be any IFS of X. Let spint(A) □ B □ A. Then A^c is an IFRGSPCS and A^c □ B^c □ spcl(A^c). Therefore B^c is an IFRGSPC, by Theorem 2.13. This implies B is an IFRGSPOS in X. Hence B □ IFRGSPO(X).

Theorem 3.6 : Let (X, τ) be an IFTS. Then for every A □ IFS(X) and for every B □ IFPO(X), B □ A □ cl(int(B)) □ A □ IFRGSPO(X).

Proof : Let B be an IFPOS. Then B □ int(cl(B)). By hypothesis, A □ cl(int(B)) □ cl(int(int(cl(B)))) = cl(int(cl(B))) □ cl(int(cl(A))) as B □ A. Therefore A is an IFβOS and by Theorem 3.2, A is an IFRGSPOS. Hence A □ IFRGSPO(X).

Theorem 3.7 : An IFS A of an IFTS (X, τ) is an IFRGSPOS if and only if F □ spint(A) whenever F is an IFRCS and F □ A.

Proof: Necessity : Suppose A is an IFRGSPOS. Let F be an IFRCS such that F □ A. Then F^c is an IFROS and A^c □ F^c. By hypothesis A^c is an IFRGSPCS, we have spcl(A^c) □ F^c. Therefore F □ spint(A).

Sufficiency : Let F be an IFRCS such that F □ A, then F □ spint(A). That is $(\text{spint}(A))^c$ □ F^c. This implies spcl(A^c) □ F^c where F^c is an IFROS. Therefore A^c is an IFRGSPCS. Hence A is an IFRGSPOS.

Theorem 3.8 : Let (X, τ) be an IFTS then for every A □ IFSPO(X) and for every IFS B in X, A □ B □ cl(A) □ B □ IFRGSPO(X).

Proof : Let A be an IFSPOS in X. Then by Definition 1.7, there exists an IFPOS, say C such that C □ A □ cl(A). By hypothesis, A □ B. Therefore C □ B. Since A □ cl(C), cl(A) □ cl(C) and B □ cl(C). Thus C □ B □ cl(C). This implies B is an IFSPOS in X. Then by Theorem 3.2, B is an IFRGSPOS. That is B □ IFRGSPO(X).

3.1 APPLICATIONS

The concept of intuitionistic fuzzy semipre $T_{1/2}$ space was introduced by Santhi, R. and Jayanthi, D[13] in 2009. In this section we have discussed some applications of intuitionistic fuzzy regular generalized semipreclosed sets.

Definition 3.1.1 : If every IFRGSPCS in (X, τ) is an IFSPCS in (X, τ), then the space can be called as an ***intuitionistic fuzzy regular semipre $T_{1/2}$ space*** ($IFRSPT_{1/2}$ in short).

Theorem 3.1.2 : An IFTS (X, τ) is an $IFRSPT_{1/2}$ space if and only if IFSPO(X) = IFRGSPO(X).

Proof : Necessity : Let A be an IFRGSPOS in (X, τ), then A^c is an IFRGSPCS in (X, τ). By hypothesis, A^c is an IFSPCS in (X, τ) and therefore A is an IFSPOS in (X, τ). Hence IFSPO(X) = IFRGSPO(X).

Sufficiency : Let A be an IFRGSPCS in (X, τ). Then A^c is an IFRGSPOS in (X, τ). By hypothesis A^c is an IFSPOS in (X, τ) and therefore A is an IFSPCS in (X, τ). Hence (X, τ) is an $IFRSPT_{1/2}$ space.

Remark 3.1.3 : Not every $IFRSPT_{1/2}$ space is an $IFT_{1/2}$ space. This can be seen easily by the following example.

Example 3.1.4 : Let X = {a, b} and G = ⟨x, (0.5, 0.4), (0.5, 0.6)⟩ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.6$. Then τ = {0~, G, 1~} is an IFT on X. Then, IFPC(X) = {0~, 1~, μ_a □ [0,1], μ_b □ [0,1], ν_a □ [0,1], ν_b □ [0,1] / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ wheneve $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Therefore, IFSPC(X) = {0~, 1~, μ_a □ [0,1], μ_b □ [0,1], ν_a □ [0,1], ν_b □ [0,1] / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Since all IFRGSPCS in X are IFSPCS in X, (X, τ) is an $IFRSPT_{1/2}$ space. But it is not a $IFT_{1/2}$ space since if A = ⟨x, (0.5, 0.7), (0.5, 0.3)⟩, then cl(A) = 1~ □ 1~ whenever A □ 1~.

Hence A is an IFGCS in X but cl(A) = 1~ ≠ A, A is not an IFCS in X. Therefore (X, τ) is not an IFT$_{1/2}$ space.

Theorem 3.1.5 : Let (X, τ) be an IFTS and X is an IFRSPT$_{1/2}$ space, then the following conditions are equivalent:

(i) A □ IFRGSPO(X) (ii) A □ cl(int(cl(A))) (iii) cl(A) □ IFRC(X).

Proof : (i) □ (ii) Let A be an IFRGSPOS. Then since X is an IFRSPT$_{1/2}$ space, A is an IFSPOS. Since every IFSPOS is an IFβOS [8] we get, A □ cl(int(cl(A))).

(ii) □ (iii) Let A □ cl(int(cl(A))). Then cl(A) □ cl(cl(int(cl(A)))) = cl(int(cl(A))) □ cl(cl(A)) = cl(A). Therefore cl(A) = cl(int(cl(A))). Hence cl(A) □ IFRC(X).

(iii) □ (i) Since cl(A) is an IFRCS, cl(A) = cl(int(cl(A))) and since A □ cl(A), A □ cl(int(cl(A))). Therefore A is an IFβOS. Hence by Theorem 3.2, A □ IFRGSPO(X).

Theorem 3.1.6 : Let (X, τ) be an IFTS and X is an IFRSPT$_{1/2}$ space, then the following conditions are equivalent:

(i) A □ IFRGSPC(X) (ii) int(cl(int(A))) □ A (iii) int(A) □ IFRO(X).

Proof : (i) □ (ii) Let A be an IFRGSPCS. Then since X is an IFRSPT$_{1/2}$ space, A is an IFSPCS. Since every IFSPCS is an IFβCS [8] we get, int(cl(int(A))) □ A.

(ii) □ (iii) Let int(cl(int(A))) □ A. Then int(A) □ int(int(cl(int(A)))) = int(cl(int(A))) □ int(int(A)) = int(A). Therefore int(cl(int(A))) = int(A). Hence int(A) □ IFRO(X).

(iii) □ (i) Since int(A) is an IFROS, int(A) = (int(cl(int(A))) and since int(A) □ (A), int(cl(int(A))) □ A. Therefore A is an IFβCS which implies A^c is an IFβOS. Hence by Theorem 3.2, A^c is an IFRGSPOS. Therefore A □ IFRGSPC(X).

Definition 3.1.7 : An IFTS (X, τ) is said to be an ***intuitionistic fuzzy regular semipre $T^*_{1/2}$ space*** (IFRSPT$^*_{1/2}$ in short) if every IFRGSPCS is an IFRCS in (X, τ).

Remark 3.1.8 : Every IFRSPT$^*_{1/2}$ space is an IFRSPT$_{1/2}$ space but not conversely.

Proof : Let (X, τ) be an IFRSPT$^*_{1/2}$ space. Let A be an IFRGSPCS in (X, τ). By hypothesis, A is an IFRCS. Since every IFRCS is an IFSPCS, A is an IFSPCS in (X, τ). Hence (X, τ) is an IFRSPT$_{1/2}$ space.

Example 3.1.9 : Let X = {a, b} and G = ⟨x, (0.5, 0.4), (0.5, 0.6)⟩ where $\mu_G(a) = 0.5$, $\mu_G(b) = 0.4$, $\nu_G(a) = 0.5$, $\nu_G(b) = 0.6$. Then τ = {0~, G, 1~} is an IFT on X. Then, IFPC(X) = {0~, 1~, μ_a □ [0,1], μ_b □ [0,1], ν_a □ [0,1], ν_b □ [0,1] / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Therefore, IFSPC(X) = {0~, 1~, μ_a □ [0,1], μ_b □ [0,1], ν_a □ [0,1], ν_b □ [0,1] / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Since all IFRGSPCS in X are IFSPCS, (X, τ) is an IFRSPT$^*_{1/2}$ space since if A = ⟨x, (0.5, 0.7), (0.5, 0.3)⟩, then spcl(A) = A □ 1~ whenever A □ 1~. Hence A is an IFRGSPCS in X but since cl(int(A)) = cl(G) = $G^c \neq$ A, A is not an IFRCS in X. Therefore (X, τ) is not an IFRSPT$^*_{1/2}$ space.

CHAPTER IV

CHAPTER IV

INTUITIONISTIC FUZZY REGULAR GENERALIZED SEMIPRE CONTINUOUS MAPPINGS

Continuity is considered to be one of the core concepts of topology. In this chapter, we have given the notion of Intuitionistic fuzzy regular generalized semipre continuous mapping. Further we have discussed the liaison of the intuitionistic fuzzy regular generalized semipre continuous mapping and few of the already existing intuitionistic fuzzy continuous mappings which is followed by the discussion of few fascinating theorems concerning intuitionistic fuzzy semipre continuous mappings in IFRGSPT$_{1/2}$ space.

Definition 4.1 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an ***intuitionistic fuzzy regular generalized semipre continuous*** (IFRGSP continuous in short) mapping if $f^{-1}(V)$ is an IFRGSPCS in (X, τ) for every IFCS V in (Y, σ).

Example 4.2 : Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ and $G_2 = \langle x, (0.6, 0.7), (0.4, 0.2)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Then, IFPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, IFSPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Now $G_2^c = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFCS in Y. Therefore $f^{-1}(G_2^c) = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$. We have $\text{spcl}(f^{-1}(G_2^c)) = f^{-1}(G_2^c)$. We have $f^{-1}(G_2^c) \subseteq G_1$. Hence $\text{spcl}(f^{-1}(G_2^c)) \subseteq G_1$, where G_1 is an IFROS in X. This implies $f^{-1}(G_2^c)$ is an IFRGSPCS in X. Therefore f is an IFRGSP continuous mapping.

Remark 4.3 : Every IF continuous mapping, IFG continuous mapping, IFS continuous mapping, IFP continuous mapping, IFSP continuous mapping, IFβ continuous mapping, IFα continuous mapping, IFGSP continuous mapping are IFRGSP continuous mapping but their converses need not be true in general.

Example 4.4 : Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ and $G_2 = \langle y, (0.6, 0.7), (0.4, 0.2)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Then, IFPC(X) = $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1],$

ν_b □ [0,1] / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Therefore, IFSPC(X) = {0~, 1~, μ_a □ [0,1], μ_b □ [0,1], ν_a □ [0,1], ν_b □ [0,1] / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}.

Now G_2^c = ⟨y, (0.4, 0.2), (0.6, 0.7)⟩ is an IFCS in Y. Therefore $f^{-1}(G_2^c)$ = ⟨x, (0.4, 0.2), (0.6, 0.7)⟩. We have spcl($f^{-1}(G_2^c)$) = $f^{-1}(G_2^c)$. We have $f^{-1}(G_2^c)$ □ G_1. Hence spcl($f^{-1}(G_2^c)$) □ G_1, where G_1 is an IFROS in X. This implies $f^{-1}(G_2^c)$ is an IFRGSPCS in X. Therefore f is an IFRGSP continuous mapping.

We have G_2 is IFOS in Y, but $f^{-1}(G_2)$ = ⟨x, (0.6, 0.7), (0.4, 0.2)⟩ is not an IFOS in X, since int($f^{-1}(G_2)$) = $G_1 \neq f^{-1}(G_2)$. Now $f^{-1}(G_2^c)$ □ G_1 but cl($f^{-1}(G_2^c)$) = $G_1^c \nsubseteq G_1$. Therefore $f^{-1}(G_2^c)$ is not an IFGCS. Hence f is an IFRGSP continuous mapping but neither IF continuous mapping nor IFG continuous mapping.

Example 4.5 : Let X = {a, b} and Y = {u, v} and G_1 = ⟨x, (0.5, 0.6), (0.5, 0.4)⟩ and G_2 = ⟨y, (0.5, 0.3), (0.5, 0.7)⟩. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are IFTs on X and Y respectively. Then, IFPC(X) = {0~, 1~, μ_a □ [0,1], μ_b □ [0,1], ν_a □ [0,1], ν_b □ [0,1] / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Therefore, IFSPC(X) = {0~, 1~, μ_a □ [0,1], μ_b □ [0,1], ν_a □ [0,1], ν_b □ [0,1] / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1$}. Now G_2^c = ⟨y, (0.5, 0.7), (0.5, 0.3)⟩ is an IFCS in Y.

Therefore $f^{-1}(G_2^c)$ = ⟨x, (0.5, 0.7), (0.5, 0.3)⟩. We have spcl($f^{-1}(G_2^c)$) = $f^{-1}(G_2^c)$. We have $f^{-1}(G_2^c)$ □ G_1. Hence spcl($f^{-1}(G_2^c)$) □ G_1, where G_1 is an IFROS in X. This implies $f^{-1}(G_2^c)$ is an IFRGSPCS in X. Therefore f is an IFRGSP continuous mapping.

We have int(cl($f^{-1}(G_2^c)$)) = int(1~) = 1~ $\nsubseteq$ $f^{-1}(G_2^c)$ which implies $f^{-1}(G_2^c)$ is not an IFSCS in X. Therefore f is not an IFS continuous mapping.

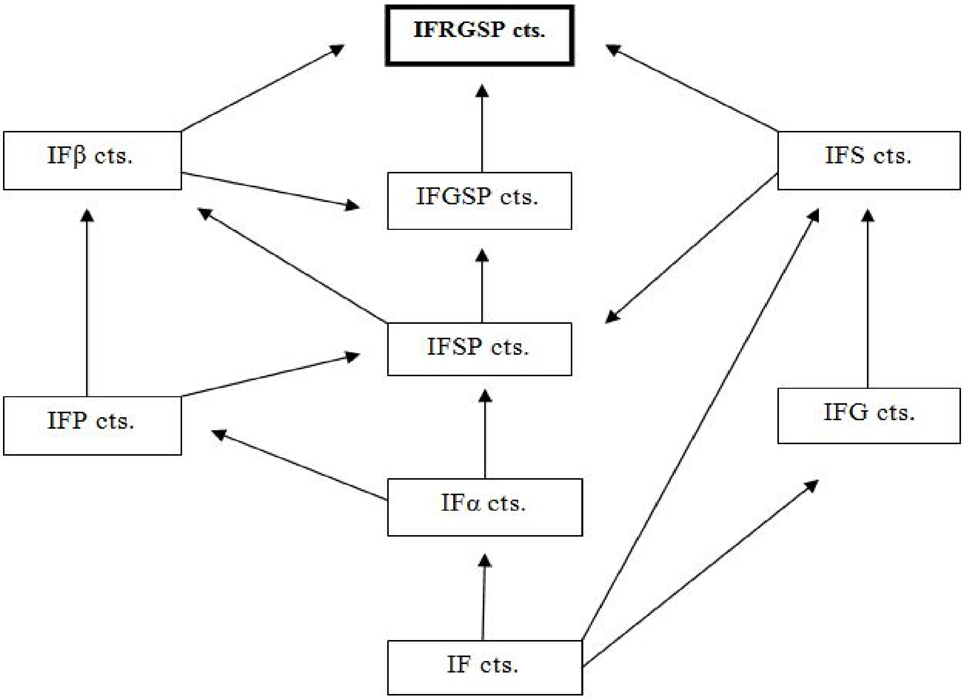
Also we have cl(int($f^{-1}(G_2^c)$)) = cl(G_1) = 1~ $\nsubseteq$ $f^{-1}(G_2^c)$ which implies $f^{-1}(G_2^c)$ is not an IFPCS in X. Hence f is not an IFP continuous mapping.

Furthermore $f^{-1}(G_2^c)$ is not an IFSPCS in X, since there exists no IFPCS B in X such that int(B) □ $f^{-1}(G_2^c)$ □ B. Therefore f is not an IFSP continuous mapping.

Also we have int(cl(int($f^{-1}(G_2^c)$))) = int(cl(G_1)) = int(1~) = 1~ $\nsubseteq$ $f^{-1}(G_2^c)$. Hence $f^{-1}(G_2^c)$ is not an IFβCS in X. Thus f is not an IFβ continuous mapping.

Further we have cl(int(cl($f^{-1}(G_2^c)$))) = cl(int(1~)) = cl(1~) = 1~ $\nsubseteq$ $f^{-1}(G_2^c)$. Therefore $f^{-1}(G_2^c)$ is not an IFαCS in X. Hence f is not an IFα continuous mapping.

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram ‘cts.’ means continuous.



Theorem 4.6 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IFRGSP continuous mapping if and only if the inverse image of each IFOS in Y is an IFRGSPOS in X.

Proof : The proof is obvious since $f^{-1}(A^c) = (f^{-1}(A))^c$.

Theorem 4.7 : If $f : (X, \tau) \to (Y, \sigma)$ is an IFRGSP continuous mapping then for each IFP $p(\alpha, \beta)$ of X and each $A \square \sigma$ such that $f(p(\alpha, \beta)) \square A$, there exists an IFRGSPOS B of X such that $p(\alpha, \beta) \square B$ and $f(B) \square A$.

Proof : Let $p(\alpha, \beta)$ be an IFP of X and $A \square \sigma$ such that $f(p(\alpha, \beta)) \square A$. Put $B = f^{-1}(A)$. Then by hypothesis B is an IFRGSPOS in X such that $p(\alpha, \beta) \square B$ and $f(B) = f(f^{-1}(A)) \square A$.

Theorem 4.8 : If $f : (X, \tau) \to (Y, \sigma)$ is an IFRGSP continuous mapping then for each IFP $p(\alpha, \beta)$ of X and each $A \square \sigma$ such that $f(p(\alpha, \beta))\ {}_q A$, there exists an IFRGSPOS of X such that $p(\alpha, \beta)\ {}_q B$ and $f(B) \square A$.

Proof : Let $p(\alpha, \beta)$ be an IFP of X and A □ σ such that $f(p(\alpha, \beta))\ _q$ A. Put $B = f^{-1}(A)$. Then by hypothesis B is an IFRGSPOS in X such that $p(\alpha, \beta)\ _q$ B and $f(B) = f(f^{-1}(A))$ □ A.

Theorem 4.9 : Let f : (X, τ) → (Y, σ) is an IFRGSP continuous mapping, then f is an IFSP continuous mapping if X is an IFRSPT$_{1/2}$ space.

Proof : Let V be an IFCS in Y. Since every IFCS is an IFGSPCS[12], V is an IFGSPCS in Y. $f^{-1}(V)$ is an IFRGSPCS in X, as f is an IFRGSP continuous mapping. Again since X is an IFRSPT$_{1/2}$ space, $f^{-1}(V)$ is an IFSPCS in X. Hence f is an IFSP continuous mapping.

Theorem 4.10 : Let f : (X, τ) → (Y, σ) is an IFRGSP continuous mapping and g : (Y, σ) → (Z, γ) is an IFG continuous mapping and Y is an IFT$_{1/2}$ space, then $g \circ f$: (X, τ) → (Z, γ) is an IFRGSP continuous mapping.

Proof : Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFGCS in Y, as g is a IFG continuous mapping. Since Y is an IFT$_{1/2}$ space, $g^{-1}(V)$ is an IFCS in Y. Therefore $f^{-1}(g^{-1}(V))$ is an IFRGSPCS in X, as f is an IFRGSP continuous mapping. Hence $g \circ f$ is an IFRGSP continuous mapping.

Theorem 4.11 : Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an IFRSPT$_{1/2}$ space:

(i) f is an IFRGSP continuous mapping

(ii) If B is an IFOS in Y then $f^{-1}(B)$ is an IFRGSPOS in X

(iii) $f^{-1}(\mathrm{int}(B))$ □ $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}(B))))$ for every IFS B in Y

Proof : (i) ⇔ (ii) is obviously true by Theorem 4.6.

(ii) □ (iii) Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(\mathrm{int}(B))$ is an IFRGSPOS in X, by hypothesis. Since X is an IFRSPT$_{1/2}$ space $f^{-1}(\mathrm{int}(B))$ is an IFSPOS in X. Therefore $f^{-1}(\mathrm{int}(B))$ □ $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}(\mathrm{int}(B)))))$ □ $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}(B))))$.

(iii) □ (i) Let B be an IFOS in Y. Then int(B) = B. By hypothesis $f^{-1}(B)$ □ $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(f^{-1}(B))))$. This implies $f^{-1}(B)$ is an IFβOS in X. Therefore it is an IFRGSPOS in X, by Theorem 3.2. Hence f is an IFRGSP continuous mapping, by Theorem 4.6.

Theorem 4.12 : Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are an IFRSPT$_{1/2}$ spaces:

(i) f is an IFRGSP continuous mapping

(ii) $int(cl(int(f^{-1}(B)))) \square f^{-1}(spcl(B))$ for each IFCS B in Y

(iii) $f^{-1}(spint(B)) \square cl(int(cl(f^{-1}(B))))$ for each IFOS B of Y

(iv) $f(int(cl(int(A)))) \square cl(f(A))$ for each IFS A of X

Proof : (i) $\square$ (ii) Let B be an IFCS in Y. This implies B is an IFSPCS, since every IFCS is an IFSPCS[16]. Therefore spcl(B) = B. Further $f^{-1}(B)$ is an IFRGSPCS in X, by hypothesis. Since X is an IFRSPT$_{1/2}$ space, $f^{-1}(B)$ is an IFSPCS in X. Furthermore since every IFSPCS is an IFβCS[8], we get $int(cl(int(f^{-1}(B)))) \square f^{-1}(B) = f^{-1}(spcl(B))$, as spcl(B) = B.

(ii) $\square$ (iii) can be easily proved by taking complement in (ii).

(iii) $\square$ (iv) Let A be an IFS in X. Then B = f(A) in Y and therefore A $\square$ $f^{-1}(B)$. Here int(f(A)) = int(B) is an IFOS in Y. Then (iii) implies that $f^{-1}(spint(int(B))) \square cl(int(cl(f^{-1}(int(B))))) \square cl(int(cl(f^{-1}(B))))$. Now $(cl(int(cl(A^c))))^c \square (cl(int(cl(f^{-1}(B)^c))))^c \square (f^{-1}(spint(int(B^c))))^c$. Therefore $int(cl(int(A))) \square f^{-1}(spcl(cl(B)))$. Now $(int(cl(int(A)))) \square f(f^{-1}(spcl(cl(B)))) \square cl(B) = cl(f(A))$.

(iv) $\square$ (i) Let B be any IFCS in Y, then $f^{-1}(B)$ is an IFS in X. By hypothesis $f(int(cl(int(f^{-1}(B))))) \square cl(f(f^{-1}(B))) \square cl(B) = B$. Now $int(cl(int(f^{-1}(B)))) \square f^{-1}(f(int(cl(int(f^{-1}(B)))))) \square f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFβCS and hence it is an IFRGSPCS in X, by Theorem 2.6. Thus f is an IFRGSP continuous mapping.

Theorem 4.13 : A mapping f : (X, τ) → (Y, σ) is an IFRGSP continuous mapping if $cl(int(cl(f^{-1}(A)))) \square f^{-1}(cl(A))$ for every IFS A in Y.

Proof : Let A be an IFOS in Y then A^c is an IFCS in Y. By hypothesis, $cl(int(cl(f^{-1}(A^c)))) \square f^{-1}(cl(A^c)) = f^{-1}(A^c)$, since A^c is an IFCS. Now $(int(cl(int(f^{-1}(A)))))^c = cl(int(cl(f^{-1}(A^c)))) \square f^{-1}(A^c) = (f^{-1}(A))^c$. This implies $f^{-1}(A) \square int(cl(int(f^{-1}(A))))$. Hence $f^{-1}(A)$ is an IFαOS in X and hence it is an IFRGSPOS in X, by Theorem 3.2. Therefore f is an IFRGSP continuous mapping, by Theorem 4.6.

Theorem 4.14 : Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y where X is an IFRSPT$_{1/2}$ space. Then f is an IFRGSP continuous mapping if and only if $\mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(A)))) \square f^{-1}(\mathrm{cl}(A))$ for every IFS A in Y.

Proof : Necessity : Let f be an IFRGSP continuous mapping then $f^{-1}(B)$ is an IFRGSPCS in X for every IFCS B in Y. Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By definition 4.1, $f^{-1}(\mathrm{cl}(A))$ is an IFRGSPCS in X. Since X is an IFRSPT$_{1/2}$ space, $f^{-1}(\mathrm{cl}(A))$ is an IFSPCS. Since every IFSPCS is an IFβCS[8],we get $f^{-1}(\mathrm{cl}(A))$ is an IFβCS in X. Therefore $\mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(\mathrm{cl}(A))))) \square f^{-1}(\mathrm{cl}(A))$. Now $\mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(A)))) \square \mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(\mathrm{cl}(A))))) \square f^{-1}(\mathrm{cl}(A))$.

Sufficiency : Let A be an IFCS in Y. By hypothesis, $\mathrm{int}(\mathrm{cl}(\mathrm{int}(f^{-1}(A)))) \square f^{-1}(\mathrm{cl}(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFβCS in X and hence it is an IFRGSPCS, by Theorem 2.6. Thus f is an IFRGSP continuous mapping.

CHAPTER V

CHAPTER V

INTUITIONISTIC FUZZY ALMOST REGULAR GENERALIZED SEMIPRE CONTINUOUS MAPPINGS

In this chapter, we have introduced the notion of intuitionistic fuzzy almost regular generalized semipre continuous mappings and some of their properties are investigated.

Definition 5.1 : A mapping $f : X \rightarrow Y$ is said to be an ***intuitionistic fuzzy almost regular generalized semipre continuous*** (IFaRGSP continuous in short) ***mapping*** if $f^{-1}(A)$ is an IFRGSPCS in X for every IFRCS A in Y.

Example 5.2 : Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ and $G_2 = \langle y, (0.2, 0.3), (0.8, 0.7)\rangle$, where $\mu_{G1}(a) = 0.5$, $\mu_{G1}(b) = 0.4$, $\nu_{G1}(a) = 0.5$, $\nu_{G1}(b) = 0.6$ and $\mu_{G2}(u) = 0.2$, $\mu_{G2}(v) = 0.3$, $\nu_{G2}(u) = 0.8$, $\nu_{G2}(v) = 0.7$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then, $IFPC(X) = \{0\sim, 1\sim, \mu_a$ □ $[0,1], \mu_b$ □ $[0,1], \nu_a$ □ $[0,1], \nu_b$ □ $[0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, $IFSPC(X) = \{0\sim, 1\sim, \mu_a$ □ $[0,1], \mu_b$ □ $[0,1], \nu_a$ □ $[0,1], \nu_b$ □ $[0,1]$ / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Now $G_2^c = \langle y, (0.8, 0.7), (0.2, 0.3)\rangle$ is an IFRCS in Y, since $cl(int(G_2^c)) = cl(G_2) = G_2^c$. We have $f^{-1}(G_2^c) = \langle x, (0.8, 0.7), (0.2, 0.3)\rangle$. Now $f^{-1}(G_2^c)$ □ $1\sim$, where $1\sim$ is an IFROS in X. Also $spcl(f^{-1}(G_2^c)) = f^{-1}(G_2^c)$ □ $1\sim$. Therefore $f^{-1}(G_2^c)$ is an IFRGSPCS in X. Thus f is an IFaRGSP continuous mapping.

Theorem 5.3 : Every IF continuous mapping is an IFaRGSP continuous mapping but not conversely.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. By Theorem 2.3, every IFCS is an IFRGSPCS. Therefore $f^{-1}(V)$ is an IFRGSPCS in X. Hence f is an IFaRGSP continuous mapping.

Example 5.4 : Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ and $G_2 = \langle y, (0.2, 0.3), (0.8, 0.7)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then, IFPC(X) = $\{0\sim, 1\sim, \mu_a$ □ $[0,1], \mu_b$ □ $[0,1], \nu_a$ □ $[0,1], \nu_b$ □ $[0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, IFSPC(X) = $\{0\sim, 1\sim, \mu_a$ □ $[0,1], \mu_b$ □ $[0,1], \nu_a$ □ $[0,1], \nu_b$ □ $[0,1]$ / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$.

Now $G_2^c = \langle y, (0.8, 0.7), (0.2, 0.3)\rangle$ is an IFRCS in Y, since $cl(int(G_2^c)) = cl(G_2) = G_2^c$. We have $f^{-1}(G_2^c) = \langle x, (0.8, 0.7), (0.2, 0.3)\rangle$. Now $f^{-1}(G_2^c)$ □ $1\sim$, where $1\sim$ is an IFROS in X. Therefore $f^{-1}(G_2^c)$ is an IFRGSPCS in X. Thus f is an IFaRGSP continuous mapping. Now $cl(f^{-1}(G_2^c)) = 1\sim \neq f^{-1}(G_2^c)$. Hence f is not an IF continuous mapping.

Theorem 5.5 : Every IFG continuous mapping is an IFaRGSP continuous mapping but not conversely.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFG continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFGCS in X. By Theorem 2.4, every IFGCS is an IFRGSPCS. Therefore $f^{-1}(V)$ is an IFRGSPCS in X. Hence f is an IFaRGSP continuous mapping.

Example 5.6 : Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.7, 0.8), (0.3, 0.2)\rangle$, $G_2 = \langle x, (0.6, 0.7), (0.4, 0.3)\rangle$, $G_3 = \langle y, (0.5, 0.4), (0.5, 0.6)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then, IFPC(X) = $\{0\sim, 1\sim, \mu_a$ □ $[0,1], \mu_b$ □ $[0,1], \nu_a$ □ $[0,1], \nu_b$ □ $[0,1]$ / $\mu_b < 0.7$ whenever $\mu_a \geq 0.6$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, IFSPC(X) = $\{0\sim, 1\sim, \mu_a$ □ $[0,1], \mu_b$ □ $[0,1], \nu_a$ □ $[0,1], \nu_b$ □ $[0,1]$ / $\mu_b < 0.7$ whenever $\mu_a \geq 0.6$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$.

Now $G_3^c = \langle y, (0.5, 0.6), (0.5, 0.4)\rangle$ is an IFRCS in Y, since $cl(int(G_3^c)) = cl(G_3) = G_3^c$. We have $f^{-1}(G_3^c) = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$ □ G_1 and $f^{-1}(G_3^c) = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$ □ G_2. Now $spcl(f^{-1}(G_3^c)) = f^{-1}(G_3^c)$. Therefore $spcl(f^{-1}(G_3^c))$ □ $1\sim$ and $spcl(f^{-1}(G_3^c))$ □ $1\sim$. Hence $f^{-1}(G_3^c)$ is an IFRGSPCS in X and hence f is an IFaRGSP continuous mapping.

We have G_3^c is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFGCS in X, since $f^{-1}(G_3^c) \square G_1$ and $f^{-1}(G_3^c) \square G_2$ but $cl(f^{-1}(G_3^c)) = 1{\sim} \nsubseteq G_1$ and $cl(f^{-1}(G_3^c)) = 1{\sim} \nsubseteq G_2$. Hence f is not an IFG continuous mapping.

Theorem 5.7 : Every IFS continuous mapping is an IFaRGSP continuous mapping but not conversely.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFS continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. By Theorem 2.7, every IFSCS is an IFRGSPCS. Therefore $f^{-1}(V)$ is an IFRGSPCS in X. Hence f is an IFaRGSP continuous mapping.

Example 5.8 : In Example 5.4 we have f is an IFaRGSP continuous mapping. Further $G_2^c = \langle y, (0.8, 0.7), (0.2, 0.3)\rangle$ is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0.8, 0.7), (0.2, 0.3)\rangle$ is not an IFSCS in X, since $int(cl(f^{-1}(G_2^c))) = int(1{\sim}) = 1{\sim} \nsubseteq f^{-1}(G_2^c)$. Hence f is not an IFS continuous mapping.

Theorem 5.9 : Every IFP continuous mapping is an IFaRGSP continuous mapping but not conversely.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFP continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. By Theorem 2.8, every IFPCS is an IFRGSPCS. Therefore $f^{-1}(V)$ is an IFRGSPCS in X. Hence f is an IFaRGSP continuous mapping.

Example 5.10 : Let X = {a, b} and Y = {u, v} and $G_1 = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$ and $G_2 = \langle y, (0.5, 0.3), (0.5, 0.7)\rangle$. Then $\tau = \{0{\sim}, G_1, 1{\sim}\}$ and $\sigma = \{0{\sim}, G_2, 1{\sim}\}$ are IFTs on X and Y respectively. Then, $IFPC(X) = \{0{\sim}, 1{\sim}, \mu_a \square [0,1], \mu_b \square [0,1], \nu_a \square [0,1], \nu_b \square [0,1] / \mu_b < 0.6$ whenever $\mu_a \geq 0.5, \mu_a < 0.5$ whenever $\mu_b \geq 0.6, \mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, $IFSPC(X) = \{0{\sim}, 1{\sim}, \mu_a \square [0,1], \mu_b \square [0,1], \nu_a \square [0,1], \nu_b \square [0,1] / \mu_b < 0.6$ whenever $\mu_a \geq 0.5, \mu_a < 0.5$ whenever $\mu_b \geq 0.6, \mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$.

Now $G_2^c = \langle y, (0.5, 0.7), (0.5, 0.3)\rangle$ is an IFRCS in Y. Therefore $f^{-1}(G_2^c) = \langle x, (0.5, 0.7), (0.5, 0.3)\rangle$ is an IFRGSPCS in X, since $f^{-1}(G_2^c) \square 1{\sim}$ and $spcl(f^{-1}(G_2^c)) = 1{\sim} \square 1{\sim}$. Hence f is an IFaRGSP continuous mapping.

We have $G_2^c = \langle y, (0.5, 0.7), (0.5, 0.3)\rangle$ is an IFCS in Y but $f^{-1}(G_2^c)$ is not an IFPCS in X, since $cl(int(f^{-1}(G_2^c))) = cl(G_1) = 1_\sim \not\subseteq f^{-1}(G_2^c)$. Hence f is not an IFP continuous mapping.

Theorem 5.11 : Every IFSP continuous mapping is an IFaRGSP continuous mapping but not conversely.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFSP continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFSPCS in X. By Theorem 2.5, every IFSPCS is IFRGSPCS. Therefore $f^{-1}(V)$ is an IFRGSPCS in X. Hence f is an IFaRGSP continuous mapping.

Example 5.12 : In Example 5.10, f is an IFaRGSP continuous mapping. Further $G_2^c = \langle y, (0.5, 0.7), (0.5, 0.3)\rangle$ is an IFCS in Y but $f^{-1}(G_2^c)$ is not an IFSPCS in X, since there exists no IFPCS B in X such that $int(f^{-1}(G_2^c))$ □ B □ $f^{-1}(G_2^c)$. Hence f is not an IFSP continuous mapping.

Theorem 5.13 : Every IFα continuous mapping is an IFaRGSP continuous mapping but not conversely.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFα continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X. By Theorem 2.9, every IFαCS is an IFRGSPCS. Therefore $f^{-1}(V)$ is an IFRGSPCS in X. Hence f is an IFaRGSP continuous mapping.

Example 5.14 : In Example 5.4, f is an IFaRGSP continuous mapping. Further $G_2^c = \langle y, (0.8, 0.7), (0.2, 0.3)\rangle$ is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0.8, 0.7), (0.2, 0.3)\rangle$ is not an IFαCS in X, since $cl(int(cl(f^{-1}(G_2^c)))) = cl(int(1_\sim)) = cl(1_\sim) = 1_\sim \not\subseteq f^{-1}(G_2^c)$. Hence f is not an IFα continuous mapping.

Theorem 5.15 : Every IFβ continuous mapping is an IFaRGSP continuous mapping but not conversely.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFβ continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFβCS in X. By

Theorem 2.6, every IFβCS is an IFRGSPCS. Therefore $f^{-1}(V)$ is an IFRGSPCS in X. Hence f is an IFaRGSP continuous mapping.

Example 5.16 : In Example 6.10, f is an IFaRGSP continuous mapping. Further $G_2^c = \langle y, (0.5, 0.7), (0.5, 0.3)\rangle$ is an IFCS in Y but $f^{-1}(G_2^c)$ is not an IFβCS in X, since $int(cl(int(f^{-1}(G_2^c)))) = int(cl(G_1)) = int(1\sim) = 1\sim \not\subseteq f^{-1}(G_2^c)$. Hence f is not an IFβ continuous mapping.

Theorem 5.17 : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping where $f^{-1}(A)$ is an IFRCS in X for every IFCS in Y. Then f is an IFaRGSP continuous mapping.

Proof : Let A be an IFRCS in Y. Since every IFRCS is an IFCS, A is an IFCS in Y. Then $f^{-1}(A)$ is an IFRCS in X. By Theorem 2.10, every IFRCS is an IFRGSPCS. Therefore $f^{-1}(A)$ is an IFRGSPCS in X. Hence f is an IFaRGSP continuous mapping.

Theorem 5.18 : Let $f : X \rightarrow Y$ be a mapping. Then the following conditions are equivalent:

(i) f is an IFaRGSP continuous mapping

(ii) $f^{-1}(A)$ is an IFRGSPOS in X for every IFROS A in Y

Proof : (i) □ (ii) Let A be an IFROS in Y. Then A^c is an IFRCS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFRGSPCS in X. That is $f^{-1}(A)^c$ is an IFRGSPCS in X. Therefore $f^{-1}(A)$ is an IFRGSPOS in X.

(ii) □ (i) Let A be an IFRCS in Y. Then A^c is an IFROS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFRGSPOS in X. That is $f^{-1}(A)^c$ is an IFRGSPOS in X. Therefore $f^{-1}(A)$ is an IFRGSPCS in X. Hence f is an IFaRGSP continuous mapping.

Theorem 5.19 : Let $f : X \rightarrow Y$ be a mapping where X is an $IFRSPT_{1/2}$ space. Then the following are equivalent:

(i) f is an IFaRGSP continuous mapping

(ii) $spcl(f^{-1}(A))$ □ $f^{-1}(cl(A))$ for every IFSPOS A in Y

(iii) $spcl(f^{-1}(A))$ □ $f^{-1}(cl(A))$ for every IFSOS A in Y

(iv) $f^{-1}(A)$ □ $spint(f^{-1}(int(cl(A))))$ for every IFPOS A in Y

Proof : (i) □ (ii) Let A be an IFSPOS in Y. Then cl(A) is an IFRCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IFRGSPCS in X. Therefore $f^{-1}(cl(A))$ is an IFSPCS in X, since X is an IFRSPT$_{1/2}$ space. This implies spcl(f^{-1}(cl(A))) = f^{-1}(cl(A)). Now spcl(f^{-1}(A)) □ spcl(f^{-1}(cl(A))) = f^{-1}(cl(A)). Thus spcl(f^{-1}(A)) □ f^{-1}(cl(A)).

(ii) □ (iii) Since every IFSOS is an IFSPOS, proof is similar as in (i) □ (ii).

(iii) □ (i) Let A be an IFRCS in Y. Then A = cl(int(A)). Therefore A is an IFSOS in Y. By hypothesis, spcl(f^{-1}(A)) □ f^{-1}(cl(A)) = f^{-1}(A). Further f^{-1}(A) □ spcl(f^{-1}(A)). Therefore spcl(f^{-1}(A)) = f^{-1}(A). This implies f^{-1}(A) is an IFSPCS in X and hence an IFRGSPCS in X, by Theorem 2.5. Thus f is an IFaRGSP continuous mapping.

(i) □ (iv) Let A be an IFPOS in Y. Then A □ int(cl(A)). Since int(cl(A)) is an IFROS in Y, by hypothesis, f^{-1}(int(cl(A))) is an IFRGSPOS in X. Since X is an IFRSPT$_{1/2}$ space, f^{-1}(int(cl(A))) is an IFSPOS in X. Therefore f^{-1}(A) □ f^{-1}(int(cl(A))) = spint(f^{-1}(int(cl(A)))).

(iv) □ (i) Let A be an IFROS in Y. Then A is an IFPOS in X. By hypothesis, f^{-1}(A) □ spint(f^{-1}(int(cl(A)))) = spint(f^{-1}(A)) □ f^{-1}(A). This implies f^{-1}(A) is an IFSPOS in X and hence is an IFRGSPOS in X, by Theorem 3.2. Therefore f is an IFaRGSP continuous mapping.

Theorem 5.20 : Let f : X → Y be a mapping. If f^{-1}(spint(B)) □ spint(f^{-1}(B)) for every IFS B in Y, then f is an IFaRGSP continuous mapping.

Proof : Let B □ Y be an IFROS. By hypothesis, f^{-1}(spint(B)) □ spint(f^{-1}(B)). Since B is an IFROS, it is an IFSPOS in Y[16]. Therefore spint(B) = B. Hence f^{-1}(B) = f^{-1}(spint(B)) □ spint(f^{-1}(B)) □ f^{-1}(B). This implies f^{-1}(B) is an IFSPOS and hence an IFRGSPOS in X. Thus f is an IFaRGSP continuous mapping.

Remark 5.21 : The converse of the above theorem is true if B □ Y is an IFROS and X is an IFRSPT$_{1/2}$ space.

Proof: Let f be an IFaRGSP continuous mapping. Let B be an IFROS in Y then it is an IFRGSPOS in Y, by Theorem 3.2. Since X is an IFRSPT$_{1/2}$ space, f^{-1}(B) is an IFSPOS in X. Therefore f^{-1}(spint(B)) □ f^{-1}(B) = spint(f^{-1}(B)).

Theorem 5.22 : Let f : X → Y be a mapping. If spcl(f^{-1}(B)) □ f^{-1}(spcl(B)) for every IFS B in Y, then f is an IFaRGSP continuous mapping.

Proof : Let B □ Y be an IFRCS. By hypothesis, f^{-1}(spcl(B)) □ spcl(f^{-1}(B)). Since B is an IFRCS, it is an IFSPCS in Y[16]. Therefore spcl(B) = B. Hence f^{-1}(B) = f^{-1}(spcl(B))

□ spcl(f^{-1}(B)). This implies f^{-1}(B) is an IFSPCS and hence an IFRGSPCS in X. Thus f is an IFaRGSP continuous mapping.

Remark 5.23 : The converse of the above theorem is true if B □ Y is an IFRCS and X is an IFRSPT$_{1/2}$ space.

Proof : Let f be an IFaRGSP continuous mapping. Let B be an IFRCS in Y then it is an IFRGSPCS in Y, by Theorem 2.10. Since X is an IFRSPT$_{1/2}$ space, f^{-1}(B) is an IFSPCS in X. Therefore spcl(f^{-1}(B)) = f^{-1}(B) □ f^{-1}(spcl(B)).

Theorem 5.24 : Let f : X → Y be a mapping. Then the following conditions are equivalent if X is an IFRSPT$_{1/2}$ space:

(i) f is an IFaRGSP continuous mapping

(ii) spcl(f^{-1}(A)) □ f^{-1}(αcl(A)) for every IFSPOS A in Y

(iii) spcl(f^{-1}(A)) □ f^{-1}(αcl(A)) for every IFSOS A in Y

(iv) f^{-1}(A) □ spint(f^{-1}(scl(A))) for every IFPOS A in Y

Proof : (i) □ (ii) Let A be an IFSPOS in Y. Then cl(A) is an IFRCS in Y. By hypothesis f^{-1}(cl(A)) is an IFRGSPCS in X and hence is an IFSPCS in X, since X is an IFRSPT$_{1/2}$ space. This implies spcl(f^{-1}(cl(A))) = f^{-1}(cl(A)). Now spcl(f^{-1}(A)) □ spcl(f^{-1}(cl(A))) = f^{-1}(cl(A)). Since cl(A) is an IFRCS, cl(int(cl(A))) = cl(A). Now spcl(f^{-1}(A)) □ f^{-1}(cl(A)) = f^{-1}(cl(int(cl(A)))) □ f^{-1}(A □ cl(int(cl(A)))) = f^{-1}(αcl(A)). Hence spcl(f^{-1}(A)) □ f^{-1}(αcl(A)).

(ii) □ (iii) Let A be an IFSOS in Y. Since every IFSOS is an IFSPOS[16], the proof is obvious.

(iii) □ (i) Let A be an IFRCS in Y. Then A =cl(int(A)). Therefore A is an IFSOS in Y . By hypothesis, spcl(f^{-1}(A)) □ f^{-1}(αcl(A)) □ f^{-1}(cl(A)) = f^{-1}(A) □ spcl(f^{-1}(A)). That is spcl(f^{-1}(A)) = f^{-1}(A). Hence f^{-1}(A) is an IFSPCS and hence is an IFRGSPCS in X. Thus f is an IFaRGSP continuous mapping.

(i) □ (iv) Let A be an IFPOS in Y . Then A □ int(cl(A)). Since int(cl(A)) is an IFROS in Y, by hypothesis, f^{-1}(int(cl(A))) is an IFRGSPOS in X. Since X is an IFRSPT$_{1/2}$ space, f^{-1}(int(cl(A))) is an IFSPOS in X. Therefore f^{-1}(A) □ f^{-1}(int(cl(A))) = spint(f^{-1}(int(cl(A)))) □ spint(f^{-1}(A □ int(cl(A)))) = spint(f^{-1}(scl(A))). That is f^{-1}(A) □ spint(f^{-1}(scl(A))).

(iv) □ (i) Let A be an IFROS in Y. Then A is an IFPOS in Y[9]. By hypothesis, f^{-1}(A) □ spint(f^{-1}(scl(A))). This implies f^{-1}(A) □ spint(f^{-1}(A □ int(cl(A)))) = spint(f^{-1}(A □ A)) = spint(f^{-1}(A)) □ f^{-1}(A). Therefore f^{-1}(A) is an IFSPOS in X and hence an IFRGSPOS in X, by Theorem 3.2. Thus f is an IFaRGSP continuous mapping.

CHAPTER VI

CHAPTER VI

CONTRA REGULAR GENERALIZED SEMIPRE CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

In this chapter, we have introduced the notion of intuitionistic fuzzy contra regular generalized semipre continuous mappings and few of its properties are discussed.

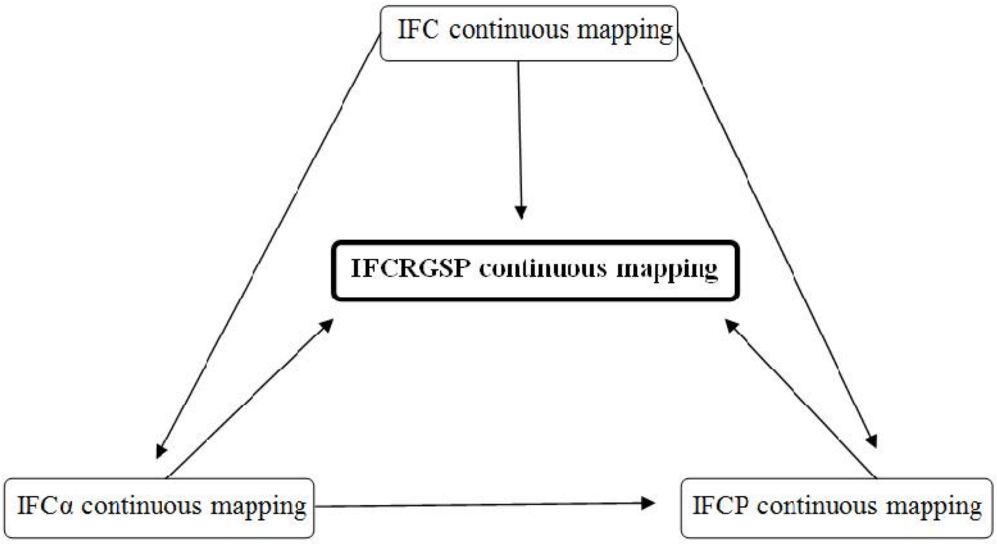
Definition 6.1 : A mapping $f : X \to Y$ is said to be an ***intuitionistic fuzzy contra regular generalized semipre continuous*** (IFCRGSP continuous in short) mapping if $f^{-1}(A)$ is an IFRGSPCS in X for every IFOS A in Y.

Example 6.2 : Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ and $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Then, $IFPC(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, \text{either } \mu_b \geq 0.6 \text{ or } \mu_b < 0.4 \text{ whenever } \mu_a \geq 0.5, \mu_a + \nu_a \leq 1 \text{ and } \mu_b + \nu_b \leq 1\}$. Therefore, $IFSPC(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, \mu_a + \nu_a \leq 1 \text{ and } \mu_b + \nu_b \leq 1\}$. Now $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFOS in Y.

Therefore $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$. We have $spcl(f^{-1}(G_2)) = f^{-1}(G_2)$. We have $f^{-1}(G_2) \subseteq G_1$. Hence $spcl(f^{-1}(G_2)) \subseteq G_1$, where G_1 is an IFROS in X. This implies $f^{-1}(G_2)$ is an IFRGSPCS in X. Therefore f is an IFCRGSP continuous mapping.

Remark 6.3 : Every IFC continuous mapping, IFCα continuous mapping, IFCP continuous mapping is an IFCRGSP continuous mapping but their converses need not hold in general.

This can be observed from the following diagram and examples.



Example 6.4 : Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6)\rangle$ and $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Then, $IFPC(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / either $\mu_b \geq 0.6$ or $\mu_b < 0.4$ whenever $\mu_a \geq 0.5$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, $IFSPC(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Now $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFOS in Y. Therefore $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$. We have $spcl(f^{-1}(G_2)) = f^{-1}(G_2)$. We have $f^{-1}(G_2) \subseteq G_1$. Hence $spcl(f^{-1}(G_2)) \subseteq G_1$, where G_1 is an IFROS in X. This implies $f^{-1}(G_2)$ is an IFRGSPCS in X. Therefore f is an IFCRGSP continuous mapping. We have $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7)\rangle$ is an IFOS in Y. But $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$ is not an IFCS in X, since $cl(f^{-1}(G_2)) = G_1^c \neq f^{-1}(G_2)$. This implies f is not an IFC continuous mapping. Further we have $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.6, 0.7)\rangle$ is not an IFαCS in X, since $cl(int(cl(f^{-1}(G_2)))) = cl(int(G_1^c)) = cl(G_1) = G_1^c \nsubseteq f^{-1}(G_2)$. Therefore f is not an IFCα continuous mapping. Hence f is an IFCRGSP continuous mapping but it is neither an IFC continuous mapping nor an IFCα continuous mapping.

Example 6.5 : Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.6), (0.5, 0.4)\rangle$ and $G_2 = \langle y, (0.5, 0.7), (0.5, 0.3)\rangle$. Then $\tau = \{0\sim, G_1, 1\sim\}$ and $\sigma = \{0\sim, G_2, 1\sim\}$ are IFTs on X and Y respectively. Then, $IFPC(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_a + \nu_a \leq 1$ and $\mu_b + \nu_b \leq 1\}$. Therefore, $IFSPC(X) = \{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]$ / $\mu_b < 0.6$ whenever $\mu_a \geq 0.5$, $\mu_a < 0.5$ whenever $\mu_b \geq 0.6$, $\mu_a + \nu_a \leq 1$ and

$\mu_b + \nu_b \leq 1\}$. Now $G_2 = \langle y, (0.5, 0.7), (0.5, 0.3)\rangle$ is an IFOS in Y. Therefore $f^{-1}(G_2) = \langle x, (0.5, 0.7), (0.5, 0.3)\rangle$. We have $spcl(f^{-1}(G_2)) = f^{-1}(G_2)$. We have $f^{-1}(G_2^c)$ □ 1~. Hence $spcl(f^{-1}(G_2^c))$ □ 1~, where 1~ is an IFROS in X. This implies $f^{-1}(G_2^c)$ is an IFRGSPCS in X. Therefore f is an IFCRGSP continuous mapping. We have $G_2 = \langle y, (0.5, 0.7), (0.5, 0.3)\rangle$ is an IFOS in Y. But since $cl(int(f^{-1}(G_2))) = cl(G_1) = 1\sim \not\subseteq f^{-1}(G_2)$, it is not an IFPCS in X. Hence f is not an IFCP continuous mapping.

Theorem 6.6 : Let f : X → Y be a mapping. Then f is an IFCRGSP continuous mapping if and only if $f^{-1}(A)$ is an IFRGSPOS in X for every IFCS A in Y.

Proof : Necessity : Let A be an IFCS in Y. Then A^c is an IFOS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFRGSPCS in X. Since, $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRGSPOS in X.

Sufficiency : Let A be an IFOS in Y. Then A^c is an IFCS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFRGSPOS in X. Since, $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRGSPCS in X. Thus f is an IFCRGSP continuous mapping.

Theorem 6.7 : Let f : X → Y be a bijective mapping. Suppose that one of the following properties hold:

(i) $f^{-1}(cl(B))$ □ $int(spcl(f^{-1}(B)))$ for each IFS B in Y

(ii) $cl(spint(f^{-1}(B)))$ □ $f^{-1}(int(B))$ for each IFS B in Y

(iii) $f(cl(spint(A)))$ □ $int(f(A))$ for each IFS A in X

(iv) $f(cl(A))$ □ $int(f(A))$ for each IFSPOS A in X

Then f is an IFCRGSP continuous mapping.

Proof : (i) □ (ii) is obvious by taking complement of (i).

(ii) □ (iii) Let A □ X. Put $B = f(A)$ in Y. This implies $A = f^{-1}(f(A)) = f^{-1}(B)$ in X. Now $cl(spint(A)) = cl(spint(f^{-1}(B))) \subseteq f^{-1}(int(B))$ by (ii). Therefore $f(cl(spint(A)))$ □ $f(f^{-1}(int(B))) = int(B) = int(f(A))$.

(iii) □ (iv) Let A □ X be an IFSPOS. Then $spint(A) = A$. By hypothesis, $f(cl(sp\ int(A)))$ □ $int(f(A))$. Therefore $f(cl(A)) = f(cl(spint(A)))$ □ $int(f(A))$.

Suppose (iv) holds. Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFS in X and $\mathrm{spint}(f^{-1}(A))$ is an IFSPOS in X. Hence by hypothesis, $f(\mathrm{cl}(\mathrm{spint}(f^{-1}(A)))) \square \mathrm{int}(f(\mathrm{spint}(f^{-1}(A)))) \square \mathrm{int}(f(f^{-1}(A))) = \mathrm{int}(A) \square A$. Therefore $\mathrm{cl}(\mathrm{spint}(f^{-1}(A))) = f^{-1}(f(\mathrm{cl}(\mathrm{spint}(f^{-1}(A))))) \square f^{-1}(A)$. Now $\mathrm{cl}(\mathrm{int}(f^{-1}(A))) \square \mathrm{cl}(\mathrm{spint}(f^{-1}(A))) \square f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFPCS in X and hence an IFRGSPCS in X, by Theorem 2.8. Thus f is an IFCRGSP continuous mapping.

Theorem 6.8 : Let $f : X \rightarrow Y$ be a mapping. Suppose that one of the following properties hold:

(i) $f(\mathrm{spcl}(A)) \square \mathrm{int}(f(A))$ for each IFS A in X

(ii) $\mathrm{spcl}(f^{-1}(B)) \square f^{-1}(\mathrm{int}(B))$ for each IFS B in Y

(iii) $f^{-1}(\mathrm{cl}(B)) \square \mathrm{spint}(f^{-1}(B))$ for each IFS B in Y

Then f is an IFCRGSP continuous mapping.

Proof : (i) □ (ii) Let B □ Y. Then $f^{-1}(B)$ is an IFS in X. By hypothesis, $f(\mathrm{spcl}(f^{-1}(B))) \square \mathrm{int}(f(f^{-1}(B))) \square \mathrm{int}(B)$. Now $\mathrm{spcl}(f^{-1}(B)) \square f^{-1}(f(\mathrm{spcl}(f^{-1}(B)))) \square f^{-1}(\mathrm{int}(B))$.

(ii) □ (iii) is obvious by taking complement in (ii).

Suppose (iii) holds. Let A be an IFCS in Y. Then cl(A) = A and $f^{-1}(A)$ is an IFS in X. Now $f^{-1}(A) = f^{-1}(\mathrm{cl}(A)) \square \mathrm{spint}(f^{-1}(A)) \square f^{-1}(A)$, by hypothesis. This implies $f^{-1}(A)$ is an IFSPOS in X and hence an IFRGSPOS in X, by Theorem 3.2. Therefore f is an IFCRGSP continuous mapping.

Theorem 6.9 : Let $f : X \rightarrow Y$ be a bijective mapping. Then f is an IFCRGSP continuous mapping if $\mathrm{cl}(f(A)) \square f(\mathrm{spint}(A))$ for every IFS A in X.

Proof : Let A be an IFCS in Y. Then cl(A) = A and $f^{-1}(A)$ is an IFS in X. By hypothesis $\mathrm{cl}(f(f^{-1}(A))) \square f(\mathrm{spint}(f^{-1}(A)))$. Since f is onto, $f(f^{-1}(A)) = A$. Therefore $A = \mathrm{cl}(A) = \mathrm{cl}(f(f^{-1}(A))) \square f(\mathrm{spint}(f^{-1}(A)))$. Now $f^{-1}(A) \square f^{-1}(f(\mathrm{spint}(f^{-1}(A)))) = \mathrm{spint}(f^{-1}(A)) \square f^{-1}(A)$. Hence $f^{-1}(A)$ is an IFSPOS in X and hence an IFRGSPOS in X, by Theorem 3.2. Thus f is an IFCRGSP continuous mapping.

Theorem 6.10 : If f : X → Y is an IFCRGSP continuous mapping, where X is an IFRSPT$_{1/2}$ space, then the following conditions hold:

(i) spcl(f^{-1}(B)) □ f^{-1}(int(spcl(B))) for every IFOS in Y

(ii) f^{-1}(cl(spint(B))) □ spint(f^{-1}(B)) for every IFCS B in Y.

Proof : (i) Let B □ Y be an IFOS. By hypothesis f^{-1}(B) is an IFRGSPCS in X. Since X is an IFRSPT$_{1/2}$ space, f^{-1}(B) is an IFSPCS in X. This implies spcl(f^{-1}(B)) = f^{-1}(B) □ f^{-1}(int(B)) □ f^{-1}(int(spcl(B))).

(ii) can be proved easily by taking the complement of (i).

Theorem 6.11 : If f : X → Y is an IFCRGSP continuous mapping and g : Y → Z is an IF continuous mapping then g ∘ f : X → Z is an IFCRGSP continuous mapping.

Proof : Let A be an IFOS in Z. Then g^{-1}(A) is an IFOS in Y, since g is an IF continuous mapping. Since f is an IFCRGSP continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFRGSPCS in X. Therefore g ∘ f is an IFCRGSP continuous mapping.

Theorem 6.12 : If f : X → Y is an IFCRGSP continuous mapping and g : Y → Z is an IFC continuous mapping then g ∘ f : X → Z is an IFRGSP continuous mapping.

Proof : Let A be an IFOS in Z. Then g^{-1}(A) is an IFCS in Y, since g is an IFC continuous mapping. Since f is an IFCRGSP continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFRGSPOS in X. Therefore g ∘ f is an IFRGSP continuous mapping.

Theorem 6.13 : For a mapping f : X → Y, where X is an IFRSPT$_{1/2}$ space, the following are equivalent:

(i) f is an IFCRGSP continuous mapping

(ii) For every IFCS A in Y and for any IFP $p(\alpha, \beta)$ □ X, if $f(p(\alpha, \beta))\ {}_q\ A$ then $p(\alpha, \beta)\ {}_q$ spint(f^{-1}(A))

(iii) For every IFCS A in Y and for any IFP $p(\alpha, \beta)$ □ X, if $f(p(\alpha, \beta))\ {}_q\ A$ then there exists an IFRGSPOS B such that $p(\alpha, \beta)\ {}_q\ B$ and f(B) □ A.

Proof : (i) □ (ii) Let f be an IFCRGSP continuous mapping. Let A □ Y be an IFCS and let $p(\alpha, \beta)$ □ X. Also let $f(p(\alpha, \beta))\ {}_q\ A$ then $p(\alpha, \beta)\ {}_q\ f^{-1}(A)$. By hypothesis f^{-1}(A) is an IFRGSPOS in X. Since X is an IFRSPT$_{1/2}$ space, $f^{-1}(A)$ is an IFSPOS in X. Hence spint($f^{-1}(A)$) = $f^{-1}(A)$. This implies p(α, β) $_q$ spint($f^{-1}(A)$).

(ii) □ (i) Let A □ Y be an IFCS then $f^{-1}(A)$ is an IFCS in X. Let p(α, β) □ X and let f(p(α, β)) $_q$ A then p(α, β) $_q$ $f^{-1}(A)$. By hypothesis this implies p(α, β) $_q$ spint($f^{-1}(A)$). That is $f^{-1}(A)$ □ spint($f^{-1}(A)$). But spint($f^{-1}(A)$) □ $f^{-1}(A)$. Therefore spint($f^{-1}(A)$) = $f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFSPOS in X and hence an IFRGSPOS in X, by Theorem 3.2. This implies f is an IFCRGSP continuous mapping.

(ii) □ (iii) Let Let A □ Y be an IFCS then $f^{-1}(A)$ is an IFCS in X. Let p(α, β) □ X. Also let f(p(α, β)) $_q$ A then p(α, β) $_q$ $f^{-1}(A)$. By hypothesis this implies p(α, β) $_q$ spint($f^{-1}(A)$). That is $f^{-1}(A)$ □ spint($f^{-1}(A)$). But spint($f^{-1}(A)$) □ $f^{-1}(A)$. Therefore spint($f^{-1}(A)$) = $f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFSPOS in X and hence an IFRGSPOS in X, by Theorem 3.2. Let $f^{-1}(A)$ = B. Therefore p(α, β) $_q$ B and f(B) □ f($f^{-1}(A)$) □ A.

(iii) □ (ii) Let A □ Y be an IFCS then $f^{-1}(A)$ is an IFCS in X. Let p(α, β) □ X. Also let f(p(α, β)) $_q$ A then p(α, β) $_q$ $f^{-1}(A)$. By hypothesis there exists an IFRGSPOS B in X such that p(α, β) $_q$ B and f(B) □ A. Let B = $f^{-1}(A)$. Since X is an IFRSPT$_{1/2}$ space, $f^{-1}(A)$ is an IFSPOS in X. Therefore p(α, β) $_q$ spint($f^{-1}(A)$).

Theorem 6.14 : A mapping f : X → Y is an IFCRGSP continuous mapping if f^{-1}(spcl(B)) □ int($f^{-1}(B)$) for every IFS B in Y.

Proof: Let B □ Y be an IFCS. Then cl(B) = B. Since every IFCS is an IFSPCS[16]. Therefore we get spcl(B) = B. Now by hypothesis, $f^{-1}(B)$ = f^{-1}(spcl(B)) □ int($f^{-1}(B)$) □ $f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFOS in X and hence an IFRGSPOS in X, by Theorem 3.2. Therefore f is an IFCRGSP continuous mapping.

Theorem 6.15 : A mapping f : X → Y is an IFCRGSP continuous mapping, where X is an IFRSPT$_{1/2}$ space if and only if f^{-1}(spcl(B)) □ spint(f^{-1}(cl((B))) for every IFS B in Y.

Proof : Necessity : Let B □ Y be an IFS. Then cl(B) is an IFCS in Y. By hypothesis, f^{-1}(cl(B)) is an IFRGSPOS in X. Since X is an IFRSPT$_{1/2}$ space, f^{-1}(cl(B)) is an IFSPOS in X. Therefore f^{-1}(spcl(B)) □ f^{-1}(cl(B)) = spint(f^{-1}(cl(B))).

Sufficiency : Let B □ Y be an IFCS. Then cl(B) = B. By hypothesis, f^{-1}(spcl(B)) □ spint(f^{-1}(cl((B))) = spint(f^{-1}(B)). But spcl(B) = B. Therefore f^{-1}(B) = f^{-1}(spcl(B)) □ spint(f^{-1}(B)) □ f^{-1}(B). This implies f^{-1}(B) is an IFSPOS in X and hence an IFRGSPOS in X, by Theorem 3.2. Hence f is an IFCRGSP continuous mapping.

Theorem 6.16 : An IF continuous mapping f : X → Y is an IFCRGSP continuous mapping if IFRGSPO(X) = IFRGSPC(X).

Proof : Let A □ Y be an IFOS. By hypothesis, f^{-1}(A) is an IFOS in X and hence is an IFRGSPOS in X, by Theorem 3.2. Thus f^{-1}(A) is an IFRGSPCS in X, since IFRGSPO(X) = IFRGSPC(X). Therefore f is an IFCRGSP continuous mapping.

SUMMARY AND CONCLUSION

SUMMARY AND CONCLUSION

In this thesis the notions of intuitionistic fuzzy regular generalized semipreclosed set, intuitionistic fuzzy regular generalized semipreopen set, intuitionistic fuzzy regular generalized semipre continuous mapping, intuitionistic fuzzy contra regular generalized semipre continuous mapping, intuitionistic fuzzy almost regular generalized semipre continuous mapping are discussed. In each of the chapters, the notions of the previously mentioned concepts are given. This is followed by their respective examples and corresponding comparisons. Furthermore, some interesting and unique theorems concerning each of the above mentioned concepts are presented.

We conclude that this particular research work made us enjoy the interesting outcomes of adding new concepts to the already existing concepts of intuitionistic fuzzy sets. The comparison of these new concepts with the already existing ones made us realize the fascinating nature of intuitionistic fuzzy sets. Additionally the pleasing theorems under each of the topics under consideration created a passion to work with the enthralling aspects of intuitionistic fuzzy sets.

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LIST OF PUBLICATIONS

PUBLICATIONS

[1] **Vaishnavy, V.,** and **Jayanthi, D.,** On Intuitionistic fuzzy regular generalized semipreclosed sets, International Journal of Engineering Sciences and Management, Vol. 5 Issue 01, January - March, 2015, 225 - 230.

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[3] **Vaishnavy, V.,** and **Jayanthi, D.,** On intuitionistic fuzzy regular generalized semipre continuous mappings. (to appear)

[4] **Vaishnavy, V.,** and **Jayanthi, D.,** On intuitionistic fuzzy almost regular generalized semipre continuous mappings. (to appear)

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