# $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces

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**By**

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# DECLARATION

I declare that the dissertation entitled **$b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces** submitted by me for the degree of **Master of Philosophy (M.Phil.)** is the record of work carried out by me during the period from August 2018 to July 2019 under the guidance of **Dr. D.Jayanthi, M.Sc., M.Phil., Ph.D.,** Assistant Professor (SS), Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for women, Coimbatore, and has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship, Titles in this University or any other University or other similar institution of Higher Learning.

S. Dhiya

**Signature of the Candidate**

# CERTIFICATE

This is to certify that the dissertation entitled **$b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces** submitted for the degree of **Master of Philosophy (M.Phil.)** by **Dhivya S.** is the record of research work carried out by her during the period from August 2018 to July 2019 under my guidance and supervision, and that this work has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship, Titles in this University or any other University or other similar institution of Higher Learning.

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31.7.19

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**Head of the Department**

31/07/19

**Signature of the Supervisor**

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# CONTENT

# Introduction

Fuzzy sets are introduced by Zadeh in 1965 as follows: a fuzzy set A in a non empty set X is a mapping from X to the unit interval [0,1] and A(x) is interpreted as the degree of membership of x in A. A fuzzy set is created to describe linguistic variables in more detail. The linguistic variable temperature for instance may have categories (members) of cold, very cold, moderate, warm and very hot. Once these categories or members are defined, the function is then developed for each member in the set. Chang (1968) introduced the notions of fuzzy topology and more basic concepts like open set, closed set, neighbourhood, interior of a set, continuity and compactness etc.

Intuitionistic fuzzy set, an extension of fuzzy set, has been introduced by Atanassov in 1986. Intuitionistic fuzzy set has been found to be more efficient in dealing with vagueness and ambiguity. Intuitionistic fuzzy sets as a generalization of fuzzy sets can be useful in situations when description of a problem by a (fuzzy) linguistic variable, given in terms of a membership function only, seems too rough.

Intuitionistic fuzzy sets take into account both the degree of membership and of non membership subject to the condition that their sum does not exceed 1. Let $E$ be the set of all countries with elective governments. Assume that we know for every country $x \in E$ the percentage of the electorate that have voted for the corresponding government. Denote it by M(x) and let $\mu(x) = M(x) \,/\, 100$ (degree of membership, validity, etc.). Let $\nu(x) = 1 - \mu(x)$. This number corresponds to the part of electorate who have not voted for the government. By fuzzy set theory alone we cannot consider this value in more detail. However, if we define $\nu(x)$ (degree of non-membership, non-validity, etc.) as the number of votes given to the parties or persons outside the government, then we can show the part of electorate who have not vote at all or who have given bad voting –paper and the corresponding number will be $\pi(x) = 1 - \mu(x) - \nu(x)$ (degree of interdeterminacy, uncertainty, etc.,). Thus we can construct the set $\{ \langle x, \mu(x), \nu(x) \rangle : x \in E\}$.

Using the notion of intuitionistic fuzzy sets, Coker [1997] has constructed the basic concepts of intuitionistic fuzzy topological spaces. After giving the fundamental definitions and the necessary examples he introduced the definitions of intuitionistic fuzzy continuity, intuitionistic fuzzy compactness, intuitionistic fuzzy connectedness and obtained several properties and some characterizations concerning intuitionistic fuzzy connectedness. The concept of $b^{\#}$ closed sets in intuitionistic fuzzy topological spaces is introduced by Gomathi and Jayanthi (2018).

In this thesis a new class of continuous mappings in intuitionistic fuzzy topological spaces namely intuitionistic fuzzy almost $b^{\#}$ continuous mapping, intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, intuitionistic fuzzy completely $b^{\#}$ continuous mapping and intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping are being introduced and their respective properties are discussed.

In Chapter I, the recent developments in intuitionistic fuzzy topology contributed by various authors are presented. This forms the basement for the remaining chapters of this thesis.

In Chapter II, some new propositions are proved for intuitionistic fuzzy $b^{\#}$ continuous mapping by defining two new spaces namely intuitionistic fuzzy $T_{cb^{\#}}$ space and intuitionistic fuzzy $T_{b^{\#}}$ space. Using these two spaces we have proved some interesting characterization theorems and relations.

In Chapter III, intuitionistic fuzzy almost $b^{\#}$ continuous mapping and intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping are introduced. The relationship between these newly introduced continuous mappings and few of the already existing intuitionistic fuzzy continuous mappings are being discussed. Further some of the characterizations of these newly introduced continuous mappings using the spaces intuitionistic fuzzy $T_{cb^{\#}}$ space and intuitionistic fuzzy $T_{b^{\#}}$ space are discussed.

In Chapter IV, we have introduced two types of intuitionistic fuzzy continuous mappings namely intuitionistic fuzzy completely $b^{\#}$ continuous mappings and intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings. As intuitionistic fuzzy $b^{\#}$ closed set stronger than many of the intuitionistic fuzzy closed sets, we have proved the relation between the newly introduced intuitionistic fuzzy continuous mappings with other already existing intuitionistic fuzzy continuous mappings using intuitionistic fuzzy $T_{cb^{\#}}$ space and intuitionistic fuzzy $T_{b^{\#}}$ space.

## REVIEW OF LITERATURE

A review of literature of recent developments on the notions of closed and open sets, continuous mappings in topological spaces, fuzzy topological spaces and intuitionistic fuzzy topological spaces is given below.

**Topological spaces**

The concepts of closed and open sets play an important role in the study of topological spaces. Stone (1937) introduced the regular open sets in topological spaces, as a stronger form of open sets and Levine (1963) initiated and studied the concept of semi open sets in topological spaces, as a weaker form of open sets.

Njastad (1965) introduced some classes of nearly open sets. Abd El- Monsef, El-Deeb and Mahmoud (1983) have introduced $\beta$ open in topological spaces. Andrijievic (1996) introduced the concept of b-open sets in general topology. Further the notion of $b^{\#}$ open set is introduced by Ushaparameswari and Thangavelu (2014). Mashhour, Abd EI-Monsef and EL-Deeb(1982) have investigated pre continuous and weak pre continuous mappings in topology. Noiri (1984) introduced alpha continuous functions in topological spaces.

Dontchev (1996) introduced a new class of mappings called contra-continuity in general topology. Also, a new weaker form of this class of mappings called contra semi continuous mappings are introduced and investigated by Dontchev and Noiri (1999).

Caldas (2000) defined and studied weak and strong forms of irresolute maps in general topology. Caldas and Jafari (2001) have investigated some properties of contra $\beta$ continuous functions. Ekici (2004) has introduced contra-continuity in topological space. Ahmad Al-Omari, Mohd.Salmi Md and Noorani (2009) have introduced some properties of contra b-continuous functions. El-magbrabi (2010) has investigated some properties of contra continuous mappings in topological spaces.

Caldas and Jafari (2007) have investigated some applications of b-open sets in topological spaces. In 2016, Kanchana and nirmalairudayam have introduced and studied the concepts of new class of maps namely generalized $^{*+}$ b-continuous and irresolute map.

**Fuzzy topological spaces**

The fundamental concept of fuzzy sets was introduced by Zadeh (1965). After the discovery of the fuzzy sets, much attention has been paid to the basic concepts of fuzzy topology. The notion of fuzzy set are naturally play a significant value in the study of fuzzy topology which was introduced by Chang (1968). In recent years fuzzy topology has been found to be very useful in solving many practical problems. Ganguly and Saha (1986) have introduced fuzzy semi open sets in fuzzy topological spaces. Singal and NitiPrakash (1991) have introduced fuzzy preopen sets.

Bin Shahna (1991) introduced and studied fuzzy continuity in fuzzy topological spaces. Benchalli and Jenifer Karnel (2010) introduced the concept of fuzzy b-open sets in fuzzy topological spaces. He proved some properties and investigated their relations with different fuzzy sets in fuzzy topological spaces.

The concept of fuzzy $b^{\#}$ closed sets was introduced by Indhumathi and Jayanthi (2018). Azad (1981) introduced some weaker forms of continuity in fuzzy topological space. He introduced fuzzy semi-continuous functions, fuzzy almost continuous functions in fuzzy topological spaces. Mukherjee and Sinha (1989) introduced and characterized the concept of fuzzy irresolute functions. Ekici and Kerre (2006) introduced the notion of fuzzy contra continuous mappings. They have analyzed some of their properties and obtained some interesting theorems.

**Intuitionistic fuzzy topological spaces**

Atanassov (1988) has introduced intuitionistic fuzzy sets and also he gave new results in intuitionistic fuzzy sets. Atanassov (1994) has defined a set of new operations on intuitionistic fuzzy sets. Intuitionistic fuzzy points are introduced by Coker and Demirci (1995). Intuitionistic fuzzy open sets, intuitionistic fuzzy closed sets are introduced by Coker (1997).

Intuitionistic fuzzy semi open sets, intuitionistic fuzzy pre open sets, intuitionistic fuzzy $\alpha$ open sets are introduced by Gurcay, Coker and Hayder (1997). Gurcay, Coker and Hayder (1997) have introduced intuitionistic fuzzy semi continuous mappings, intuitionistic fuzzy pre continuous mappings. Intuitionistic fuzzy $\alpha$ continuous mappings and intuitionistic fuzzy $\beta$

continuous mappings in intuitionistic fuzzy topological spaces. Jun, Kang and Song (2005) have introduced intuitionistic fuzzy continuous and irresolute mapping.

Hanafy (2009) has introduced intuitionistic fuzzy $\gamma$ continuity in intuitionistic fuzzy topological spaces. Krsteska and Ekici (2007) have introduced intuitionistic fuzzy contra pre continuity. The concepts of nowhere dense in intuitionistic fuzzy topological space was introduced by Thakur and Dhavaseelan (2015).The concept of intuitionistic fuzzy $b^{\#}$ closed sets was introduced by Gomathi and Jayanthi (2018).

## 1. FUZZY SETS

[Zadeh, 1965]

In this article, the authors has introduced fuzzy sets which are characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. Further the author has provided the notions of inclusion, union, intersection, complement, etc. with respect to fuzzy sets.

## 2. GENERAL TOPOLOGICAL SPACES

[Bourbaki, 1996]

In this book, important classes of topological spaces are studied, uniform structures are introduced and applied to topological groups. Real numbers are constructed and their properties are established.

## 3. INTUITIONISTIC FUZZY SETS

[Atanassov, 1986]

In this article, the author has provided the notion of intuitionistic fuzzy sets. This is considered to be the generalization of fuzzy sets. The highlight of this particular article is that some relations and operations concerning classical sets are extended to intuitionistic fuzzy sets.

### 4. FUZZY TOPOLOGICAL SPACES

[Chang, 1968]

In this article, the author has introduced fuzzy topological spaces. This concept is considered to be the generalization of general topological spaces. In brief, the basic concepts such as fuzzy open set, fuzzy closed set, fuzzy neighbourhood, fuzzy continuity etc., are discussed.

### 5. INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Dogan Coker, 1997]

In this article, the author has introduced intuitionistic fuzzy topological space. The notions of intuitionistic fuzzy interior and intuitionistic fuzzy closure are being provided and this is followed by the discussion of some important properties concerning them. Furthermore, the notion of intuitionistic fuzzy continuity is provided.

### 6. ON FUZZY CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Gurcay, Coker and Haydar, 1997]

This article consists of the notions of intuitionistic fuzzy semiopen set, intuitionistic fuzzy preopen set, intuitionistic fuzzy α-open set, intuitionistic fuzzy β open set and their corresponding closed sets. Further the relationship between these sets are established.

# CHAPTER 1

## Preliminaries

In this chapter, the basic definitions and results of intuitionistic fuzzy sets, intuitionistic fuzzy continuous mappings and intuitionistic fuzzy contra continuous mappings in intuitionistic fuzzy topological space that are used to accomplish the present study are given in detail.

## 1.1 Intuitionistic fuzzy sets

**Definition 1.1.1:** [Zadeh 1965]

Let X be a non empty set. A fuzzy set A in X can be described in the form

$A = \{\langle x, \mu_A(x)\rangle / x \in X\}$

Where the function $\mu_A: X \to [0,1]$ is called the membership function and $\mu_A(x)$ denotes the degree to which $x \in A$ and $0 \le \mu_A(x) \le 1$ for each $x \in X$.

**Definition 1.1.2: [**Atanassov 1986]

An intuitionistic fuzzy set A is an object having the form

$A= \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$

Where the functions $\mu_A: X \to [0, 1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively , and $0\le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X) , the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set of A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$.

**Definition 1.1.3:** [Atanassov 1986]

Let A and B be two intuitionistic fuzzy sets of the form

$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$ and $B = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$. Then the following properties hold:

i. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

ii. $A=B$ if and only if $A \subseteq B$ and $A \supseteq B$,

iii. $A^c = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$,

iv. $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X\}$,

v. $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_{\sim} = \langle x, 0, 1 \rangle$ and $1_{\sim} = \langle x, 1, 0 \rangle$ are respectively the empty set and whole set of X.

**Definition 1.1.4:** [Coker, 1995]

Let $(X, \tau)$ be an intuitionistic fuzzy topological space and A, B be intuitionistic fuzzy sets in X. Then the following properties hold:

i. $int(A) = A$

ii. $A \subseteq cl(A)$

iii. $A \subseteq B \Rightarrow int(A) \subseteq int(B)$

iv. $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$

v. $int(int(A)) = int(A)$

vi. $cl(cl(A)) = cl(A)$

vii. $int(A \cap B) = int(A) \cap int(B)$

viii. $cl(A \cup B) = cl(A) \cup cl(B)$

ix. $int(1_{\sim}) = 1_{\sim}$

x. $cl(0_{\sim}) = 0_{\sim}$

**Definition 1.1.5:** [Coker, 1995]

Let A, B and C be intuitiontistic fuzzy sets in an intuitionistic fuzzy topological space X. Then

i. $(A \subseteq B)$ and $(C \subseteq D) \Rightarrow (A \cup C) \subseteq (B \cup D)$ and $(A \cap C) \subseteq (B \cap D)$

ii. $A \subseteq B$ and $A \subseteq C \Rightarrow A \subseteq (B \cap C)$

iii. $A \subseteq C$ and $B \subseteq C \Rightarrow (A \cup B) \subseteq C$

iv. $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$

v. $(A \cup B)^c = A^c \cap B^c$

vi. $(A \cap B)^c = A^c \cup B^c$

vii. $A \subseteq B \Rightarrow B^c \subseteq A^c$

viii. $(A^c)^c = A$

ix. $(0_\sim)^c = 1_\sim$

x. $(1_\sim)^c = 0_\sim$

**Definition 1.1.6:** [Coker, 1997]

An intuitionistic fuzzy topology on X is a family $\tau$ of intuitionistic fuzzy sets in X satisfying the following axioms:

i. $0_\sim, 1_\sim \in \tau$

ii. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

iii. $\cup G_i \in \tau$ for any $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called the **intuitionistic topological space** and any intuitionistic fuzzy set in $\tau$ is known as an **intuitionistic fuzzy open set** in X. Then the complement $A^c$ of an intuitionistic fuzzy open set A in an intuitionistic fuzzy topological space $(X, \tau)$ is called an **intuitionistic fuzzy closed set** in X.

**Definition 1.1.7:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said be an **intuitionistic fuzzy semi closed set** if $int(cl(A)) \subseteq A$.

**Definition 1.1.8:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **intuitionistic fuzzy pre closed set** if $cl(int(A)) \subseteq A$.

**Definition 1.1.9:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **intuitionistic fuzzy regular closed set** if $cl(int(A)) = A$.

**Definition 1.1.10:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **Intuitionistic fuzzy α closed set** if $cl(int(cl(A))) \subseteq A$.

**Definition 1.1.11:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **intuitionistic fuzzy β closed set** if $int(cl(int(A))) \subseteq A$.

**Definition 1.1.12:** [Hanafy, 2009]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be **intuitionistic fuzzy γ closed set** (equivalently intuitionistic fuzzy b closed set) if

$$int(cl(A)) \cap cl(int(A)) \subseteq A.$$

**Definition1.1.13:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **intuitionistic fuzzy semi open set** if $A \subseteq cl(int(A))$.

**Definition 1.1.14:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A =\langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **intuitionistic fuzzy pre open set** if $A \subseteq int(cl(A))$.

**Definition 1.1.15:** [Gurcay, Coker and Hayder, 1997]

An intuitionist fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **intuitionistic fuzzy regular open set** if $int(cl(A)) = A$

**Definition 1.1.16:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **intuitionistic fuzzy α open set** if $A \subseteq int(cl(int(A)))$.

**Definition 1.1.17:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **intuitionistic fuzzy β open set** if $A \subseteq cl(int(cl(A)))$.

**Definition 1.1.18:** [Hanafy, 2009]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be **intuitionistic fuzzy γ open set** if $A \subseteq int(cl(A)) \cup cl(int(A))$.

**Definition 1.1.19:** [Thakur and Rekha Chaturvedi, 2008]

Two intuitionistic fuzzy sets A and B are said to be **q-coincident** ($A_q B$) if and only if there exist an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

**Definition 1.1.20:** [Coker, 1997]

Let $(X, \tau)$ be an intuitionistic fuzzy topological space and $A = \langle x, \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy set in X.

Then the **intuitionistic fuzzy interior** and **intuitionistic fuzzy closure** are defined by int(A) = ∪ {G/G is an intuitionistic fuzzy open set in X and G ⊆A},

cl(A) = ∩ {K/K is an intuitionistic fuzzy closed set in X and A ⊆ K}.

**Definition 1.1.21:** [Rajarajeswari, and Krishna Moorthy, 2013]

Let (X, τ) be an intuitionistic fuzzy topological space and A= ⟨x, $\mu_A$, $\nu_A$⟩ be an intuitionistic fuzzy set in X.

Then the **intuitionistic fuzzy b-interior** and **intuitionistic fuzzy b-closure** are defined by

b-int(A) = ∪ {G/G is an intuitinistic fuzzy b open set in X and G ⊆ A},

b-cl(A) = ∩ {K/K is an intuitionistic fuzzy b closed set in X and A ⊆ K}.

Note that for an intuitionistic fuzzy set A in (X, τ), we have bcl($A^c$)= (bint(A))$^c$ and

bint(A)$^c$ = (bcl(A))$^c$.

**Definition 1.1.22:** [Thakur and Rekha Chaturvedi, 2008]

Two intuitionistic fuzzy set A and B are said to be **not q-coincident** if and only if A ⊆ $B^c$.

**Definition 1.1.23:** [Coker and Demirci, 1995]

**Intuitionistic fuzzy point**, written as $p_{(\alpha,\beta)}$ ,is defined to be an intuitionistic fuzzy set of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha,\beta) & \text{if } x=p \\ (0,1) & \text{otherwise} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha, \beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

**Definition 1.1.24:** [Seok Jong Lee and Eun Pyo Lee, 2000]

Let $p_{(\alpha, \beta)}$ be an intuitionistic fuzzy point in $(X, \tau)$. An intuitionistic fuzzy set A of X is called an **intuitionistic fuzzy neighbourhood** of $p_{(\alpha, \beta)}$ if there exist an intuitionistic fuzzy open set B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

**Definition 1.1.25:** [Gomathi and Jayanthi, 2018]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be **intuitionistic fuzzy $b^{\#}$ closed set** if $int(cl(A)) \cap cl(int(A)) = A$.

**Definition 1.1.26:** [Gomathi and Jayanthi, 2018]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be **intuitionistic fuzzy $b^{\#}$ open set** if $A = int(cl(A)) \cup cl(int(A))$ .

## 1.2 Intuitionistic fuzzy continuous mappings

In this section we have listed the definitions of some previously existing intuitionistic fuzzy continuous mappings.

**Definition 1.2.1:** [Coker, 1997]

Let X and Y be two non empty sets and $f: X \to Y$ be a mapping.

If $A = \{\langle x, \mu_A(x), \nu_A(x) / x \in X\rangle\}$ is an intuitionistic fuzzy set in X, then the image of A under f, denoted by f(A), is the intuitionistic fuzzy set in Y defined by

$$f(A) = \{\langle y, f(\mu_A)(y), f\_(\nu_A)(y) / y \in Y)\rangle\}$$

Where $f\_(\nu_A) = 1 - f(1- \nu_A)$.

**Definition 1.2.2:** [Coker, 1997]

Let X and Y be two non empty sets and $f: X \to Y$ be a mapping.

If $B = \{\langle y, \mu_B(y), \nu_B(y) / y \in Y\rangle\}$ is an intuitionistic fuzzy set in Y, then the preimage of B under f is denoted and defined by

$$f^{-1}(B)= \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) / x \in X \rangle \}$$

Where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ for every $x \in X$.

**Definition 1.2.3:** [Gurcay, Coker and Hayder, 1997]

Let f be a mapping from an intuitionistic fuzzy topological space $(X, \tau)$ into an intuitionistic fuzzy topological space $(Y, \sigma)$. Then f said to be an **intuitionistic fuzzy continuous mapping** if $f^{-1}(V)$ is an intuitionistic fuzzy closed set in $(X, \tau)$ for every intuitionistic fuzzy closed set V of $(Y, \sigma)$.

**Definition 1.2.4:** [Joung Kon Jeon, 2005]

Let f be a mapping from an intuitionistic fuzzy topological space $(X, \tau)$ into an intuitionistic fuzzy topological space $(Y, \sigma)$. Then f said to be an **intuitionistic fuzzy semi continuous mapping** if $f^{-1}(V)$ is an intuitionistic fuzzy semi closed set in $(X, \tau)$ for every intuitionistic fuzzy closed set V of $(Y, \sigma)$.

**Definition 1.2.5:** [Joung Kon Jeon, 2005]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy α continuous mapping** if $f^{-1}(V)$ is an intuitionistic fuzzy α closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ).

**Definition 1.2.6:** [Joung Kon Jeon, 2005]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f said to be an **intuitionistic fuzzy pre continuous mapping** if $f^{-1}(V)$ is an intuitionistic fuzzy pre closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ).

**Definition 1.2.7:** [ Joung Kon Jeon, 2005]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy β continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy β closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ).

**Definition 1.2.8:** [Hanafy, 2009]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy γ continuous mapping** (equivalently intuitionistic fuzzy b continuous mapping) if $f^{-1}(V)$ is an intuitionistic fuzzy γ closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ).

**Definition 1.2.9:** [Krsteska and Ekici, 2007]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ) .

**Definition 1.2.10:** [Krsteska and Ekici, 2007]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra α continuous mapping** if $f^{-1}$(V) is an intuitionistic fuzzy α closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

**Definition 1.2.11: [**Krsteska and Ekici, 2007]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra pre continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy pre closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

**Definition 1.2.12:** [Krsteska and Ekici, 2007]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra semi continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy semi closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

**Definition 1.2.13:** [Krsteska and Ekici, 2007]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra β continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy β closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

**Definition 1.2.14:** [Hanafy, 2009]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra γ continuous mapping** (equivalently intuitionistic fuzzy contra b continuous mapping) if $f^{-1}$ (V) is an intuitionistic fuzzy γ closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

**Definition 1.2.15:** [Joung Kon Jeon, 2005]

An intuitionistic fuzzy set A is said to be an **intuitionistic fuzzy dense** in another intuitionistic fuzzy set B in an intuitionistic fuzzy topological space (X, τ), if cl(A) = B.

**Definition 1.2.16:** [Hanafy and El-Arish, 2003]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy completely continuous mapping** if $f^{-1}$(V) is an intuitionistic fuzzy regular open set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

**Definition 1.2.17:** [Gomathi and Jayanthi, 2018]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy $b^{\#}$ continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy $b^{\#}$ closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ).

**Definition 1.2.18:** [Gomathi and Jayanthi, 2018]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy $b^{\#}$ irresolute mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy $b^{\#}$ closed set in (X, τ) for every intuitionistic fuzzy $b^{\#}$ closed set V of (Y, σ).

**Definition 1.2.19:** [Gomathi and Jayanthi, 2018]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra $b^{\#}$ continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy $b^{\#}$ closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

# CHAPTER 2

## Intuitionistic fuzzy $T_{cb^{\#}}$ spaces

## 2.1 Introduction

In this chapter we have introduced two new spaces called intuitionistic fuzzy $T_{cb^{\#}}$ space and intuitionistic fuzzy $T_{b^{\#}}$ space. Intuitionistic fuzzy $b^{\#}$ closed sets are independent with intuitionistic fuzzy closed sets and intuitionistic fuzzy $b^{\#}$ continuous mappings are independent with any other already existing mapping, so to remove this difficulty and to prove the interrelation we have introduced two new spaces called intuitionistic fuzzy $T_{cb^{\#}}$ space and intuitionistic fuzzy $T_{b^{\#}}$ space.

## 2.2 Intuitionistic fuzzy $b^{\#}$ continuous mappings

Gomathi and Jayanthi (2018) have introduced intuitionistic fuzzy $b^{\#}$ continuous mappings and intuitionistic fuzzy contra $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces. In this section we have proved the interrelation between intuitionistic fuzzy $b^{\#}$ continuous mappings with some of the other already existing mappings like intuitionistic fuzzy continuous mappings, intuitionistic fuzzy pre continuous mappings etc. using intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Definition 2.2.1:**

If every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in $(X, \tau)$, then the space is called as an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Example 2.2.2:**

Let X={a, b} and then $\tau = \{0_{\sim}, G_1, G_2\ 1_{\sim}\}$ is an intuitionistic fuzzy topology on X, where,

$$G_1 = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle \quad \text{and}$$

$$G_2 = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle.$$

Then $(X, \tau)$ is an intuitionistic fuzzy topological space.

The intuitionistic fuzzy sets $G_1^c$ and $G_2^c$ are intuitionistic fuzzy $b^\#$ closed set in (X, τ) as

$$int(cl(G_1^c)) \cap cl(int(G_1^c)) = G_1^c \cap G_2$$

$$= G_1^c \quad \text{and}$$

$$int(cl(G_2^c)) \cap cl(int(G_2^c)) = G_2^c \cap G_1$$

$$= G_2^c.$$

Then intuitionistic fuzzy sets $G_1^c = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle$ and

$$G_2^c = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle$$

are intuitionistic fuzzy closed sets in X as,

$$cl(G_1^c) = G_1^c \quad \text{and}$$

$$cl(G_2^c) = G_2^c.$$

Since every intuitionistic fuzzy $b^\#$ closed set is an intuitionistic fuzzy $b^\#$ closed set in (X, τ), the space (X, τ) is as an intuitionistic fuzzy $T_{cb^\#}$ space.

**Definition 2.2.3:**

If every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^\#$ closed set in (X, τ), then the space is called as an intuitionistic fuzzy $T_{b^\#}$ space.

**Example 2.2.4:**

Let X = {a, b} and then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ is an intuitionistic fuzzy topology on X, where,

$$G_1 = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle \quad \text{and}$$

$$G_2 = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle.$$

Then (X, τ) is an intuitionistic fuzzy topological space.

The intuitionistic fuzzy sets $G_1^c = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle$ and

$$G_2^c = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle$$

are intuitionistic fuzzy closed sets in X as,

$$cl(G_1^c) = G_1^c \quad \text{and}$$

$$cl(G_2^c) = G_2^c.$$

Then $G_1^c$ and $G_2^c$ are intuitionistic fuzzy $b^\#$ closed set in $(X, \tau)$ as

$$int(cl(G_1^c)) \cap cl(int(G_1^c)) = G_1^c \cap G_2$$

$$= G_1^c \quad \text{and}$$

$$int(cl(G_2^c)) \cap cl(int(G_2^c)) = G_2^c \cap G_1$$

$$= G_2^c.$$

Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^\#$ closed set in $(X, \tau)$, the space $(X, \tau)$ is as an intuitionistic fuzzy $T_{b^\#}$ space.

**Proposition 2.2.5:**

Every intuitionistic fuzzy $b^\#$ continuous mapping $f: X \rightarrow Y$ is an intuitionistic fuzzy semi continuous mapping if X is an intuitionistic fuzzy $T_{cb^\#}$ space.

**Proof:**

Let $f: X \rightarrow Y$ be an intuitionistic fuzzy $b^\#$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^\#}$ space. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^\#$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^\#}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semi closed set, $f^{-1}(A)$ is an intuitionistic fuzzy semi closed set in X. Hence f is an intuitionistic fuzzy semi continuous mapping.

**Proposition 2.2.6:**

Every intuitionistic fuzzy $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy pre continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy pre closed set, $f^{-1}(A)$ is an intuitionistic fuzzy pre closed set in X. Hence f is an intuitionistic fuzzy pre continuous mapping.

**Proposition 2.2.7:**

Every intuitionistic fuzzy $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy $\beta$ continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\beta$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\beta$ closed set in X. Hence f is an intuitionistic fuzzy $\beta$ continuous mapping.

**Proposition 2.2.8:**

Every intuitionistic fuzzy $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy $\alpha$ continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\alpha$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\alpha$ closed set in X. Hence f is an intuitionistic fuzzy $\alpha$ continuous mapping.

**Proposition 2.2.9:**

Every intuitionistic fuzzy $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy $\gamma$ continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\gamma$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\gamma$ closed set in X. Hence f is an intuitionistic fuzzy $\gamma$ continuous mapping.

**Proposition 2.2.10:**

Every intuitionistic fuzzy $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy semipre continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semipre closed set, $f^{-1}(A)$ is an intuitionistic fuzzy semipre closed set in X. Hence f is an intuitionistic fuzzy semipre continuous mapping.

## 2.3 Intuitionistic fuzzy contra $b^{\#}$ continuous mappings

In this section we have proved the interrelation between intuitionistic fuzzy contra $b^{\#}$ continuous mappings with some of the other already existing mappings like intuitionistic fuzzy contra continuous mappings, intuitionistic fuzzy contra pre continuous mappings etc. using intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proposition 2.3.1:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy contra continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Therefore f is an intuitionistic fuzzy contra continuous mapping.

**Proposition 2.3.2:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy contra semi continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semi closed set, $f^{-1}(A)$ is an intuitionistic fuzzy semi closed set in X. Therefore f is an intuitionistic fuzzy contra semi continuous mapping.

**Proposition 2.3.3:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy contra pre continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy pre closed set, $f^{-1}(A)$ is an intuitionistic fuzzy pre closed set in X. Therefore f is an intuitionistic fuzzy contra pre continuous mapping.

**Proposition 2.3.4:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping $f : X \to Y$ is an intuitionistic fuzzy contra $\beta$ continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let $f : X \to Y$ be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\beta$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\beta$ closed set in X. Therefore f is an intuitionistic fuzzy contra $\beta$ continuous mapping.

**Proposition 2.3.5:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping $f : X \to Y$ is an intuitionistic fuzzy contra $\alpha$ continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let $f : X \to Y$ be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\alpha$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\alpha$ closed set in X. Therefore f is an intuitionistic fuzzy contra $\alpha$ continuous mapping.

**Proposition 2.3.6:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping $f : X \to Y$ is an intuitionistic fuzzy contra $\gamma$ continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\gamma$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\gamma$ closed set in X. Therefore f is an intuitionistic fuzzy contra $\gamma$ continuous mapping.

**Proposition 2.3.7:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy contra semipre continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semipre closed set, $f^{-1}(A)$ is an intuitionistic fuzzy semipre closed set in X. Therefore f is an intuitionistic fuzzy contra semipre continuous mapping.

# CHAPTER 3

## Almost $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces

## 3.1 Introduction

In this chapter we have introduced two types of $b^{\#}$ continuous mappings namely intuitionistic fuzzy almost $b^{\#}$ continuous mappings and intuitionistic fuzzy almost contra $b^{\#}$ continuous mappings. Also we have provided some interesting results based on these continuous mappings.

## 3.2 Intuitionistic fuzzy almost $b^{\#}$ continuous mappings

In this section we have introduced intuitionistic fuzzy almost $b^{\#}$ continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced continuous mappings and the already existing continuous mappings in intuitionistic fuzzy topological spaces.

**Definition 3.2.1:**

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy almost $b^{\#}$continuous mapping if $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in $(X, \tau)$ for every intuitionistic fuzzy regular closed set V of $(Y, \sigma)$.

**Example 3.2.2:**

Let $X = \{a, b\}$, $Y = \{u, v\}$.Then $\tau = \{0_\sim , G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim , G_3, G_4, 1_\sim\}$ are intuitionistic fuzzy topologies on X and Y respectively, where,

$$G_1 = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle,$$

$$G_2 = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle,$$

$$G_3 = \langle y, (0.7_u, 0.6_v), (0.2_u, 0.3_v)\rangle \quad \text{and}$$

$$G_4 = \langle y, (0.2_u, 0.3_v), (0.7_u, 0.6_v)\rangle.$$

Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The intuitionistic fuzzy sets,

$$G_3^c = \langle y, (0.2_u, 0.3_v), (0.7_u, 0.6_v)\rangle \quad \text{and}$$

$$G_4^c = \langle y, (0.7_u, 0.6_v), (0.2_u, 0.3_v)\rangle$$

are intuitionistic fuzzy regular closed sets in Y.

Then $f^{-1}(G_3^c) = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle$ is an intuitionistic fuzzy $b^{\#}$ closed set in $(X, \tau)$, as,

$$\begin{aligned} int(cl(f^{-1}(G_3^c)))\cap cl(int(f^{-1}(G_3^c))) &= int(G_2^c)\cap cl(G_1) \\ &= G_1\cap G_2^c \\ &= f^{-1}(G_3^c). \end{aligned}$$

Similarly $f^{-1}(G_4^c) = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle$ is an intuitionistic fuzzy $b^{\#}$ closed set in $(X, \tau)$, as,

$$\begin{aligned} int(cl(f^{-1}(G_4^c)))\cap cl(int(f^{-1}(G_4^c))) &= int(G_1^c)\cap cl(G_2^c) \\ &= G_2\cap G_1^c \\ &= f^{-1}(G_4^c). \end{aligned}$$

Therefore f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 3.2.3:**

A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping if and only if the inverse image of each intuitionistic fuzzy regular open set in Y is an intuitionistic fuzzy $b^{\#}$ open set in X.

**Proof:**

**Necessity:**

Let A be an intuitionistic fuzzy regular open set in Y. Then $A^c$ is an intuitionistic fuzzy regular closed set in Y. Since f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, $f^{-1}(A^c)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ open set in X.

**Sufficiency:**

Let A be an intuitionistic fuzzy regular closed set in Y. Then $A^c$ is an intuitionistic fuzzy regular open set in Y. By hypothesis $f^{-1}(A^c)$ is an intuitionistic fuzzy $b^{\#}$ open set in X.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 3.2.4:**

If f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then for each intuitionistic fuzzy point $p_{(\alpha,\ \beta)}$ of X and each A ∈ σ such that $f(p_{(\alpha,\ \beta)}) \in A$, there exists an intuitionistic fuzzy $b^{\#}$ open set B of X such that $p_{(\alpha,\ \beta)} \in B$ and $f(B) \subseteq A$.

**Proof:**

Let $p_{(\alpha,\ \beta)}$ be an intuitionistic fuzzy point of X and A ∈ σ such that $f(p_{(\alpha,\ \beta)}) \in A$, then

$p_{(\alpha,\ \beta)} \in f^{-1}(A)$. Put $B = f^{-1}(A)$.

Then by hypothesis, B is an intuitionistic fuzzy $b^{\#}$ open set in X such that

$$p_{(\alpha,\ \beta)} \in B \quad \text{and} \quad f(B) = f(f^{-1}(A)) \subseteq A.$$

**Proposition 3.2.5:**

If f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping then for each intuitionistic fuzzy point $p_{(\alpha,\ \beta)}$ of X and each A ∈ σ such that $f(p_{(\alpha,\ \beta)})_q A$, there exists an intuitionistic fuzzy $b^{\#}$ open set B of X such that

$$(p_{(\alpha,\ \beta)})_q B \quad \text{and} \quad f(B) \subseteq A.$$

**Proof:**

Let $p_{(\alpha,\ \beta)}$ be an intuitionistic fuzzy point of X and A ∈ σ such that $f(p_{(\alpha,\ \beta)})_q A$. Then $p_{(\alpha,\ \beta)q}\ f^{-1}(A)$ put $B = f^{-1}(A)$. Then by hypothesis, B is an intuitionistic fuzzy $b^{\#}$ open set in X such that $p_{(\alpha,\ \beta)q} B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

**Proposition 3.2.6:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then $f^{-1}(int(cl(B))) \subseteq cl(int(f^{-1}(cl(B)))) \cup int(cl((f^{-1}(cl(B))))$ for every intuitionistic fuzzy set B in Y.

**Proof:**

Let B be any intuitionistic fuzzy set in Y. Then int(cl(B)) is an intuitionistic fuzzy regular open set in Y. By hypothesis $f^{-1}(int(cl(B)))$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy b open set, $f^{-1}(int(cl(B)))$ is an intuitionistic fuzzy b open set in X. Therefore

$$f^{-1}(int(cl(B))) \subseteq cl(int(f^{-1}(int(cl(B))))) \cup int(cl(f^{-1}(int(cl(B)))))$$

$$\subseteq cl(int(f^{-1}(cl(B)))) \cup int(cl(f^{-1}(cl(B)))).$$

**Proposition 3.2.7:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then $cl(int(f^{-1}(int(B)))) \cap int(cl(f^{-1}(int(B)))) \subseteq f^{-1}(cl(int(B)))$ for each intuitionistic fuzzy regular closed set B of Y.

**Proof:**

Let B be any intuitionistic fuzzy set in Y. Then cl(int(B)) is an intuitionistic fuzzy regular closed set in Y. By hypothesis $f^{-1}(cl(int(B)))$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy b closed set, $f^{-1}(cl(int(B)))$ is an intuitionistic fuzzy b closed set in X. Therefore

$$cl(int(f^{-1}(int(B))))\cap int(cl(f^{-1}(int(B)))) \subseteq cl(int(f^{-1}(cl(int(B))))) \cap int(cl(f^{-1}(cl(int(B)))))$$

$$\subseteq f^{-1}(cl(int(B))).$$

**Proposition 3.2.8:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy continuous mapping, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping where X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let A be an intuitionistic fuzzy regular closed set in Y. Every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set. Therefore A is an intuitionistic fuzzy closed set in Y. Since f is an intuitionistic fuzzy continuous mapping, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space. Therefore $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy Almost $b^{\#}$ continuous mapping.

**Proposition 3.2.9:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost continuous mapping, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping where X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let A be an intuitionistic fuzzy regular closed set in Y. Since f is an intuitionistic fuzzy almost continuous mapping, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space. Therefore $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 3.2.10:**

Let f: (X, τ) → (Y, σ) be an mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. If f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then $(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))) \subseteq f^{-1}(cl(B)$ for every intuitionistic fuzzy regular closed set B in Y.

**Proof:**

Let B ⊆ Y be an intuitionistic fuzzy regular closed set. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Therefore $cl(f^{-1}(B) = f^{-1}(B)$. Now

$$(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))) \subseteq f^{-1}(B) \cup (int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B)))$$

$$\subseteq cl(f^{-1}(B)$$

$$= f^{-1}(B)$$

$$\subseteq f^{-1}(cl(B).$$

Hence $(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))) \subseteq f^{-1}(cl(B)$.

**Proposition 3.2.11:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, then for each intuitionistic fuzzy point $p_{(\alpha,\ \beta)}$ in X and each intuitionistic fuzzy regular open set A in Y such that $f(p_{(\alpha,\ \beta)}) \in A$, $int(f^{-1}(cl(A)))$ is an intuitionistic fuzzy neighbourhood of $p_{(\alpha,\ \beta)}$ in X.

**Proof:**

Let $p_{(\alpha,\ \beta)} \in X$ and let A be an intuitionistic fuzzy regular open set in Y such that $f(p_{(\alpha,\ \beta)}) \in A$, $p_{(\alpha,\ \beta)} \in f^{-1}(A)$. By hypothesis $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy open set in X.

Now $p_{(\alpha,\ \beta)} \in f^{-1}(A) = int(f^{-1}(A)) \subseteq int(f^{-1}(cl(A)))$.

Hence $intf^{-1}(cl(A))$ is an intuitionistic fuzzy neighbourhood of $p_{(\alpha,\ \beta)}$ in X.

**Proposition 3.2.12:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Then $cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every intuitionistic fuzzy semi open set A in Y.

**Proof:**

Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping and let A be an intuitionistic fuzzy semi open set in Y. Then

$A \subseteq cl(int(A))$. Now,

$$cl(A) \subseteq cl(cl(int(A)))$$

$$\subseteq cl(int(cl(A)))$$

$$\subseteq cl(cl(A))$$

$$\subseteq cl(A).$$

Therefore cl(A) = cl(int(cl(A))). This implies cl(A) is an intuitionistic fuzzy regular closed set in Y. By hypothesis, $f^{-1}$(cl(A)) is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}$(cl(A)) is an intuitionistic fuzzy closed set in X. Therefore $cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$. Now,

$$cl(f^{-1}(A)) \subseteq cl(f^{-1}(cl(A)))$$

$$= f^{-1}(cl(A)).$$

Thus $cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$.

**Proposition 3.2.13:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Then $f^{-1}(A) \subseteq int(f^{-1}(int(cl(A))))$ for every intuitionistic fuzzy pre open set A in Y.

**Proof:**

Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping and let A be an intuitionistic fuzzy pre open set in Y. Then A ⊆ int(cl(A)) . Since int(cl(A)) is an intuitionistic fuzzy regular open set in Y , by hypothesis, $f^{-1}$ (int(cl(A))) is an intuitionistic fuzzy $b^{\#}$ open set in X.

Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy open set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}$ (int(cl(A))) is an intuitionistic fuzzy open set in X.

Therefore $f^{-1}(A) \subseteq f^{-1}(int(cl(A)))$

$$= int(f^{-1}(int(cl(A)))).$$

**Proposition 3.2.14:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping then $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for every intuitionistic fuzzy set B in Y where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping. Let B be an intuitionistic fuzzy regular open set in Y. By hypothesis $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy open set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}$ (B) is an intuitionistic fuzzy open set in X.

Therefore $f^{-1}(int(B)) \subseteq f^{-1}(B) = int(f^{-1}(B))$.

**Proposition 3.2.15:**

Let f: (X, τ) → (Y, σ) be a mapping where X is an intuitionistic fuzzy $T_{b^{\#}}$ space. If $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for every intuitionistic fuzzy set B in Y, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proof:**

Let B be an intuitionistic fuzzy regular open set.

By hypothesis, $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ . Since B is an intuitionistic fuzzy regular open set, it is an intuitionistic fuzzy open set in Y. Therefore int(B) = B.

Hence $f^{-1}(B) = f^{-1}(int(B)) \subseteq int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an intuitionistic fuzzy open set in X and hence $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ open set in X as X is an intuitionistic fuzzy $T_{b^{\#}}$ space. Thus f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 3.2.16:**

Let f: (X, τ) → (Y, σ) be a mapping where X is an intuitionistic fuzzy $T_{b^{\#}}$ space. If $cl(f^{-1}(B)) \subseteq (f^{-1}(cl(B))$ for every intuitionistic fuzzy set B in Y, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proof:**

Let B be an intuitionistic fuzzy regular closed set.

By hypothesis, $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ . Since B is an intuitionistic fuzzy regular closed set, it is an intuitionistic fuzzy closed set in Y. Therefore cl(B) = B.

Hence $f^{-1}(B) = f^{-1}(cl(B)) \supseteq cl(f^{-1}(B)) \supseteq f^{-1}(B)$.

This implies $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X and hence $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X as X is an intuitionistic fuzzy $T_{b^{\#}}$ space. Thus f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 3.2.17:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy almost semi continuous mapping where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy regular closed set in Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X.

Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X, as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. As every intuitionistic fuzzy closed set is an intuitionistic fuzzy semi closed set, $f^{-1}(B)$ is an intuitionistic fuzzy semi closed set in X. Therefore f is an intuitionistic fuzzy almost semi continuous mapping.

**Proposition 3.2.18:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy almost $\beta$ continuous mapping where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy regular closed set in Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X.

Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X, as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. As every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\beta$ closed set, $f^{-1}(B)$ is an intuitionistic fuzzy $\beta$ closed set in X. Therefore f is an intuitionistic fuzzy almost $\beta$ continuous mapping.

**Proposition 3.2.19:**

Let f : (X, τ) → (Y, σ) be a mapping where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. If f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(cl(B))$ for every intuitionistic fuzzy regular closed set B in Y.

**Proof:**

Let B $\subseteq$ Y be an intuitionistic fuzzy regular closed set. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Therefore $cl(f^{-1}(B)) = f^{-1}(B)$. Now,

$$int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(B) \cup int(cl(int(f^{-1}(B))))$$

$$\subseteq cl(f^{-1}(B))$$

$= f^{-1}(B)$

$= f^{-1}(cl(B))$.

Hence $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(cl(B))$.

**Proposition 3.2.20:**

Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. If f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then, $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(B))))$ for every intuitionistic fuzzy regular open set B in Y.

**Proof:**

Let $B \subseteq Y$ be an intuitionistic fuzzy regular open set. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy open set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy open set in X. Therefore $int(f^{-1}(B)) = f^{-1}(B)$. Now,

$cl(int(cl(f^{-1}(B)))) \supseteq f^{-1}(B) \cap cl(int(cl(f^{-1}(B))))$

$\supseteq int(f^{-1}(B))$

$= f^{-1}(B)$

$= f^{-1}(int(B))$.

Hence $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(B))))$.

**Proposition 3.2.21:**

Let $f : (X, \tau) \to (Y, \sigma)$ be an intuitionistic fuzzy $b^{\#}$ continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ be an intuitionistic fuzzy continuous mapping then $g \circ f$ is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proof:**

Let B be intuitionistic fuzzy regular closed set in Z. Since every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, B is an intuitionistic fuzzy closed set in Z.

Then by hypothesis $g^{-1}(B)$ is an intuitionistic fuzzy closed set in Y. Since f is an intuitionistic fuzzy $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence $g \circ f$ is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

## 3.3 Intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping

In this section we have introduced intuitionistic fuzzy almost contra $b^{\#}$ continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced continuous mappings and the already existing continuous mappings in intuitionistic fuzzy topological spaces.

**Definition 3.3.1:**

A mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping if $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in $(X, \tau)$ for every intuitionistic fuzzy regular open set V of $(Y, \sigma)$.

**Example 3.3.2:**

Let X = {a, b}, Y = {u, v}. Then $\tau = \{0_{\sim}, G_1, G_2\ 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_3, G_4\ 1_{\sim}\}$ are intuitionistic fuzzy topologies on X and Y respectively, where,

$$G_1 = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle,$$

$$G_2 = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle,$$

$G_3 = \langle\, y, (0.7_u, 0.6_v), (0.2_u, 0.3_v)\rangle$ and

$G_4 = \langle\, y, (0.2_u, 0.3_v), (0.7_u, 0.6_v)\rangle$ .

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

The intuitionistic fuzzy sets $G_3 = \langle\, y, (0.7_u, 0.6_v), (0.2_u, 0.3_v)\rangle$ and

$G_4 = \langle\, y, (0.2_u, 0.3_v), (0.7_u, 0.6_v)\rangle$

are intuitionistic fuzzy regular open sets in Y as

$$\begin{aligned} int(cl(f^{-1}(G_3))) &= int(G_4^c) \\ &= G_3 \quad \text{and} \end{aligned}$$

$$\begin{aligned} int(cl(f^{-1}(G_4))) &= int(G_3^c) \\ &= G_4. \end{aligned}$$

Then $f^{-1}(G_3) = \langle\, x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle$ is an intuitionistic fuzzy $b^{\#}$ closed set in (X, τ), as

$$\begin{aligned} int(cl(f^{-1}(G_3)))\cap cl(int(f^{-1}(G_3))) &= int(G_2^c)\cap cl(G_1) \\ &= G_1\cap G_2^c \\ &= f^{-1}(G_3). \end{aligned}$$

Similarly $f^{-1}(G_4) = \langle\, x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle$ is an intuitionistic fuzzy $b^{\#}$ closed set in (X, τ), as

$$\begin{aligned} int(cl(f^{-1}(G_4)))\cap cl(int(f^{-1}(G_4))) &= int(G_1^c)\cap cl(G_1) \\ &= G_2\cap G_1^c \\ &= f^{-1}(G_4). \end{aligned}$$

Therefore f is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping.

**Proposition 3.3.3:**

A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping if and only if the inverse image of each intuitionistic fuzzy regular closed set in Y is an intuitionistic fuzzy $b^{\#}$ open set in X.

**Proof:**

**Necessity:**

Let A be an intuitionistic fuzzy regular closed set in Y. Then $A^c$ is an intuitionistic fuzzy regular open set in Y. Then $f^{-1}(A^c)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X, by hypothesis.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$,

$f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ open set in X.

**Sufficiency:**

Let A be an intuitionistic fuzzy regular open set in Y. Then $A^c$ is an intuitionistic fuzzy regular closed set in Y. By hypothesis $f^{-1}(A^c)$ is an intuitionistic fuzzy $b^{\#}$ open set in X.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an intuitionistic fuzzy $b^{\#}$ open set in X.

Therefore $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping.

**Proposition 3.3.4:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping. Then for every intuitionistic fuzzy regular closed set A in Y and for every intuitionistic fuzzy point $p_{(\alpha,\ \beta)} \in X$, if $f(p_{(\alpha,\ \beta)})\ _q\ A$

Then $p_{(\alpha,\ \beta)}\ _q\ bint(f^{-1}(A))$.

**Proof:**

Let f be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping. Let $A \subseteq Y$ be an intuitionistic fuzzy regular closed set and let $p_{(\alpha,\ \beta)} \in X$. Also let $f(p_{(\alpha,\ \beta)})\ _q\ A$, then $p_{(\alpha,\ \beta)}\ _q\ f^{-1}(A)$. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy b open set, $f^{-1}(A)$ is an intuitionistic fuzzy b open set in X.

Hence $bint(f^{-1}(A)) = f^{-1}(A)$ and

$p_{(\alpha,\ \beta)}\ _q bint(f^{-1}(A))$.

**Proposition 3.3.5:**

If f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping $f^{-1}(bcl(int(B))) \subseteq bint(f^{-1}(cl(int(B))))$ for every intuitionistic fuzzy set B in Y.

**Proof:**

Let $B \subseteq Y$ be an intuitionistic fuzzy set. Then cl(int(B)) is an intuitionistic fuzzy regular closed set in Y. By hypothesis, $f^{-1}(cl(int(B)))$ is an intuitionistic fuzzy $b^{\#}$ open set in X.

Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy b open set, $f^{-1}(cl(int(B)))$ is an intuitionistic fuzzy b open set in X. Therefore

$$f^{-1}(bcl(int(B))) \subseteq f^{-1}(cl(int(B)))$$

$$= bint(f^{-1}(cl(int(B)))).$$

**Proposition 3.3.6:**

If f : (X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, then for each intuitionistic fuzzy point $\square_{(\square,\ \square)} \in X$ and for each intuitionistic fuzzy regular closed set B containing $f(\square_{(\square,\ \square)})$, there exists an intuitionistic fuzzy b open set $A \subseteq X$ and $\square_{(\square,\ \square)} \in A$ such that $A \subseteq f^{-1}(B)$.

**Proof:**

Let B be an intuitionistic fuzzy regular closed set in Y. Let $\square_{(\square,\ \square)}$ be an intuitionistic fuzzy point in X such that $f(\square_{(\square,\ \square)}) \in B$. Then

$$\square_{(\square,\ \square)} \in f^{-1}(f(\square_{(\square,\ \square)})) \in f^{-1}(B).$$

By hypothesis $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy b open set, $f^{-1}(B)$ is an intuitionistic fuzzy b open set in X. Now,

$$\text{let } A = bint(f^{-1}(B)) \subseteq f^{-1}(B). \text{ Therefore } A \subseteq f^{-1}(B).$$

**Proposition 3.3.7:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, then

$$f^{-1}(cl(int(B))) \supseteq cl(int(f^{-1}(int(B)))) \cup int(cl((f^{-1}(int(B))))$$

for every intuitionistic fuzzy set B in Y.

**Proof:**

Let B be any intuitionistic fuzzy set in Y. Then cl(int(B)) is an intuitionistic fuzzy regular closed set in Y. By hypothesis $f^{-1}(cl(int(B))$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy b open set, $f^{-1}(cl(int(B))$ is an intuitionistic fuzzy b open set in X.

$$\text{Therefore } f^{-1}(cl(int(B)) \supseteq cl(int(f^{-1}(cl(int(B))))) \cup int(cl(f^{-1}(cl(int(B)))))$$

$$\supseteq cl(int(f^{-1}(int(B)))) \cup int(cl(f^{-1}(int(B)))).$$

**Proposition 3.3.8:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, then

$$cl(int(f^{-1}(cl(B)))) \cap int(cl(f^{-1}(cl(B)))) \subseteq f^{-1}(int(cl(B)))$$

for each intuitionistic fuzzy set B of Y.

**Proof:**

Let B be any intuitionistic fuzzy set in Y. Then int(cl(B)) is an intuitionistic fuzzy regular open set in Y.

By hypothesis $f^{-1}(int(cl(B)))$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy b closed set,

$f^{-1}(int(cl(B)))$ is an intuitionistic fuzzy b closed set in X. Therefore $cl(int(f^{-1}(cl(B))))\cap int(cl(f^{-1}(cl(B))))\subseteq cl(int(f^{-1}(int(cl(B)))))\cap int(cl(f^{-1}(int(cl(B)))))$

$$\subseteq f^{-1}(int(cl(B))).$$

**Proposition 3.3.9:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping where X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in Y. Then V is an intuitionistic fuzzy open set in Y as every intuitionistic fuzzy regular open set is an intuitionistic fuzzy open set. By hypothesis, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in X as X is an intuitionistic fuzzy $T_{b^{\#}}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping.

**Proposition 3.3.10:**

Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy contra continuous mapping, then f is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping where X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in Y. Then V is an intuitionistic fuzzy open set in Y as every intuitionistic fuzzy regular open set is an intuitionistic fuzzy open set. By hypothesis, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in X, as X is an intuitionistic fuzzy $T_{b^{\#}}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping.

**Proposition 3.3.11:**

Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, then f is an intuitionistic fuzzy almost contra pre continuous mapping.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in X. Then by hypothesis $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy pre closed set, $f^{-1}(V)$ is an intuitionistic fuzzy pre closed set in X. Hence f is an intuitionistic fuzzy almost contra pre continuous mapping.

**Proposition 3.3.12:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, then f is an intuitionistic fuzzy almost contra □ continuous mapping.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in X. Then by hypothesis $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy □ closed set, $f^{-1}(V)$ is an intuitionistic fuzzy □ closed set in X. Hence f is an intuitionistic fuzzy almost contra □ continuous mapping.

**Proposition 3.3.13:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, then f is an intuitionistic fuzzy almost contra □ continuous mapping.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in X. Then by hypothesis $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy □ closed set, $f^{-1}(V)$ is an intuitionistic fuzzy □ closed set in X. Hence f is an intuitionistic fuzzy almost contra □ continuous mapping.

**Proposition 3.3.14:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^\#$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^\#}$ space, then f is an intuitionistic fuzzy almost contra □ continuous mapping.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in X. Then by hypothesis, $f^{-1}(V)$ is an intuitionistic fuzzy $b^\#$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^\#}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy □ closed set, $f^{-1}(V)$ is an intuitionistic fuzzy □ closed set in X. Hence f is an intuitionistic fuzzy almost contra □ continuous mapping.

**Proposition 3.3.15:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^\#$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^\#}$ space, then f is an intuitionistic fuzzy almost contra semipre continuous mapping.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in X. Then by hypothesis, $f^{-1}(V)$ is an intuitionistic fuzzy $b^\#$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^\#}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semipre closed set, $f^{-1}(V)$ is an intuitionistic fuzzy semipre closed set in X. Hence f is an intuitionistic fuzzy almost contra semipre continuous mapping.

**Proposition 3.3.16:**

If f : (X, τ) → (Y, σ) is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, then the following conditions hold:

i. $cl(f^{-1}(B)) \subseteq f^{-1}(\text{int}(cl(B)))$ for every intuitionistic fuzzy regular open set in Y.

ii. $f^{-1}(cl(\text{int}(B))) \subseteq \text{int}(f^{-1}(B))$ for every intuitionistic fuzzy regular closed set in Y.

**Proof:**

(i) Let B ⊆ Y be an intuitionistic fuzzy regular open set. By, hypothesis $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X.

This implies $cl(f^{-1}(B)) = f^{-1}(B) = f^{-1}(\text{int}(B)) \subseteq f^{-1}(\text{int}(cl(B)))$.

(ii) Let B ⊆ Y be an intuitionistic fuzzy regular closed set. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy open set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy open set in X.

This implies $\text{int}(f^{-1}(B)) = f^{-1}(B) = f^{-1}(cl(B)) \supseteq f^{-1}cl(\text{int}(B))$.

# CHAPTER 4

## Completely $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces

### 4.1 Introduction

In this chapter we have introduced two types of $b^{\#}$ continuous mappings namely intuitionistic fuzzy completely $b^{\#}$ continuous mappings and intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings. Also we have provided some interesting results based on these continuous mappings.

### 4.2 Intuitionistic fuzzy completely $b^{\#}$ continuous mapping

In this section we have introduced intuitionistic fuzzy completely $b^{\#}$ continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced continuous mappings and the already existing continuous mappings in intuitionistic fuzzy topological spaces.

**Definition 4.2.1:**

A mapping f: (X, □) → (Y, □) is called an intuitionistic fuzzy completely $b^{\#}$ continuous mapping if $f^{-1}(V)$ is an intuitionistic fuzzy regular closed set in (X, □) for every intuitionistic fuzzy $b^{\#}$ closed set V of (Y, □).

**Example 4.2.2:**

Let X = {a, b}, Y = {u, v}. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4\ 1_\sim\}$ are intuitionistic fuzzy topologies on X and Y respectively, where,

$G_1 = \langle\, x, (0.2_a, 0.3_b), (0.4_a, 0.5_b)\rangle$ ,

$G_2 = \langle\, x, (0.4_a, 0.5_b), (0.2_a, 0.3_b)\rangle$ ,

$G_3 = \langle\, y, (0.2_u, 0.3_v), (0.4_u, 0.5_v)\rangle$ and

$G_4 = \langle\, y, (0.4_u, 0.5_v), (0.2_u, 0.3_v)\rangle$ .

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Now $G_3^c = \langle\, y, (0.4_u, 0.5_v), (0.2_u, 0.3_v)\rangle$ is an intuitionistic fuzzy $b^{\#}$ closed set in (Y, τ), as

$$\begin{aligned} int(cl(G_3^c)) \cap cl(int(G_3^c)) &= int(G_3^c) \cap cl(G_4) \\ &= G_4 \cap G_3^c \\ &= G_3^c. \end{aligned}$$

Similarly $G_4^c = \langle\, y,(0.2_u,0.3_v),(0.4_u,0.5_v)\rangle$ is an intuitionistic fuzzy $b^{\#}$ closed set in (Y, τ), as

$$\begin{aligned} int(cl(G_4)) \cap cl(int(G_4)) &= int(G_4^c) \cap cl(G_3) \\ &= G_3 \cap G_4^c \\ &= G_4^c. \end{aligned}$$

Then $f^{-1}(G_3^c) = \langle\, x, (0.4_a , 0.5_b), (0.2_a , 0.3_b)\rangle$ is an intuitionistic fuzzy regular closed set in (X, τ), as

$$\begin{aligned} cl(int(f^{-1}(G_3^c))) &= cl(G_2) \\ &= G_1^c \\ &= f^{-1}(G_3^c). \end{aligned}$$

Similarly $f^{-1}(G_4^c) = \langle\, x, (0.2_a, 0.3_b), (0.4_a, 0.5_b)\rangle$ is an intuitionistic fuzzy regular closed set in (X, τ), as

$$\begin{aligned} cl(int(f^{-1}(G_4^c))) &= cl(G_1) \\ &= G_2^c \end{aligned}$$

$= f^{-1}(G_4^c)$.

Therefore f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

**Proposition 4.2.3:**

A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy completely continuous mapping where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy regular closed set in X. Hence f is an intuitionistic completely continuous mapping.

**Proposition 4.2.4:**

A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping if and only if the inverse image of each intuitionistic fuzzy $b^{\#}$ open set in Y is an intuitionistic fuzzy regular open set in X.

**Proof:**

**Necessity:**

Let A be an intuitionistic fuzzy $b^{\#}$ open set in Y. Then $A^c$ is an intuitionistic fuzzy $b^{\#}$ closed set in Y. Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(A^c)$ is an intuitionistic fuzzy regular closed set in X.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$,

$f^{-1}(A)$ is an intuitionistic fuzzy regular open set in X.

**Sufficiency:**

Let A be an intuitionistic fuzzy $b^{\#}$ closed set in Y. Then $A^c$ is an intuitionistic fuzzy $b^{\#}$ open set in Y. By hypothesis, $f^{-1}(A^c)$ is an intuitionistic fuzzy regular open set in X.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X.

Hence f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

**Proposition 4.2.5:**

A mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy $b^{\#}$ continuous mapping where X and Y are intuitionistic fuzzy $T_{b^{\#}}$ spaces.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y, as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy regular closed set in X.

Since every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Here $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X, as X is an intuitionistic fuzzy $T_{b^{\#}}$ space. Hence f is an intuitionistic fuzzy $b^{\#}$ continuous mapping.

**Proposition 4.2.6:**

A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping where X and Y are intuitionistic fuzzy $T_{b^{\#}}$ spaces.

**Proof:**

Let B be an intuitionistic fuzzy regular closed set in Y. Since every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, B is an intuitionistic fuzzy closed set in Y. Since Y is an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}$ (B) is an intuitionistic fuzzy regular closed set in X. We know that every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, $f^{-1}$ (B) is an intuitionistic fuzzy closed set in X. As X is an intuitionistic fuzzy $T_{b^{\#}}$ space,

$f^{-1}$(B) is an intuitionistic fuzzy $b^{\#}$ closed set in X.

Hence f is intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 4.2.7:**

If f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space, then for each intuitionistic fuzzy point $\square_{(\square, \square)}$ ∈ X and for every intuitionistic fuzzy neighbourhood A of f($\square_{(\square,\ \square)}$), there exists an intuitionistic fuzzy regular open set B of X such that

$$\square_{(\square,\ \square)} \in B \text{ and } f(B) \subseteq A.$$

**Proof:**

Let $\square_{(\square,\ \square)}$ be an intuitionistic fuzzy point of X and let A be an intuitionistic fuzzy neighbourhood of f($\square_{(\square,\ \square)}$) such that f($\square_{(\square,\ \square)}$) ∈ C ⊆ A, where C is an intuitionistic fuzzy open set in X. Since every intuitionistic fuzzy open set is an intuitionistic fuzzy $b^{\#}$ open set in an intuitionistic fuzzy $T_{b^{\#}}$ space, C is an intuitionistic fuzzy $b^{\#}$ open set in Y as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Hence by hypothesis, $f^{-1}(C)$ is an intuitionistic fuzzy regular open set in X and $\square_{(\square,\ \square)} \in f^{-1}(C)$. Put $B = f^{-1}(C)$. Therefore

$$\square_{(\square,\ \square)} \in B = f^{-1}(C) \subseteq f^{-1}(A).\text{Thus}$$

$$f(B) \subseteq f(f^{-1}(A)) \subseteq A.$$

That is $f(B) \subseteq A$.

**Proposition 4.2.8:**

A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping then

$$cl(int(f^{-1}(cl(B)))) \supseteq f^{-1}(B)$$

for every intuitionistic fuzzy set B in Y where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B ⊆ Y be an intuitionistic fuzzy set. Then cl(B) is an intuitionistic fuzzy closed set in Y and hence an intuitionistic fuzzy $b^{\#}$ closed set in Y as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. By hypothesis, $f^{-1}(cl(B))$ is an intuitionistic fuzzy regular closed set in X.

Hence $cl(int(f^{-1}(cl(B)))) = f^{-1}(cl(B)) \supseteq f^{-1}(B)$.

**Proposition 4.2.9:**

Let f: (X, τ) → (Y, σ) be an mapping. Then the following are equivalent:

i. f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping
ii. $f^{-1}(V)$ is an intuitionistic fuzzy regular open set in X for every intuitionistic fuzzy $b^{\#}$ open set V in Y
iii. for every intuitionistic fuzzy point $p_{(\alpha,\ \beta)} \in X$ and for every intuitionistic fuzzy $b^{\#}$ open set B in Y such that $f(p_{(\alpha,\ \beta)}) \in B$ there exists an intuitionistic fuzzy regular open set in X such that $p_{(\alpha,\ \beta)} \in A$ and f(A) ⊆ B

**Proof:**

(i)⇒(ii): Let V be an intuitionistic fuzzy $b^{\#}$ open set in Y. Then $V^c$ is an intuitionistic fuzzy $b^{\#}$ closed set in Y. Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(V^c)$ is an intuitionistic fuzzy regular closed set in X.

Since $f^{-1}(V^c) = (f^{-1}(V))^c$, $f^{-1}(V)$ is an intuitionistic fuzzy regular open set in X.

(ii)⇒(iii): Let $\square_{(\square,\ \square)} \in \square$ and B ⊆Y such that $f(\square_{(\alpha,\ \square)}) \in B$. This implies $\square_{(\square,\square)} \in f^{-1}(B)$. Since B is an intuitionistic fuzzy $b^{\#}$ open set in Y, by hypothesis $f^{-1}(B)$ is an intuitionistic fuzzy regular open set in X. Let $A = f^{-1}(B)$.Then

$$\square_{(\square,\ \square)} \in f^{-1}(f(\square_{(\square,\ \square)})) \in f^{-1}(B) = A.$$

Therefore $\square_{(\square,\ \square)} \in \square$ and $f(A) = f(f^{-1}(B)) \subseteq B$.This implies f(A) ⊆ B.

(iii)⇒(ii): Let B ⊆ Y be an intuitionistic fuzzy $b^{\#}$ open set. Let $\square_{(\square,\ \square)} \in \square$ and $f(\square_{(\square,\ \square)}) \in B$. By hypothesis, there exists an intuitionistic fuzzy regular open set C in X such that $\square_{(\square,\ \square)} \in \square$ and f(C) ⊆ B. This implies $C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B)$.

Therefore $\square_{(\square,\ \square)} \in \square \subseteq f^{-1}(B)$.

That is $f^{-1}(B) = \bigcup_{p(\alpha,\beta)\in f^{-1}(B)} p_{(\alpha,\beta)} \subseteq \bigcup_{p(\alpha,\beta)\in f^{-1}(B)} C \subseteq f^{-1}(B)$.

This implies $f^{-1}(B) = \bigcup_{p(\alpha,\beta)\in f^{-1}(B)} C$ .

Since the union intuitionistic fuzzy regular open sets is an intuitionistic fuzzy regular open set, $f^{-1}(B)$ is an intuitionistic fuzzy regular open set in X.

Hence f is intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

**Proposition 4.2.10:**

A mapping $f : X \rightarrow Y$ is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping then the following are equivalent:

i. for any intuitionistic fuzzy $b^{\#}$ open set A in Y and for any intuitionistic fuzzy point $p_{(\alpha,\ \beta)} \in X$, if $f(p_{(\alpha,\ \beta)})_q$ A, then $p_{(\alpha,\ \beta)q}$ $int(f^{-1}(A))$.

ii. for any intuitionistic fuzzy $b^{\#}$ open set A in Y and for any $p_{(\alpha,\ \beta)} \in X$, if $f(p_{(\alpha,\ \beta)})_q$ A , then there exists an intuitionistic fuzzy open set B such that $p_{(\alpha,\ \beta)q}$ B and $f(B) \subseteq A$.

**Proof:**

(i)$\Rightarrow$(ii): Let $A \subseteq Y$ be an intuitionistic fuzzy $b^{\#}$ open set and let $\square_{(\square,\ \square)} \in \square$. Let $f(\square_{(\square,\ \square)})_q$ A. Then $\square_{(\square,\ \square)q} f^{-1}(A)$ (i) implies that $\square_{(\square,\ \square)q}$ $int(f^{-1}(A))$ where $int(f^{-1}(A))$ is an intuitionistic fuzzy open set in X. Let $B = int(f^{-1}(A))$. Since $int(f^{-1}(A)) \subseteq f^{-1}(A)$ , $B \subseteq f^{-1}(A)$. Then $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(ii)$\Rightarrow$(i): Let $A \subseteq Y$ be an intuitionistic fuzzy $b^{\#}$ open set and let $\square_{(\square,\ \square)} \in \square$. Suppose $f(\square_{(\square,\ \square)})_q$ A, then by (ii) there exists an intuitionistic fuzzy open set B in X such that $\square_{(\square,\ \square)q}$ B and $f(B) \subseteq A$. Now $B \subseteq f^{-1}(f(B)) \subseteq f^{-1}(A)$.

That is $B = int(B) \subseteq int(f^{-1}(A))$.

Therefore $\square_{(\square,\ \square)q}$ B implies $\square_{(\square,\ \square)q}$ $int(f^{-1}(A))$.

**Proposition 4.2.11:**

Let $f_1$: (X, □) → (Y, □) and $f_2$ : (X, □) → (Y, □) be any two intuitionistic fuzzy completely $b^\#$ continuous mappings. Then the mapping $(f_1,f_2)$ : (X, □) → (□ × □, □ × □) is also an intuitionistic fuzzy completely $b^\#$ continuous mapping.

**Proof:**

Let □ × □ be an intuitionistic fuzzy $b^\#$ closed set of □ × □.

Then $(f_1, f_2)^{-1}$(□ × □)(x) = (□ × □)($f_1(x), f_2(x)$)

$$= \langle x, \min(\mu_A f_1(x), \mu_B f_2(x))), \max(\nu_A(f_1(x)), \nu_B(f_2(x)))\rangle$$

$$= \langle x, \min(f_1^{-1}(\mu_A)(x), f_2^{-1}(\mu_B)(x)), \max(f_1^{-1}(\nu_A)(x), f_2^{-1}(\nu_B)(x))\rangle$$

$$= f_1^{-1}(A) \cap f_2^{-1}(B)(x).$$

Since $f_1$ and $f_2$ are an intuitionistic fuzzy completely $b^\#$ continuous mapping, $f^{-1}(A)$ and $f^{-1}(B)$ are intuitionistic fuzzy regular open sets in X. Since the intersection of two intuitionistic fuzzy regular open sets is an intuitionistic fuzzy regular open set, $f_1^{-1}(A) \cap f_2^{-1}(B)$ is an intuitionistic fuzzy regular open set in X. Hence $(f_1, f_2)$ is an intuitionistic fuzzy completely $b^\#$ continuous mapping.

**Proposition 4.2.12:**

Let f : X → Y be an intuitionistic fuzzy completely $b^\#$ continuous mapping, then f is an intuitionistic fuzzy $b^\#$ irresolute mapping, where X is an intuitionistic fuzzy $T_{b^\#}$ space.

**Proof:**

Let B be an intuitionistic fuzzy $b^{\#}$ closed set in Y. Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy regular closed set in X. Since every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in X, as X is an intuitionistic fuzzy $T_{b^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy $b^{\#}$ irresolute mapping.

**Proposition 4.2.13:**

Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be any two mappings. If f and g are intuitionistic fuzzy completely $b^{\#}$ continuous mapping, then $g \circ f$ is also an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy $b^{\#}$ closed set in Z. Since g is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $g^{-1}(B)$ is an intuitionistic fuzzy regular closed set in Y. Since every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, $g^{-1}(B)$ is an intuitionistic fuzzy closed set in Y. As Y is an intuitionistic fuzzy $T_{b^{\#}}$ space, $g^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Now as f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping,

$f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is an intuitionistic fuzzy regular closed set in X.

Hence $g \circ f$ is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

**Proposition 4.2.14:**

Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be any two mappings. If f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping and g is an intuitionistic fuzzy $b^{\#}$ irresolute mapping then $g \circ f$ is also an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

**Proof:**

Let B be an intuitionistic fuzzy $b^{\#}$ closed set in Z. Since g is an intuitionistic fuzzy $b^{\#}$ irresolute mapping, $g^{-1}$ (B) is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Also, since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an intuitionistic fuzzy regular closed set in X.

Since $(g \circ f)^{-1}$ (B) = $f^{-1}(g^{-1}(B))$ , g ∘ f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

**Proposition 4.2.15:**

Let f : X → Y and g : Y → Z be any two mappings. If f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping and g is an intuitionistic fuzzy $b^{\#}$ continuous mapping then g ∘ f is also an intuitionistic fuzzy completely continuous mapping.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Z. Since g is an intuitionistic fuzzy $b^{\#}$ continuous mapping, $g^{-1}$ (B) is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Also, since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an intuitionistic fuzzy regular closed set in X.

Since $(g \circ f)^{-1}$ (B) = $f^{-1}(g^{-1}(B))$ , g ∘ f is an intuitionistic fuzzy completely continuous mapping.

**Proposition 4.2.16:**

Let f : X → Y be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping and then f is an intuitionistic fuzzy completely continuous mapping if Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in Y, as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy regular closed set in X. Hence f is an intuitionistic fuzzy completely continuous mapping.

**Proposition 4.2.17:**

Let f : X → Y and g : Y → Z be any two mappings. If f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping and g is an intuitionistic fuzzy $b^{\#}$ continuous mapping then g ∘ f is also an intuitionistic fuzzy completely continuous mapping.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Z. Since g is an intuitionistic fuzzy $b^{\#}$ continuous mapping, $g^{-1}$ (B) is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Also, since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an intuitionistic fuzzy regular closed set in X.

Since $(g \circ f)^{-1} (B) = f^{-1}(g^{-1}(B))$ , g ∘ f is an intuitionistic fuzzy completely continuous mapping.

**Proposition 4.2.18:**

Let f : X → Y and g : Y → Z be any two mappings. If f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping and g is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping then g ∘ f is also an intuitionistic fuzzy $b^{\#}$ irresolute mapping.

**Proof:**

Let B be an intuitionistic fuzzy $b^\#$ closed set in Z. Since g is an intuitionistic fuzzy completely $b^\#$ continuous mapping, $g^{-1}$ (B) is an intuitionistic fuzzy regular closed set in Y.

Also, since f is an intuitionistic fuzzy almost $b^\#$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an intuitionistic fuzzy $b^\#$ closed set in X.

Since $(g \circ f)^{-1}$ (B) = $f^{-1}(g^{-1}(B))$ , $g \circ f$ is an intuitionistic fuzzy $b^\#$ irresolute mapping.

**Proposition 4.2.19:**

Every intuitionistic fuzzy completely $b^\#$ continuous mapping $f : X \to Y$ is an intuitionistic fuzzy semi continuous mapping if Y is an intuitionistic fuzzy $T_{b^\#}$ space.

**Proof:**

Let $f : X \to Y$ be an intuitionistic fuzzy completely $b^\#$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^\#}$ space. Let A be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^\#$ closed set in an intuitionistic fuzzy $T_{b^\#}$ space, A is an intuitionistic fuzzy $b^\#$ closed set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X. Since every regular closed set in an intuitionistic fuzzy semi closed set, $f^{-1}(A)$ is an intuitionistic fuzzy semi closed set in X. Therefore f is an intuitionistic fuzzy semi continuous mapping.

**Proposition 4.2.20:**

Every intuitionistic fuzzy completely $b^\#$ continuous mapping $f : X \to Y$ is an intuitionistic fuzzy pre continuous mapping if Y is an intuitionistic fuzzy $T_{b^\#}$ space.

**Proof:**

Let $f : X \to Y$ be an intuitionistic fuzzy completely $b^\#$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^\#}$ space. Let A be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, A is an intuitionistic fuzzy $b^{\#}$ closed set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X. Since every regular closed set in an intuitionistic fuzzy pre closed set, $f^{-1}(A)$ is an intuitionistic fuzzy pre closed set in X. Therefore f is an intuitionistic fuzzy pre continuous mapping.

**Proposition 4.2.21:**

Every intuitionistic fuzzy completely $b^{\#}$ continuous mapping $f : X \rightarrow Y$ is an intuitionistic fuzzy □ continuous mapping if Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let $f : X \rightarrow Y$ be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, A is an intuitionistic fuzzy $b^{\#}$ closed set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X. Since every regular closed set in an intuitionistic fuzzy □ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy □ closed set in X. Therefore f is an intuitionistic fuzzy □ continuous mapping.

**Proposition 4.2.22:**

Every intuitionistic fuzzy completely $b^{\#}$ continuous mapping $f : X \rightarrow Y$ is an intuitionistic fuzzy □ continuous mapping if Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let $f : X \rightarrow Y$ be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, A is an intuitionistic fuzzy $b^{\#}$ closed set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X. Since every regular closed set in an intuitionistic fuzzy □ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy □ closed set in X. Therefore f is an intuitionistic fuzzy □ continuous mapping

**Proposition 4.2.23:**

Every intuitionistic fuzzy completely $b^{\#}$ continuous mapping $f : X \rightarrow Y$ is an intuitionistic fuzzy □ continuous mapping if Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let $f : X \rightarrow Y$ be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, A is an intuitionistic fuzzy $b^{\#}$ closed set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X. Since every regular closed set in an intuitionistic fuzzy □ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy □ closed set in X. Therefore f is an intuitionistic fuzzy □ continuous mapping.

**Proposition 4.2.24:**

Every intuitionistic fuzzy completely $b^{\#}$ continuous mapping $f : X \rightarrow Y$ is an intuitionistic fuzzy semipre continuous mapping if Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let $f : X \rightarrow Y$ be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$

space, A is an intuitionistic fuzzy $b^{\#}$ closed set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X. Since every regular closed set in an intuitionistic fuzzy semipre closed set, $f^{-1}(A)$ is an intuitionistic fuzzy semipre closed set in X. Therefore f is an intuitionistic fuzzy semipre continuous mapping.

## 4.3 Intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings

In this section we have introduced intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced continuous mappings and the already existing continuous mappings in intuitionistic fuzzy topological spaces.

**Definition 4.3.1:**

A mapping f: (X, □) → (Y, □) is called an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping if $f^{-1}(V)$ is an intuitionistic fuzzy clopen set in (X, □) for every intuitionistic fuzzy $b^{\#}$ closed set V of (Y, □).

**Example 4.3.2:**

Let X = {a, b}, Y = {u, v}. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4\ 1_\sim\}$ are intuitionistic fuzzy topologies on X and Y respectively, where,

$$G_1 = \langle x, (0.2_a, 0.3_b), (0.4_a, 0.5_b)\rangle,$$

$$G_2 = \langle x, (0.4_a, 0.5_b), (0.2_a, 0.3_b)\rangle,$$

$$G_3 = \langle y, (0.2_u, 0.3_v), (0.4_u, 0.5_v)\rangle \quad \text{and}$$

$$G_4 = \langle y, (0.4_u, 0.5_v), (0.2_u, 0.3_v)\rangle.$$

Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v.

Here $G_3^c = \langle y, (0.4_u, 0.5_v), (0.2_u, 0.3_v)\rangle$ is an intuitionistic fuzzy $b^\#$ closed set in (Y, τ), as

$$\begin{aligned} int(cl(G_3^c))\cap cl(int(G_3^c)) &= int(G_3^c)\cap cl(G_4) \\ &= G_4\cap G_3^c \\ &= G_3^c. \end{aligned}$$

Similarly $G_4^c = \langle y, (0.2_u, 0.3_v), (0.4_u, 0.5_v)\rangle$ is an intuitionistic fuzzy $b^\#$ closed set in (Y, τ), as

$$\begin{aligned} int(cl(G_4))\cap cl(int(G_4)) &= int(G_4^c)\cap cl(G_3) \\ &= G_3\cap G_4^c \\ &= G_4^c. \end{aligned}$$

Now $f^{-1}(G_3^c) = \langle x, (0.4_a , 0.5_b), (0.2_a , 0.3_b)\rangle$ is an intuitionistic fuzzy clopen set in (X, τ), as

$$\begin{aligned} cl(f^{-1}(G_3^c)) &= G_1^c \\ &= f^{-1}(G_3^c) \quad \text{and} \\ int(f^{-1}(G_3^c)) &= G_2 \\ &= f^{-1}(G_3^c). \end{aligned}$$

Similarly $f^{-1}(G_4^c) = \langle x, (0.2_a, 0.3_b), (0.4_a, 0.5_b)\rangle$ is an intuitionistic fuzzy clopen set in (X, τ), as

$$\begin{aligned} cl(f^{-1}(G_4^c)) &= G_2^c \\ &= f^{-1}(G_4^c) \quad \text{and} \\ int(f^{-1}(G_4^c)) &= G_1 \\ &= f^{-1}(G_4^c). \end{aligned}$$

Therefore f is an intuitionistic fuzzy perfectly $b^\#$ continuous mapping.

**Proposition 4.3.3:**

A mapping f:(X, □) → (Y, □) is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping if and only if the inverse image of each intuitionistic fuzzy $b^{\#}$ open set in Y is an intuitionistic fuzzy clopen in X.

**Proof:**

**Neccessity:**

Let a mapping f:(X, □) → (Y,□) be an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping. Let A be an intuitionistic fuzzy $b^{\#}$ open set in Y. Then $A^c$ is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping and

$f^{-1}(A^c) = (f^{-1}(A))^c$ , we have $f^{-1}(A)$ is an intuitionistic fuzzy clopen set in X.

**Sufficiency:**

Let B be an intuitionistic fuzzy $b^{\#}$ closed set in Y. then $B^c$ is an intuitionistic fuzzy $b^{\#}$ open set in Y. By hypothesis, $f^{-1}(B^c)$ is an intuitionistic fuzzy clopen set in X. Which implies $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X, as

$$f^{-1}(B^c) = (f^{-1}(B))^c.$$

Therefore f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping.

**Proposition 4.3.4:**

A mapping f:(X, □) → (Y, □) is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy continuous mapping where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y, as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X.Thus$f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Hence f is an intuitionistic fuzzy continuous mapping.

**Proposition 4.3.5:**

A mapping f:(X, □) → (Y, □) is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, where X and Y are intuitionistic fuzzy $T_{b^{\#}}$ spaces.

**Proof:**

Let B be an intuitionistic fuzzy regular closed set in Y. Since every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, B is an intuitionistic fuzzy closed set in Y. Since Y is an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Thus $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X.

Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X, as X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Hence f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 4.3.6:**

A mapping f:(X, □) → (Y, □) is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping and then f is an intuitionistic fuzzy $b^{\#}$ continuous mapping where X and Y are intuitionistic fuzzy $T_{b^{\#}}$ spaces.

**Proof**:

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y, as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X.

Thus $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X, as X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Hence f is an intuitionistic fuzzy $b^{\#}$ continuous mapping.

**Proposition 4.3.7:**

A mapping f:(X, □) → (Y, □) is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy semi continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X.

Thus $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semi closed set, $f^{-1}(B)$ is an intuitionistic fuzzy semi closed set in X.

Hence f is an intuitionistic fuzzy semi continuous mapping.

**Proposition 4.3.8:**

A mapping f :(X, □) → (Y, □) is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy □ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y, as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}$(B) is an intuitionistic fuzzy clopen set in X. Thus $f^{-1}$(B) is an intuitionistic fuzzy closed set in X.

Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy □ closed set, $f^{-1}$(B) is an intuitionistic fuzzy □ closed set in X. Hence f is an intuitionistic fuzzy □ continuous mapping.

**Proposition 4.3.9:**

A mapping f : (X, □) → (Y, □) is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy pre continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space. B is an intuitionistic fuzzy $b^{\#}$ closed set in Y as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}$(B) is an intuitionistic fuzzy clopen set in X.

Thus $f^{-1}$(B) is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy pre closed set, $f^{-1}$(B) is an intuitionistic fuzzy pre closed set in X. Hence f is an intuitionistic fuzzy pre continuous mapping.

**Proposition 4.3.10:**

Let f : X → Y and g : Y → Z be any two intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Then their composition g ∘ f : X → Z is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping.

**Proof:**

Let A be an intuitionistic fuzzy $b^{\#}$ closed set in Z. Then by hypothesis, $g^{-1}(A)$ is an intuitionistic fuzzy clopen set in Y. Since Y is an intuitionistic fuzzy $T_{b^{\#}}$ space, $g^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in Y. Again by hypothesis, $f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy clopen set in X. Since $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$, $(g \circ f)^{-1}(A)$ is an intuitionistic fuzzy clopen set in X. Hence g ∘ f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping.

## Summary and conclusion

Throughout the research work, the concepts of intuitionistic fuzzy almost $b^{\#}$ continuous mappings, intuitionistic fuzzy almost contra $b^{\#}$ continuous mappings, intuitionistic fuzzy completely $b^{\#}$ continuous mappings and intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings are introduced and studied. They are compared with the already existing intuitionistic fuzzy continuous mappings in intuitionistic fuzzy topological spaces. Also intuitionistic fuzzy $T_{b^{\#}}$ space and intuitionistic fuzzy $T_{cb^{\#}}$ space, are introduced and investigated to remove some difficulties arise at the time of comparison of our newly defined continuous mappings and already existing intuitionistic fuzzy continuous mappings. Many properties and characterizations of the newly defined continuous mappings are obtained and analyzed with the help of the newly introduced spaces.

The future research directions based on this research work may be extended as follows:

1. Various types of intuitionistic fuzzy $b^{\#}$ continuous mappings can be studied for homeomorphisms, compactness in intuitionistic fuzzy topological spaces.
2. Various types of intuitionistic fuzzy $b^{\#}$ closed mappings, intuitionistic fuzzy $b^{\#}$ open mappings can be introduced.
3. The notion of intuitionistic fuzzy $b^{\#}$ closed sets can be extended to bitopological spaces, supra topological spaces and nano topological spaces.

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# Publications

1. **Dhivya, S.,** and **Jayanthi, D.,** Almost $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces, International Organization of Scientific Research Journal of Mathematics, (to be appeared).
2. **Dhivya, S.,** and **Jayanthi, D.,** Completely $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces, International Journal of Engineering Research and Technology, (to be appeared).

# CONTENT

# Introduction

Fuzzy sets are introduced by Zadeh in 1965 as follows: a fuzzy set A in a non empty set X is a mapping from X to the unit interval [0,1] and A(x) is interpreted as the degree of membership of x in A. A fuzzy set is created to describe linguistic variables in more detail. The linguistic variable temperature for instance may have categories (members) of cold, very cold, moderate, warm and very hot. Once these categories or members are defined, the function is then developed for each member in the set. Chang (1968) introduced the notions of fuzzy topology and more basic concepts like open set, closed set, neighbourhood, interior of a set, continuity and compactness etc.

Intuitionistic fuzzy set, an extension of fuzzy set, has been introduced by Atanassov in 1986. Intuitionistic fuzzy set has been found to be more efficient in dealing with vagueness and ambiguity. Intuitionistic fuzzy sets as a generalization of fuzzy sets can be useful in situations when description of a problem by a (fuzzy) linguistic variable, given in terms of a membership function only, seems too rough.

Intuitionistic fuzzy sets take into account both the degree of membership and of non membership subject to the condition that their sum does not exceed 1. Let $E$ be the set of all countries with elective governments. Assume that we know for every country $x \in E$ the percentage of the electorate that have voted for the corresponding government. Denote it by M(x) and let $\mu(x) = M(x) / 100$ (degree of membership, validity, etc.). Let $\nu(x) = 1 - \mu(x)$. This number corresponds to the part of electorate who have not voted for the government. By fuzzy set theory alone we cannot consider this value in more detail. However, if we define $\nu(x)$ (degree of non-membership, non-validity, etc.) as the number of votes given to the parties or persons outside the government, then we can show the part of electorate who have not vote at all or who have given bad voting –paper and the corresponding number will be $\pi(x) = 1 - \mu(x) - \nu(x)$ (degree of interdeterminacy, uncertainty, etc.,). Thus we can construct the set $\{\langle\, x, \mu(x), \nu(x) \,\rangle : x \in E\}$.

Using the notion of intuitionistic fuzzy sets, Coker [1997] has constructed the basic concepts of intuitionistic fuzzy topological spaces. After giving the fundamental definitions and the necessary examples he introduced the definitions of intuitionistic fuzzy continuity, intuitionistic fuzzy compactness, intuitionistic fuzzy connectedness and obtained several properties and some characterizations concerning intuitionistic fuzzy connectedness. The concept of $b^{\#}$ closed sets in intuitionistic fuzzy topological spaces is introduced by Gomathi and Jayanthi (2018).

In this thesis a new class of continuous mappings in intuitionistic fuzzy topological spaces namely intuitionistic fuzzy almost $b^{\#}$ continuous mapping, intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, intuitionistic fuzzy completely $b^{\#}$ continuous mapping and intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping are being introduced and their respective properties are discussed.

In Chapter I, the recent developments in intuitionistic fuzzy topology contributed by various authors are presented. This forms the basement for the remaining chapters of this thesis.

In Chapter II, some new propositions are proved for intuitionistic fuzzy $b^{\#}$ continuous mapping by defining two new spaces namely intuitionistic fuzzy $T_{cb^{\#}}$ space and intuitionistic fuzzy $T_{b^{\#}}$ space. Using these two spaces we have proved some interesting characterization theorems and relations.

In Chapter III, intuitionistic fuzzy almost $b^{\#}$ continuous mapping and intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping are introduced. The relationship between these newly introduced continuous mappings and few of the already existing intuitionistic fuzzy continuous mappings are being discussed. Further some of the characterizations of these newly introduced continuous mappings using the spaces intuitionistic fuzzy $T_{cb^{\#}}$ space and intuitionistic fuzzy $T_{b^{\#}}$ space are discussed.

In Chapter IV, we have introduced two types of intuitionistic fuzzy continuous mappings namely intuitionistic fuzzy completely $b^{\#}$ continuous mappings and intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings. As intuitionistic fuzzy $b^{\#}$ closed set stronger than many of the intuitionistic fuzzy closed sets, we have proved the relation between the newly introduced intuitionistic fuzzy continuous mappings with other already existing intuitionistic fuzzy continuous mappings using intuitionistic fuzzy $T_{cb^{\#}}$ space and intuitionistic fuzzy $T_{b^{\#}}$ space.

# REVIEW OF LITERATURE

A review of literature of recent developments on the notions of closed and open sets, continuous mappings in topological spaces, fuzzy topological spaces and intuitionistic fuzzy topological spaces is given below.

## Topological spaces

The concepts of closed and open sets play an important role in the study of topological spaces. Stone (1937) introduced the regular open sets in topological spaces, as a stronger form of open sets and Levine (1963) initiated and studied the concept of semi open sets in topological spaces, as a weaker form of open sets.

Njastad (1965) introduced some classes of nearly open sets. Abd El- Monsef, El-Deeb and Mahmoud (1983) have introduced $\beta$ open in topological spaces. Andrijievic (1996) introduced the concept of b-open sets in general topology. Further the notion of $b^{\#}$ open set is introduced by Ushaparameswari and Thangavelu (2014). Mashhour, Abd EI-Monsef and EL-Deeb(1982) have investigated pre continuous and weak pre continuous mappings in topology. Noiri (1984) introduced alpha continuous functions in topological spaces.

Dontchev (1996) introduced a new class of mappings called contra-continuity in general topology. Also, a new weaker form of this class of mappings called contra semi continuous mappings are introduced and investigated by Dontchev and Noiri (1999).

Caldas (2000) defined and studied weak and strong forms of irresolute maps in general topology. Caldas and Jafari (2001) have investigated some properties of contra $\beta$ continuous functions. Ekici (2004) has introduced contra-continuity in topological space. Ahmad Al-Omari, Mohd.Salmi Md and Noorani (2009) have introduced some properties of contra b-continuous functions. El-magbrabi (2010) has investigated some properties of contra continuous mappings in topological spaces.

Caldas and Jafari (2007) have investigated some applications of b-open sets in topological spaces. In 2016, Kanchana and nirmalairudayam have introduced and studied the concepts of new class of maps namely generalized $^{*+}$ b-continuous and irresolute map.

**Fuzzy topological spaces**

The fundamental concept of fuzzy sets was introduced by Zadeh (1965). After the discovery of the fuzzy sets, much attention has been paid to the basic concepts of fuzzy topology. The notion of fuzzy set are naturally play a significant value in the study of fuzzy topology which was introduced by Chang (1968). In recent years fuzzy topology has been found to be very useful in solving many practical problems. Ganguly and Saha (1986) have introduced fuzzy semi open sets in fuzzy topological spaces. Singal and NitiPrakash (1991) have introduced fuzzy preopen sets.

Bin Shahna (1991) introduced and studied fuzzy continuity in fuzzy topological spaces. Benchalli and Jenifer Karnel (2010) introduced the concept of fuzzy b-open sets in fuzzy topological spaces. He proved some properties and investigated their relations with different fuzzy sets in fuzzy topological spaces.

The concept of fuzzy $b^{\#}$ closed sets was introduced by Indhumathi and Jayanthi (2018). Azad (1981) introduced some weaker forms of continuity in fuzzy topological space. He introduced fuzzy semi-continuous functions, fuzzy almost continuous functions in fuzzy topological spaces. Mukherjee and Sinha (1989) introduced and characterized the concept of fuzzy irresolute functions. Ekici and Kerre (2006) introduced the notion of fuzzy contra continuous mappings. They have analyzed some of their properties and obtained some interesting theorems.

**Intuitionistic fuzzy topological spaces**

Atanassov (1988) has introduced intuitionistic fuzzy sets and also he gave new results in intuitionistic fuzzy sets. Atanassov (1994) has defined a set of new operations on intuitionistic fuzzy sets. Intuitionistic fuzzy points are introduced by

Coker and Demirci (1995). Intuitionistic fuzzy open sets, intuitionistic fuzzy closed sets are introduced by Coker (1997).

Intuitionistic fuzzy semi open sets, intuitionistic fuzzy pre open sets, intuitionistic fuzzy $\alpha$ open sets are introduced by Gurcay, Coker and Hayder (1997). Gurcay, Coker and Hayder (1997) have introduced intuitionistic fuzzy semi continuous mappings, intuitionistic fuzzy pre continuous mappings. Intuitionistic fuzzy $\alpha$ continuous mappings and intuitionistic fuzzy $\beta$ continuous mappings in intuitionistic fuzzy topological spaces. Jun, Kang and Song (2005) have introduced intuitionistic fuzzy continuous and irresolute mapping.

Hanafy (2009) has introduced intuitionistic fuzzy $\gamma$ continuity in intuitionistic fuzzy topological spaces. Krsteska and Ekici (2007) have introduced intuitionistic fuzzy contra pre continuity. The concepts of nowhere dense in intuitionistic fuzzy topological space was introduced by Thakur and Dhavaseelan (2015).The concept of intuitionistic fuzzy $b^{\#}$ closed sets was introduced by Gomathi and Jayanthi (2018).

## 1. FUZZY SETS

[Zadeh, 1965]

In this article, the authors has introduced fuzzy sets which are characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. Further the author has provided the notions of inclusion, union, intersection, complement, etc. with respect to fuzzy sets.

## 2. GENERAL TOPOLOGICAL SPACES

[Bourbaki, 1996]

In this book, important classes of topological spaces are studied, uniform structures are introduced and applied to topological groups. Real numbers are constructed and their properties are established.

## 3. INTUITIONISTIC FUZZY SETS

[Atanassov, 1986]

In this article, the author has provided the notion of intuitionistic fuzzy sets. This is considered to be the generalization of fuzzy sets. The highlight of this particular article is that some relations and operations concerning classical sets are extended to intuitionistic fuzzy sets.

## 4. FUZZY TOPOLOGICAL SPACES

[Chang, 1968]

In this article, the author has introduced fuzzy topological spaces. This concept is considered to be the generalization of general topological spaces. In brief, the basic concepts such as fuzzy open set, fuzzy closed set, fuzzy neighbourhood, fuzzy continuity etc., are discussed.

## 5. INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Dogan Coker, 1997]

In this article, the author has introduced intuitionistic fuzzy topological space. The notions of intuitionistic fuzzy interior and intuitionistic fuzzy closure are being provided and this is followed by the discussion of some important properties concerning them. Furthermore, the notion of intuitionistic fuzzy continuity is provided.

## 6. ON FUZZY CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Gurcay, Coker and Haydar, 1997]

This article consists of the notions of intuitionistic fuzzy semiopen set, intuitionistic fuzzy preopen set, intuitionistic fuzzy α-open set, intuitionistic fuzzy β open set and their corresponding closed sets. Further the relationship between these sets are established.

# CHAPTER 1

## Preliminaries

In this chapter, the basic definitions and results of intuitionistic fuzzy sets, intuitionistic fuzzy continuous mappings and intuitionistic fuzzy contra continuous mappings in intuitionistic fuzzy topological space that are used to accomplish the present study are given in detail.

### 1.1 Intuitionistic fuzzy sets

**Definition 1.1.1:** [Zadeh 1965]

Let X be a non empty set. A fuzzy set A in X can be described in the form

$A = \{\langle x, \mu_A(x)\rangle / x \in X\}$

Where the function $\mu_A$: X → [0,1] is called the membership function and $\mu_A(x)$ denotes the degree to which x ∈ A and $0 \leq \mu_A(x) \leq 1$ for each x ∈ X.

**Definition 1.1.2: [**Atanassov 1986]

An intuitionistic fuzzy set A is an object having the form

$$A= \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$$

Where the functions $\mu_A$: X → [0, 1] and $\nu_A$: X →[0,1] denote the degree of membership and the degree of non-membership of each element x ∈ X to the set A respectively , and $0\leq \mu_A(x) + \nu_A(x) \leq 1$ for each x ∈ X. Denote by IFS(X) , the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set of A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle: x \in X\}$.

**Definition 1.1.3:** [Atanassov 1986]

Let A and B be two intuitionistic fuzzy sets of the form

$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$. Then the following properties hold:

i. $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

ii. $A=B$ if and only if $A \subseteq B$ and $A \supseteq B$,

iii. $A^c = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$,

iv. $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X\}$,

v. $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_{\sim} = \langle x, 0, 1\rangle$ and $1_{\sim} = \langle x, 1, 0\rangle$ are respectively the empty set and whole set of X.

**Definition 1.1.4:** [Coker, 1995]

Let $(X, \tau)$ be an intuitionistic fuzzy topological space and A, B be intuitionistic fuzzy sets in X. Then the following properties hold:

i. $int(A) = A$

ii. $A \subseteq cl(A)$

iii. $A \subseteq B \Rightarrow int(A) \subseteq int(B)$

iv. $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$

v. $int(int(A)) = int(A)$

vi. $cl(cl(A)) = cl(A)$

vii. $int(A \cap B) = int(A) \cap int(B)$

viii. $cl(A \cup B) = cl(A) \cup cl(B)$

ix. $int(1_{\sim}) = 1_{\sim}$

x. $cl(0_{\sim}) = 0_{\sim}$

**Definition 1.1.5:** [Coker, 1995]

Let A, B and C be intuitiontistic fuzzy sets in an intuitionistic fuzzy topological space X. Then

i. $(A \subseteq B)$ and $(C \subseteq D) \Rightarrow (A \cup C) \subseteq (B \cup D)$ and $(A \cap C) \subseteq (B \cap D)$
ii. $A \subseteq B$ and $A \subseteq C \Rightarrow A \subseteq (B \cap C)$
iii. $A \subseteq C$ and $B \subseteq C \Rightarrow (A \cup B) \subseteq C$
iv. $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$
v. $(A \cup B)^c = A^c \cap B^c$
vi. $(A \cap B)^c = A^c \cup B^c$
vii. $A \subseteq B \Rightarrow B^c \subseteq A^c$
viii. $(A^c)^c = A$
ix. $(0_\sim)^c = 1_\sim$
x. $(1_\sim)^c = 0_\sim$

**Definition 1.1.6:** [Coker, 1997]

An intuitionistic fuzzy topology on X is a family $\tau$ of intuitionistic fuzzy sets in X satisfying the following axioms:

i. $0_\sim, 1_\sim \in \tau$
ii. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
iii. $\cup G_i \in \tau$ for any $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair $(X, \tau)$ is called the **intuitionistic topological space** and any intuitionistic fuzzy set in $\tau$ is known as an **intuitionistic fuzzy open set** in X. Then the complement $A^c$ of an intuitionistic fuzzy open set A in an intuitionistic fuzzy topological space $(X, \tau)$ is called an **intuitionistic fuzzy closed set** in X.

**Definition 1.1.7:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X,\tau)$ is said be an **intuitionistic fuzzy semi closed set** if $int(cl(A)) \subseteq A$.

**Definition 1.1.8:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **intuitionistic fuzzy pre closed set** if $cl(int(A)) \subseteq A$.

**Definition 1.1.9:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **intuitionistic fuzzy regular closed set** if $cl(int(A)) = A$.

**Definition 1.1.10:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **Intuitionistic fuzzy α closed set** if $cl(int(cl(A))) \subseteq A$.

**Definition 1.1.11:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **intuitionistic fuzzy β closed set** if $int(cl(int(A))) \subseteq A$.

**Definition 1.1.12:** [Hanafy, 2009]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be **intuitionistic fuzzy γ closed set** (equivalently intuitionistic fuzzy b closed set) if

$$int(cl(A)) \cap cl(int(A)) \subseteq A.$$

**Definition1.1.13:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space $(X, \tau)$ is said to be an **intuitionistic fuzzy semi open set** if $A \subseteq cl(int(A))$.

**Definition 1.1.14:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set A =⟨x, $\mu_A$, $\nu_A$⟩ in an intuitionistic fuzzy topological space (X, τ) is said to be an **intuitionistic fuzzy pre open set** if A ⊆ int(cl(A)).

**Definition 1.1.15:** [Gurcay, Coker and Hayder, 1997]

An intuitionist fuzzy set A = ⟨x, $\mu_A$, $\nu_A$⟩ in an intuitionistic fuzzy topological space (X, τ) is said to be an **intuitionistic fuzzy regular open set** if int(cl(A)) = A

**Definition 1.1.16:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set A =⟨x, $\mu_A$, $\nu_A$⟩ in an intuitionistic fuzzy topological space (X, τ) is said to be an **intuitionistic fuzzy α open set** if A ⊆ int(cl(int(A))).

**Definition 1.1.17:** [Gurcay, Coker and Hayder, 1997]

An intuitionistic fuzzy set A =⟨x, $\mu_A$, $\nu_A$⟩ in an intuitionistic fuzzy topological space (X, τ) is said to be an **intuitionistic fuzzy β open set** if A ⊆ cl(int(cl(A))).

**Definition 1.1.18:** [Hanafy, 2009]

An intuitionistic fuzzy set A=⟨x, $\mu_A$, $\nu_A$⟩ in an intuitionistic fuzzy topological space (X, τ) is said to be **intuitionistic fuzzy γ open set** if A ⊆ int(cl(A)) ∪ cl(int(A)).

**Definition 1.1.19:** [Thakur and Rekha Chaturvedi, 2008]

Two intuitionistic fuzzy sets A and B are said to be **q-coincident** ($A\ _q\ B$) if and only if there exist an element x ∈ X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

**Definition 1.1.20:** [Coker, 1997]

Let (X, $\tau$) be an intuitionistic fuzzy topological space and A = $\langle x, \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy set in X.

Then the **intuitionistic fuzzy interior** and **intuitionistic fuzzy closure** are defined by

int(A) = ∪ {G/G is an intuitionistic fuzzy open set in X and G ⊆A},

cl(A) = ∩ {K/K is an intuitionistic fuzzy closed set in X and A ⊆ K}.

**Definition 1.1.21:** [Rajarajeswari, and Krishna Moorthy, 2013]

Let (X, τ) be an intuitionistic fuzzy topological space and A= $\langle x, \mu_A, \nu_A \rangle$ be an intuitionistic fuzzy set in X.

Then the **intuitionistic fuzzy b-interior** and **intuitionistic fuzzy b-closure** are defined by

b-int(A) = ∪ {G/G is an intuitinistic fuzzy b open set in X and G ⊆ A},

b-cl(A) = ∩ {K/K is an intuitionistic fuzzy b closed set in X and A ⊆ K}.

Note that for an intuitionistic fuzzy set A in (X, τ), we have $bcl(A^c)= (bint(A))^c$ and $bint(A)^c = (bcl(A))^c$.

**Definition 1.1.22:** [Thakur and Rekha Chaturvedi, 2008]

Two intuitionistic fuzzy set A and B are said to be **not q-coincident** if and only if A ⊆ $B^c$.

**Definition 1.1.23:** [Coker and Demirci, 1995]

**Intuitionistic fuzzy point**, written as $p_{(\alpha, \beta)}$ ,is defined to be an intuitionistic fuzzy set of X given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & if\ x=p \\ (0,1) & otherwise \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha, \beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

**Definition 1.1.24:** [Seok Jong Lee and Eun Pyo Lee, 2000]

Let $p_{(\alpha, \beta)}$ be an intuitionistic fuzzy point in (X, τ). An intuitionistic fuzzy set A of X is called an **intuitionistic fuzzy neighbourhood** of $p_{(\alpha, \beta)}$ if there exist an intuitionistic fuzzy open set B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

**Definition 1.1.25:** [Gomathi and Jayanthi, 2018]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space (X, τ) is said to be **intuitionistic fuzzy b$^{\#}$ closed set** if $int(cl(A)) \cap cl(int(A)) = A$.

**Definition 1.1.26:** [Gomathi and Jayanthi, 2018]

An intuitionistic fuzzy set $A = \langle x, \mu_A, \nu_A \rangle$ in an intuitionistic fuzzy topological space (X, τ) is said to be **intuitionistic fuzzy b$^{\#}$ open set** if $A = int(cl(A)) \cup cl(int(A))$ .

## 1.2 Intuitionistic fuzzy continuous mappings

In this section we have listed the definitions of some previously existing intuitionistic fuzzy continuous mappings.

**Definition 1.2.1:** [Coker, 1997]

Let X and Y be two non empty sets and f: X→Y be a mapping.

If A = {⟨x, $\mu_A(x)$, $\nu_A(x)$ / x∈ X⟩} is an intuitionistic fuzzy set in X, then the image of A under f, denoted by f(A), is the intuitionistic fuzzy set in Y defined by

$$f(A) = \{\langle y, f(\mu_A)(y), f\_(\nu_A)(y) / y \in Y)\rangle\}$$

Where $f\_(\nu_A) = 1 - f(1- \nu_A)$.

**Definition 1.2.2:** [Coker, 1997]

Let X and Y be two non empty sets and f: X →Y be a mapping.

If B = {⟨y, $\mu_B(y)$, $\nu_B(y)$ / y ∈ Y⟩} is an intuitionistic fuzzy set in Y, then the preimage of B under f is denoted and defined by

$$f^{-1}(B)= \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) / x \in X \rangle \}$$

Where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ for every x ∈ X.

**Definition 1.2.3:** [Gurcay, Coker and Hayder, 1997]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f said to be an **intuitionistic fuzzy continuous mapping** if $f^{-1}(V)$ is an intuitionistic fuzzy closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ).

**Definition 1.2.4:** [Joung Kon Jeon, 2005]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f said to be an **intuitionistic fuzzy semi continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy semi closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ).

**Definition 1.2.5:** [Joung Kon Jeon, 2005]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy α continuous mapping** if $f^{-1}$(V) is an intuitionistic fuzzy α closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ).

**Definition 1.2.6:** [Joung Kon Jeon, 2005]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f said to be an **intuitionistic fuzzy pre continuous mapping** if $f^{-1}$(V) is an intuitionistic fuzzy pre closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ).

**Definition 1.2.7:** [ Joung Kon Jeon, 2005]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy β continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy β closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ).

**Definition 1.2.8:** [Hanafy, 2009]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy γ continuous mapping** (equivalently intuitionistic fuzzy b continuous mapping) if $f^{-1}(V)$ is an intuitionistic fuzzy γ closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ).

**Definition 1.2.9:** [Krsteska and Ekici, 2007]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ) .

**Definition 1.2.10:** [Krsteska and Ekici, 2007]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra α continuous mapping** if $f^{-1}(V)$ is an intuitionistic fuzzy α closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

**Definition 1.2.11: [**Krsteska and Ekici, 2007]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra pre continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy pre closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

**Definition 1.2.12:** [Krsteska and Ekici, 2007]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra semi continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy semi closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

**Definition 1.2.13:** [Krsteska and Ekici, 2007]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra β continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy β closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

**Definition 1.2.14:** [Hanafy, 2009]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra γ continuous mapping** (equivalently intuitionistic fuzzy contra b continuous mapping) if $f^{-1}$ (V) is an intuitionistic fuzzy γ closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

**Definition 1.2.15:** [Joung Kon Jeon, 2005]

An intuitionistic fuzzy set A is said to be an **intuitionistic fuzzy dense** in another intuitionistic fuzzy set B in an intuitionistic fuzzy topological space (X, τ), if cl(A) = B.

**Definition 1.2.16:** [Hanafy and El-Arish, 2003]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy completely continuous mapping** if $f^{-1}(V)$ is an intuitionistic fuzzy regular open set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

**Definition 1.2.17:** [Gomathi and Jayanthi, 2018]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy $b^{\#}$ continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy $b^{\#}$ closed set in (X, τ) for every intuitionistic fuzzy closed set V of (Y, σ).

**Definition 1.2.18:** [Gomathi and Jayanthi, 2018]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy $b^{\#}$ irresolute mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy $b^{\#}$ closed set in (X, τ) for every intuitionistic fuzzy $b^{\#}$ closed set V of (Y, σ).

**Definition 1.2.19:** [Gomathi and Jayanthi, 2018]

Let f be a mapping from an intuitionistic fuzzy topological space (X, τ) into an intuitionistic fuzzy topological space (Y, σ). Then f is said to be an **intuitionistic fuzzy contra $b^{\#}$ continuous mapping** if $f^{-1}$ (V) is an intuitionistic fuzzy $b^{\#}$ closed set in (X, τ) for every intuitionistic fuzzy open set V of (Y, σ).

# CHAPTER 2

## Intuitionistic fuzzy T $_{cb^{\#}}$ spaces

### 2.1 Introduction

In this chapter we have introduced two new spaces called intuitionistic fuzzy T $_{cb^{\#}}$ space and intuitionistic fuzzy T $_{b^{\#}}$ space. Intuitionistic fuzzy $b^{\#}$ closed sets are independent with intuitionistic fuzzy closed sets and intuitionistic fuzzy $b^{\#}$ continuous mappings are independent with any other already existing mapping, so to remove this difficulty and to prove the interrelation we have introduced two new spaces called intuitionistic fuzzy T $_{cb^{\#}}$ space and intuitionistic fuzzy T $_{b^{\#}}$ space.

### 2.2 Intuitionistic fuzzy $b^{\#}$ continuous mappings

Gomathi and Jayanthi (2018) have introduced intuitionistic fuzzy $b^{\#}$ continuous mappings and intuitionistic fuzzy contra $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces. In this section we have proved the interrelation between intuitionistic fuzzy $b^{\#}$ continuous mappings with some of the other already existing mappings like intuitionistic fuzzy continuous mappings, intuitionistic fuzzy pre continuous mappings etc. using intuitionistic fuzzy T $_{cb^{\#}}$ space.

**Definition 2.2.1:**

If every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in (X, τ), then the space is called as an intuitionistic fuzzy T $_{cb^{\#}}$ space.

**Example 2.2.2:**

Let X={a, b} and then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ is an intuitionistic fuzzy topology on X, where,

$$G_1 = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle \quad \text{and}$$

$$G_2 = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle.$$

Then (X, τ) is an intuitionistic fuzzy topological space.

The intuitionistic fuzzy sets $G_1^c$ and $G_2^c$ are intuitionistic fuzzy $b^\#$ closed set in (X, τ) as

$$int(cl(G_1^c))\cap cl(int(G_1^c)) = G_1^c \cap G_2$$

$$= G_1^c \quad \text{and}$$

$$int(cl(G_2^c))\cap cl(int(G_2^c)) = G_2^c \cap G_1$$

$$= G_2^c.$$

Then intuitionistic fuzzy sets $G_1^c = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle$ and

$$G_2^c = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle$$

are intuitionistic fuzzy closed sets in X as,

$$cl(G_1^c) = G_1^c \quad \text{and}$$

$$cl(G_2^c) = G_2^c.$$

Since every intuitionistic fuzzy $b^\#$ closed set is an intuitionistic fuzzy $b^\#$ closed set in (X, τ), the space (X, τ) is as an intuitionistic fuzzy $T_{cb^\#}$ space.

**Definition 2.2.3:**

If every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^\#$ closed set in (X, τ), then the space is called as an intuitionistic fuzzy $T_{b^\#}$ space.

**Example 2.2.4:**

Let X = {a, b} and then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ is an intuitionistic fuzzy topology on X, where,

$$G_1 = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle \quad \text{and}$$

$$G_2 = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle.$$

Then (X, τ) is an intuitionistic fuzzy topological space.

The intuitionistic fuzzy sets $G_1^c = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle$ and

$$G_2^c = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle$$

are intuitionistic fuzzy closed sets in X as,

$$cl(G_1^c) = G_1^c \quad \text{and}$$

$$cl(G_2^c) = G_2^c.$$

Then $G_1^c$ and $G_2^c$ are intuitionistic fuzzy $b^\#$ closed set in (X, τ) as

$$int(cl(G_1^c)) \cap cl(int(G_1^c)) = G_1^c \cap G_2$$

$$= G_1^c \quad \text{and}$$

$$int(cl(G_2^c)) \cap cl(int(G_2^c)) = G_2^c \cap G_1$$

$$= G_2^c.$$

Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^\#$ closed set in (X, τ), the space (X, τ) is as an intuitionistic fuzzy $T_{b^\#}$ space.

**Proposition 2.2.5:**

Every intuitionistic fuzzy $b^\#$ continuous mapping f: X → Y is an intuitionistic fuzzy semi continuous mapping if X is an intuitionistic fuzzy $T_{cb^\#}$ space.

**Proof:**

Let f: X → Y be an intuitionistic fuzzy $b^\#$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^\#}$ space. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^\#$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^\#}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semi closed set, $f^{-1}(A)$ is an intuitionistic fuzzy semi closed set in X. Hence f is an intuitionistic fuzzy semi continuous mapping.

**Proposition 2.2.6:**

Every intuitionistic fuzzy $b^\#$ continuous mapping f : X → Y is an intuitionistic fuzzy pre continuous mapping if X is an intuitionistic fuzzy $T_{cb^\#}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy $b^\#$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^\#}$ space. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^\#$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^\#}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy pre closed set, $f^{-1}(A)$ is an intuitionistic fuzzy pre closed set in X. Hence f is an intuitionistic fuzzy pre continuous mapping.

**Proposition 2.2.7:**

Every intuitionistic fuzzy $b^\#$ continuous mapping f : X → Y is an intuitionistic fuzzy $\beta$ continuous mapping if X is an intuitionistic fuzzy $T_{cb^\#}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\beta$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\beta$ closed set in X. Hence f is an intuitionistic fuzzy $\beta$ continuous mapping.

**Proposition 2.2.8:**

Every intuitionistic fuzzy $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy $\alpha$ continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\alpha$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\alpha$ closed set in X. Hence f is an intuitionistic fuzzy $\alpha$ continuous mapping.

**Proposition 2.2.9:**

Every intuitionistic fuzzy $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy $\gamma$ continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\gamma$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\gamma$ closed set in X. Hence f is an intuitionistic fuzzy $\gamma$ continuous mapping.

**Proposition 2.2.10:**

Every intuitionistic fuzzy $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy semipre continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Then by hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semipre closed set, $f^{-1}(A)$ is an intuitionistic fuzzy semipre closed set in X. Hence f is an intuitionistic fuzzy semipre continuous mapping.

## 2.3 Intuitionistic fuzzy contra $b^{\#}$ continuous mappings

In this section we have proved the interrelation between intuitionistic fuzzy contra $b^{\#}$ continuous mappings with some of the other already existing mappings like intuitionistic fuzzy contra continuous mappings, intuitionistic fuzzy contra pre continuous mappings etc. using intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proposition 2.3.1:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy contra continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Therefore f is an intuitionistic fuzzy contra continuous mapping.

**Proposition 2.3.2:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy contra semi continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semi closed set, $f^{-1}(A)$ is an intuitionistic fuzzy semi closed set in X. Therefore f is an intuitionistic fuzzy contra semi continuous mapping.

**Proposition 2.3.3:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy contra pre continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy pre closed set, $f^{-1}(A)$ is an intuitionistic fuzzy pre closed set in X. Therefore f is an intuitionistic fuzzy contra pre continuous mapping.

**Proposition 2.3.4:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy contra $\beta$ continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\beta$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\beta$ closed set in X. Therefore f is an intuitionistic fuzzy contra $\beta$ continuous mapping.

**Proposition 2.3.5:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy contra $\alpha$ continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\alpha$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\alpha$ closed set in X. Therefore f is an intuitionistic fuzzy contra $\alpha$ continuous mapping.

**Proposition 2.3.6:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy contra $\gamma$ continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\gamma$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\gamma$ closed set in X. Therefore f is an intuitionistic fuzzy contra $\gamma$ continuous mapping.

**Proposition 2.3.7:**

Every intuitionistic fuzzy contra $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy contra semipre continuous mapping if X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Let A be an intuitionistic fuzzy open set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semipre closed set, $f^{-1}(A)$ is an intuitionistic fuzzy semipre closed set in X. Therefore f is an intuitionistic fuzzy contra semipre continuous mapping.

# CHAPTER 3

## Almost $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces

### 3.1 Introduction

In this chapter we have introduced two types of $b^{\#}$ continuous mappings namely intuitionistic fuzzy almost $b^{\#}$ continuous mappings and intuitionistic fuzzy almost contra $b^{\#}$ continuous mappings. Also we have provided some interesting results based on these continuous mappings.

### 3.2 Intuitionistic fuzzy almost $b^{\#}$ continuous mappings

In this section we have introduced intuitionistic fuzzy almost $b^{\#}$ continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced continuous mappings and the already existing continuous mappings in intuitionistic fuzzy topological spaces.

**Definition 3.2.1:**

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy almost $b^{\#}$continuous mapping if $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in $(X, \tau)$ for every intuitionistic fuzzy regular closed set V of $(Y, \sigma)$.

**Example 3.2.2:**

Let $X = \{a, b\}$, $Y = \{u, v\}$.Then $\tau = \{0_{\sim}, G_1, G_2, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_3, G_4, 1_{\sim}\}$ are intuitionistic fuzzy topologies on X and Y respectively, where,

$$G_1 = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle,$$

$G_2 = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle$,

$G_3 = \langle y, (0.7_u, 0.6_v), (0.2_u, 0.3_v)\rangle$ and

$G_4 = \langle y, (0.2_u, 0.3_v), (0.7_u, 0.6_v)\rangle$.

Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. The intuitionistic fuzzy sets,

$G_3^c = \langle y, (0.2_u, 0.3_v), (0.7_u, 0.6_v)\rangle$ and

$G_4^c = \langle y, (0.7_u, 0.6_v), (0.2_u, 0.3_v)\rangle$

are intuitionistic fuzzy regular closed sets in Y.

Then $f^{-1}(G_3^c) = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle$ is an intuitionistic fuzzy $b^\#$ closed set in $(X, \tau)$, as,

$$\begin{aligned} int(cl(f^{-1}(G_3^c)))\cap cl(int(f^{-1}(G_3^c))) &= int(G_2^c)\cap cl(G_1) \\ &= G_1\cap G_2^c \\ &= f^{-1}(G_3^c). \end{aligned}$$

Similarly $f^{-1}(G_4^c) = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle$ is an intuitionistic fuzzy $b^\#$ closed set in $(X, \tau)$, as,

$$\begin{aligned} int(cl(f^{-1}(G_4^c)))\cap cl(int(f^{-1}(G_4^c))) &= int(G_1^c)\cap cl(G_2^c) \\ &= G_2\cap G_1^c \\ &= f^{-1}(G_4^c). \end{aligned}$$

Therefore f is an intuitionistic fuzzy almost $b^\#$ continuous mapping.

**Proposition 3.2.3:**

A mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy almost $b^\#$ continuous mapping if and only if the inverse image of each intuitionistic fuzzy regular open set in Y is an intuitionistic fuzzy $b^\#$ open set in X.

**Proof:**

**Necessity:**

Let A be an intuitionistic fuzzy regular open set in Y. Then $A^c$ is an intuitionistic fuzzy regular closed set in Y. Since f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, $f^{-1}(A^c)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ open set in X.

**Sufficiency:**

Let A be an intuitionistic fuzzy regular closed set in Y. Then $A^c$ is an intuitionistic fuzzy regular open set in Y. By hypothesis $f^{-1}(A^c)$ is an intuitionistic fuzzy $b^{\#}$ open set in X.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 3.2.4:**

If f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then for each intuitionistic fuzzy point $p_{(\alpha,\ \beta)}$ of X and each A ∈ σ such that $f(p_{(\alpha,\ \beta)}) \in A$, there exists an intuitionistic fuzzy $b^{\#}$ open set B of X such that $p_{(\alpha,\ \beta)} \in B$ and $f(B) \subseteq A$.

**Proof:**

Let $p_{(\alpha,\ \beta)}$ be an intuitionistic fuzzy point of X and A ∈ σ such that $f(p_{(\alpha,\ \beta)}) \in A$, then

$$p_{(\alpha,\ \beta)} \in f^{-1}(A). \quad \text{Put } B = f^{-1}(A).$$

Then by hypothesis, B is an intuitionistic fuzzy $b^{\#}$ open set in X such that

$$p_{(\alpha,\ \beta)} \in B \quad \text{and} \quad f(B) = f(f^{-1}(A)) \subseteq A.$$

**Proposition 3.2.5:**

If f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping then for each intuitionistic fuzzy point $p_{(\alpha,\ \beta)}$ of X and each A ∈ σ such that $f(p_{(\alpha,\ \beta)})_q A$, there exists an intuitionistic fuzzy $b^{\#}$ open set B of X such that

$$(p_{(\alpha,\ \beta)})_q B \quad \text{and} \quad f(B) \subseteq A.$$

**Proof:**

Let $p_{(\alpha,\ \beta)}$ be an intuitionistic fuzzy point of X and A ∈ σ such that $f(p_{(\alpha,\ \beta)})_q A$. Then $p_{(\alpha,\ \beta)q}$ $f^{-1}(A)$ put B = $f^{-1}(A)$. Then by hypothesis, B is an intuitionistic fuzzy $b^{\#}$ open set in X such that $p_{(\alpha,\ \beta)q}B$ and f(B) = $f(f^{-1}(A)) \subseteq A$.

**Proposition 3.2.6:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then $f^{-1}(int(cl(B))) \subseteq cl(int(f^{-1}(cl(B)))) \cup int(cl((f^{-1}(cl(B))))$ for every intuitionistic fuzzy set B in Y.

**Proof:**

Let B be any intuitionistic fuzzy set in Y. Then int(cl(B)) is an intuitionistic fuzzy regular open set in Y. By hypothesis $f^{-1}(int(cl(B)))$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy b open set, $f^{-1}(int(cl(B)))$ is an intuitionistic fuzzy b open set in X. Therefore

$$f^{-1}(int(cl(B))) \subseteq cl(int(f^{-1}(int(cl(B))))) \cup int(cl(f^{-1}(int(cl(B)))))$$

$$\subseteq cl(int(f^{-1}(cl(B)))) \cup int(cl(f^{-1}(cl(B)))).$$

**Proposition 3.2.7:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then $cl(int(f^{-1}(int(B)))) \cap int(cl(f^{-1}(int(B)))) \subseteq f^{-1}(cl(int(B)))$ for each intuitionistic fuzzy regular closed set B of Y.

**Proof:**

Let B be any intuitionistic fuzzy set in Y. Then cl(int(B)) is an intuitionistic fuzzy regular closed set in Y. By hypothesis $f^{-1}(cl(int(B)))$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy b closed set, $f^{-1}(cl(int(B)))$ is an intuitionistic fuzzy b closed set in X. Therefore

$$cl(int(f^{-1}(int(B))))\cap int(cl(f^{-1}(int(B)))) \subseteq cl(int(f^{-1}(cl(int(B))))) \cap int(cl(f^{-1}(cl(int(B)))))$$

$$\subseteq f^{-1}(cl(int(B))).$$

**Proposition 3.2.8:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy continuous mapping, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping where X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let A be an intuitionistic fuzzy regular closed set in Y. Every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set. Therefore A is an intuitionistic fuzzy closed set in Y. Since f is an intuitionistic fuzzy continuous mapping, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space. Therefore $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy Almost $b^{\#}$ continuous mapping.

**Proposition 3.2.9:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost continuous mapping, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping where X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let A be an intuitionistic fuzzy regular closed set in Y. Since f is an intuitionistic fuzzy almost continuous mapping, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space. Therefore $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 3.2.10:**

Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be an mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. If f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then $(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))) \subseteq f^{-1}(cl(B)$ for every intuitionistic fuzzy regular closed set B in Y.

**Proof:**

Let $B \subseteq Y$ be an intuitionistic fuzzy regular closed set. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Therefore $cl(f^{-1}(B) = f^{-1}(B)$. Now

$$(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))) \subseteq f^{-1}(B) \cup (int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B)))$$

$$\subseteq cl(f^{-1}(B)$$

$$= f^{-1}(B)$$

$$\subseteq f^{-1}(cl(B).$$

Hence $(int(cl(f^{-1}(B))) \cap cl(int(f^{-1}(B))) \subseteq f^{-1}(cl(B)$.

**Proposition 3.2.11:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, then for each intuitionistic fuzzy point $p_{(\alpha, \ \beta)}$ in X and each intuitionistic fuzzy regular open set A in Y such that $f(p_{(\alpha, \ \beta)}) \in A$, $int(f^{-1}(cl(A)))$ is an intuitionistic fuzzy neighbourhood of $p_{(\alpha, \ \beta)}$ in X.

**Proof:**

Let $p_{(\alpha, \ \beta)} \in X$ and let A be an intuitionistic fuzzy regular open set in Y such that $f(p_{(\alpha, \ \beta)}) \in A$, $p_{(\alpha, \ \beta)} \in f^{-1}(A)$. By hypothesis $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(A)$ is an intuitionistic fuzzy open set in X.

Now $p_{(\alpha, \ \beta)} \in f^{-1}(A) = int(f^{-1}(A)) \subseteq int(f^{-1}(cl(A)))$.

Hence $intf^{-1}(cl(A))$ is an intuitionistic fuzzy neighbourhood of $p_{(\alpha, \ \beta)}$ in X.

**Proposition 3.2.12:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Then $cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every intuitionistic fuzzy semi open set A in Y.

**Proof:**

Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping and let A be an intuitionistic fuzzy semi open set in Y. Then

$A \subseteq cl(int(A))$. Now,

$$
\begin{aligned}
cl(A) &\subseteq cl(cl(int(A))) \\
&\subseteq cl(int(cl(A))) \\
&\subseteq cl(cl(A))
\end{aligned}
$$

$\subseteq$ cl(A).

Therefore cl(A) = cl(int(cl(A))). This implies cl(A) is an intuitionistic fuzzy regular closed set in Y. By hypothesis, $f^{-1}$(cl(A)) is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}$(cl(A)) is an intuitionistic fuzzy closed set in X. Therefore cl($f^{-1}$(cl(A))) = $f^{-1}$(cl(A)). Now,

$$cl(f^{-1}(A)) \subseteq cl(f^{-1}(cl(A)))$$

$$= f^{-1}(cl(A)).$$

Thus $cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$.

**Proposition 3.2.13:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. Then $f^{-1}(A) \subseteq int(f^{-1}(int(cl(A))))$ for every intuitionistic fuzzy pre open set A in Y.

**Proof:**

Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping and let A be an intuitionistic fuzzy pre open set in Y. Then A $\subseteq$ int(cl(A)) . Since int(cl(A)) is an intuitionistic fuzzy regular open set in Y , by hypothesis, $f^{-1}$ (int(cl(A))) is an intuitionistic fuzzy $b^{\#}$ open set in X.

Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy open set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}$ (int(cl(A))) is an intuitionistic fuzzy open set in X.

Therefore $f^{-1}(A) \subseteq f^{-1}(int(cl(A)))$

$$= int(f^{-1}(int(cl(A)))).$$

**Proposition 3.2.14:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping then $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for every intuitionistic fuzzy set B in Y where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let f be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping. Let B be an intuitionistic fuzzy regular open set in Y. By hypothesis $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy open set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}$ (B) is an intuitionistic fuzzy open set in X.

Therefore $f^{-1}(int(B)) \subseteq f^{-1}(B) = int(f^{-1}(B))$.

**Proposition 3.2.15:**

Let f: (X, τ) → (Y, σ) be a mapping where X is an intuitionistic fuzzy $T_{b^{\#}}$ space. If $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for every intuitionistic fuzzy set B in Y, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proof:**

Let B be an intuitionistic fuzzy regular open set.

By hypothesis, $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ . Since B is an intuitionistic fuzzy regular open set, it is an intuitionistic fuzzy open set in Y. Therefore int(B) = B.

Hence $f^{-1}(B) = f^{-1}(int(B)) \subseteq int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an intuitionistic fuzzy open set in X and hence $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ open set in X as X is an intuitionistic fuzzy $T_{b^{\#}}$ space. Thus f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 3.2.16:**

Let f: (X, τ) → (Y, σ) be a mapping where X is an intuitionistic fuzzy $T_{b^{\#}}$ space. If $cl(f^{-1}(B)) \subseteq (f^{-1}(cl(B))$ for every intuitionistic fuzzy set B in Y, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proof:**

Let B be an intuitionistic fuzzy regular closed set.

By hypothesis, $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ . Since B is an intuitionistic fuzzy regular closed set, it is an intuitionistic fuzzy closed set in Y. Therefore cl(B) = B.

Hence $f^{-1}(B) = f^{-1}(cl(B)) \supseteq cl(f^{-1}(B)) \supseteq f^{-1}(B)$.

This implies $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X and hence $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X as X is an intuitionistic fuzzy $T_{b^{\#}}$ space. Thus f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 3.2.17:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy almost semi continuous mapping where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy regular closed set in Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X.

Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X, as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. As every intuitionistic fuzzy closed set is an intuitionistic fuzzy semi closed set, $f^{-1}(B)$ is an intuitionistic fuzzy semi closed set in X. Therefore f is an intuitionistic fuzzy almost semi continuous mapping.

**Proposition 3.2.18:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy almost $\beta$ continuous mapping where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy regular closed set in Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X.

Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X, as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. As every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\beta$ closed set, $f^{-1}(B)$ is an intuitionistic fuzzy $\beta$ closed set in X. Therefore f is an intuitionistic fuzzy almost $\beta$ continuous mapping.

**Proposition 3.2.19:**

Let f : (X, τ) → (Y, σ) be a mapping where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. If f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(cl(B))$ for every intuitionistic fuzzy regular closed set B in Y.

**Proof:**

Let B $\subseteq$ Y be an intuitionistic fuzzy regular closed set. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Therefore $cl(f^{-1}(B)) = f^{-1}(B)$. Now,

$$int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(B) \cup int(cl(int(f^{-1}(B))))$$

$$\subseteq cl(f^{-1}(B))$$

$$= f^{-1}(B)$$

$= f^{-1}(cl(B))$.

Hence $int(cl(int(f^{-1}(B)))) \subseteq f^{-1}(cl(B))$.

**Proposition 3.2.20:**

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space. If f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, then, $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(B))))$ for every intuitionistic fuzzy regular open set B in Y.

**Proof:**

Let $B \subseteq Y$ be an intuitionistic fuzzy regular open set. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy open set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy open set in X. Therefore $int(f^{-1}(B)) = f^{-1}(B)$. Now,

$$cl(int(cl(f^{-1}(B)))) \supseteq f^{-1}(B) \cap cl(int(cl(f^{-1}(B))))$$

$$\supseteq int(f^{-1}(B))$$

$$= f^{-1}(B)$$

$$= f^{-1}(int(B)).$$

Hence $f^{-1}(int(B)) \subseteq cl(int(cl(f^{-1}(B))))$.

**Proposition 3.2.21:**

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic fuzzy $b^{\#}$ continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ be an intuitionistic fuzzy continuous mapping then $g \circ f$ is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proof:**

Let B be intuitionistic fuzzy regular closed set in Z. Since every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, B is an intuitionistic fuzzy closed set in Z.

Then by hypothesis $g^{-1}(B)$ is an intuitionistic fuzzy closed set in Y. Since f is an intuitionistic fuzzy $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence $g \circ f$ is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

## 3.3 Intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping

In this section we have introduced intuitionistic fuzzy almost contra $b^{\#}$ continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced continuous mappings and the already existing continuous mappings in intuitionistic fuzzy topological spaces.

**Definition 3.3.1:**

A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping if $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in $(X, \tau)$ for every intuitionistic fuzzy regular open set V of $(Y, \sigma)$.

**Example 3.3.2:**

Let X = {a, b}, Y = {u, v}. Then $\tau = \{0_{\sim}, G_1, G_2\ 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_3, G_4\ 1_{\sim}\}$ are intuitionistic fuzzy topologies on X and Y respectively, where,

$$G_1 = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle,$$

$$G_2 = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle,$$

$$G_3 = \langle y, (0.7_u, 0.6_v), (0.2_u, 0.3_v)\rangle \quad \text{and}$$

$$G_4 = \langle y, (0.2_u, 0.3_v), (0.7_u, 0.6_v)\rangle.$$

Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$.

The intuitionistic fuzzy sets $G_3 = \langle y, (0.7_u, 0.6_v), (0.2_u, 0.3_v)\rangle$ and

$$G_4 = \langle y, (0.2_u, 0.3_v),(0.7_u, 0.6_v)\rangle$$

are intuitionistic fuzzy regular open sets in Y as

$$\begin{aligned} int(cl(f^{-1}(G_3))) &= int(G_4^c) \\ &= G_3 \quad \text{and} \\ int(cl(f^{-1}(G_4))) &= int(G_3^c) \\ &= G_4. \end{aligned}$$

Then $f^{-1}(G_3) = \langle x, (0.7_a, 0.6_b), (0.2_a, 0.3_b)\rangle$ is an intuitionistic fuzzy $b^\#$ closed set in $(X, \tau)$, as

$$\begin{aligned} int(cl(f^{-1}(G_3))) \cap cl(int(f^{-1}(G_3))) &= int(G_2^c) \cap cl(G_1) \\ &= G_1 \cap G_2^c \\ &= f^{-1}(G_3). \end{aligned}$$

Similarly $f^{-1}(G_4) = \langle x, (0.2_a, 0.3_b), (0.7_a, 0.6_b)\rangle$ is an intuitionistic fuzzy $b^\#$ closed set in $(X, \tau)$, as

$$\begin{aligned} int(cl(f^{-1}(G_4))) \cap cl(int(f^{-1}(G_4))) &= int(G_1^c) \cap cl(G_1) \\ &= G_2 \cap G_1^c \\ &= f^{-1}(G_4). \end{aligned}$$

Therefore f is an intuitionistic fuzzy almost contra $b^\#$ continuous mapping.

**Proposition 3.3.3:**

A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping if and only if the inverse image of each intuitionistic fuzzy regular closed set in Y is an intuitionistic fuzzy $b^{\#}$ open set in X.

**Proof:**

**Necessity:**

Let A be an intuitionistic fuzzy regular closed set in Y. Then $A^c$ is an intuitionistic fuzzy regular open set in Y. Then $f^{-1}(A^c)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X, by hypothesis.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$,

$f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ open set in X.

**Sufficiency:**

Let A be an intuitionistic fuzzy regular open set in Y. Then $A^c$ is an intuitionistic fuzzy regular closed set in Y. By hypothesis $f^{-1}(A^c)$ is an intuitionistic fuzzy $b^{\#}$ open set in X.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an intuitionistic fuzzy $b^{\#}$ open set in X.

Therefore $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping.

**Proposition 3.3.4:**

Let f: (X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping. Then for every intuitionistic fuzzy regular closed set A in Y and for every intuitionistic fuzzy point $p_{(\alpha, \beta)} \in X$, if $f(p_{(\alpha, \beta)})\ q\ A$

Then $p_{(\alpha, \beta)}\ q\ bint(f^{-1}(A))$.

**Proof:**

Let f be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping. Let $A \subseteq Y$ be an intuitionistic fuzzy regular closed set and let $p_{(\alpha,\ \beta)} \in X$. Also let $f(p_{(\alpha,\ \beta)})\ _q\ A$, then $p_{(\alpha,\ \beta)}\ _q\ f^{-1}(A)$. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy b open set, $f^{-1}(A)$ is an intuitionistic fuzzy b open set in X.

Hence $bint(f^{-1}(A)) = f^{-1}(A)$ and

$p_{(\alpha,\ \beta)}\ _q bint(f^{-1}(A))$.

**Proposition 3.3.5:**

If f: (X, τ) → (Y, σ) is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping $f^{-1}(bcl(int(B))) \subseteq bint(f^{-1}(cl(int(B))))$ for every intuitionistic fuzzy set B in Y.

**Proof:**

Let $B \subseteq Y$ be an intuitionistic fuzzy set. Then cl(int(B)) is an intuitionistic fuzzy regular closed set in Y. By hypothesis, $f^{-1}(cl(int(B)))$ is an intuitionistic fuzzy $b^{\#}$ open set in X.

Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy b open set, $f^{-1}(cl(int(B)))$ is an intuitionistic fuzzy b open set in X. Therefore

$$f^{-1}(bcl(int(B))) \subseteq f^{-1}(cl(int(B)))$$

$$= bint(f^{-1}(cl(int(B)))).$$

**Proposition 3.3.6:**

If f : (X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, then for each intuitionistic fuzzy point $p_{(\alpha,\ \beta)} \in$ X and for each intuitionistic fuzzy regular closed set B containing f($p_{(\alpha,\ \beta)}$), there exists an intuitionistic fuzzy b open set A ⊆ X and $p_{(\alpha,\ \beta)} \in$ A such that A ⊆ $f^{-1}$(B).

**Proof:**

Let B be an intuitionistic fuzzy regular closed set in Y. Let $p_{(\alpha,\ \beta)}$ be an intuitionistic fuzzy point in X such that f($p_{(\alpha,\ \beta)}$) ∈ B. Then

$$p_{(\alpha,\ \beta)} \in f^{-1}(f\,(p_{(\alpha,\ \beta)})) \in f^{-1}(B).$$

By hypothesis $f^{-1}$(B) is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy b open set, $f^{-1}$(B) is an intuitionistic fuzzy b open set in X. Now,

let A = bint($f^{-1}$(B)) ⊆ $f^{-1}$(B). Therefore A⊆ $f^{-1}$(B).

**Proposition 3.3.7:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, then

$$f^{-1}(cl(int(B))) \supseteq cl(int(f^{-1}(int(B))))\cup int(cl((f^{-1}(int(B))))$$

for every intuitionistic fuzzy set B in Y.

**Proof:**

Let B be any intuitionistic fuzzy set in Y. Then cl(int(B)) is an intuitionistic fuzzy regular closed set in Y. By hypothesis $f^{-1}$(cl(int(B)) is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy b open set, $f^{-1}$(cl(int(B)) is an intuitionistic fuzzy b open set in X.

Therefore $f^{-1}(cl(int(B)) \supseteq cl(int(f^{-1}(cl(int(B)))))\cup int(cl(f^{-1}(cl(int(B)))))$

$$\supseteq cl(int(f^{-1}(int(B)))) \cup int(cl(f^{-1}(int(B)))).$$

**Proposition 3.3.8:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, then

$$cl(int(f^{-1}(cl(B)))) \cap int(cl(f^{-1}(cl(B)))) \subseteq f^{-1}(int(cl(B)))$$

for each intuitionistic fuzzy set B of Y.

**Proof:**

Let B be any intuitionistic fuzzy set in Y. Then int(cl(B)) is an intuitionistic fuzzy regular open set in Y.

By hypothesis $f^{-1}(int(cl(B)))$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy b closed set,

$f^{-1}(int(cl(B)))$ is an intuitionistic fuzzy b closed set in X. Therefore $cl(int(f^{-1}(cl(B))))\cap int(cl(f^{-1}(cl(B)))) \subseteq cl(int(f^{-1}(int(cl(B)))))\cap int(cl(f^{-1}(int(cl(B)))))$

$$\subseteq f^{-1}(int(cl(B))).$$

**Proposition 3.3.9:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy contra $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping where X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in Y. Then V is an intuitionistic fuzzy open set in Y as every intuitionistic fuzzy regular open set is an intuitionistic fuzzy open set. By hypothesis, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in X as X is an intuitionistic fuzzy $T_{b^{\#}}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping.

**Proposition 3.3.10:**

Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy contra continuous mapping, then f is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping where X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in Y. Then V is an intuitionistic fuzzy open set in Y as every intuitionistic fuzzy regular open set is an intuitionistic fuzzy open set. By hypothesis, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in X, as X is an intuitionistic fuzzy $T_{b^{\#}}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping.

**Proposition 3.3.11:**

Let f : (X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, then f is an intuitionistic fuzzy almost contra pre continuous mapping.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in X. Then by hypothesis $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy pre closed set, $f^{-1}(V)$ is an intuitionistic fuzzy pre closed set in X. Hence f is an intuitionistic fuzzy almost contra pre continuous mapping.

**Proposition 3.3.12:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, then f is an intuitionistic fuzzy almost contra $\alpha$ continuous mapping.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in X. Then by hypothesis $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\alpha$ closed set, $f^{-1}(V)$ is an intuitionistic fuzzy $\alpha$ closed set in X. Hence f is an intuitionistic fuzzy almost contra $\alpha$ continuous mapping.

**Proposition 3.3.13:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, then f is an intuitionistic fuzzy almost contra $\gamma$ continuous mapping.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in X. Then by hypothesis $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\gamma$ closed set, $f^{-1}(V)$ is an intuitionistic fuzzy $\gamma$ closed set in X. Hence f is an intuitionistic fuzzy almost contra $\gamma$ continuous mapping.

**Proposition 3.3.14:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, then f is an intuitionistic fuzzy almost contra $\beta$ continuous mapping.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in X. Then by hypothesis, $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\beta$ closed set, $f^{-1}(V)$ is an intuitionistic fuzzy $\beta$ closed set in X. Hence f is an intuitionistic fuzzy almost contra $\beta$ continuous mapping.

**Proposition 3.3.15:**

Let f:(X, τ) → (Y, σ) be an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, then f is an intuitionistic fuzzy almost contra semipre continuous mapping.

**Proof:**

Let V be an intuitionistic fuzzy regular open set in X. Then by hypothesis, $f^{-1}(V)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(V)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semipre closed set, $f^{-1}(V)$ is an intuitionistic fuzzy semipre closed set in X. Hence f is an intuitionistic fuzzy almost contra semipre continuous mapping.

**Proposition 3.3.16:**

If f : (X, τ) → (Y, σ) is an intuitionistic fuzzy almost contra $b^{\#}$ continuous mapping, where X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, then the following conditions hold:

i. $cl(f^{-1}(B)) \subseteq f^{-1}(\text{int}(cl(B)))$ for every intuitionistic fuzzy regular open set in Y.

ii. $f^{-1}(cl(\text{int}(B))) \subseteq \text{int}(f^{-1}(B))$ for every intuitionistic fuzzy regular closed set in Y.

**Proof:**

(i) Let B ⊆ Y be an intuitionistic fuzzy regular open set. By, hypothesis $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Since every intuitionistic fuzzy $b^{\#}$ closed set is an intuitionistic fuzzy closed set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X.

This implies $cl(f^{-1}(B)) = f^{-1}(B) = f^{-1}(\text{int}(B)) \subseteq f^{-1}(\text{int}(cl(B)))$.

(ii) Let B ⊆ Y be an intuitionistic fuzzy regular closed set. By hypothesis, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ open set in X. Since every intuitionistic fuzzy $b^{\#}$ open set is an intuitionistic fuzzy open set in X as X is an intuitionistic fuzzy $T_{cb^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy open set in X.

This implies $\text{int}(f^{-1}(B)) = f^{-1}(B) = f^{-1}(cl(B)) \supseteq f^{-1}cl(\text{int}(B))$.

# CHAPTER 4

## Completely $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces

### 4.1 Introduction

In this chapter we have introduced two types of $b^{\#}$ continuous mappings namely intuitionistic fuzzy completely $b^{\#}$ continuous mappings and intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings. Also we have provided some interesting results based on these continuous mappings.

### 4.2 Intuitionistic fuzzy completely $b^{\#}$ continuous mapping

In this section we have introduced intuitionistic fuzzy completely $b^{\#}$ continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced continuous mappings and the already existing continuous mappings in intuitionistic fuzzy topological spaces.

**Definition 4.2.1:**

A mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy completely $b^{\#}$ continuous mapping if $f^{-1}(V)$ is an intuitionistic fuzzy regular closed set in $(X, \tau)$ for every intuitionistic fuzzy $b^{\#}$ closed set V of $(Y, \sigma)$.

**Example 4.2.2:**

Let X = {a, b}, Y = {u, v}. Then $\tau = \{0_{\sim}, G_1, G_2\ 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_3, G_4\ 1_{\sim}\}$ are intuitionistic fuzzy topologies on X and Y respectively, where,

$$G_1 = \langle x, (0.2_a, 0.3_b), (0.4_a, 0.5_b)\rangle,$$

$$G_2 = \langle x, (0.4_a, 0.5_b), (0.2_a, 0.3_b)\rangle,$$

$$G_3 = \langle y, (0.2_u, 0.3_v), (0.4_u, 0.5_v)\rangle \quad \text{and}$$

$$G_4 = \langle y, (0.4_u, 0.5_v), (0.2_u, 0.3_v)\rangle.$$

Define a mapping $f: (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v.

Now $G_3^c = \langle y, (0.4_u, 0.5_v), (0.2_u, 0.3_v)\rangle$ is an intuitionistic fuzzy $b^{\#}$ closed set in $(Y, \tau)$, as

$$\begin{aligned} int(cl(G_3^c)) \cap cl(int(G_3^c)) &= int(G_3^c) \cap cl(G_4) \\ &= G_4 \cap G_3^c \\ &= G_3^c. \end{aligned}$$

Similarly $G_4^c = \langle y,(0.2_u,0.3_v),(0.4_u,0.5_v)\rangle$ is an intuitionistic fuzzy $b^{\#}$ closed set in $(Y, \tau)$, as

$$\begin{aligned} int(cl(G_4)) \cap cl(int(G_4)) &= int(G_4^c) \cap cl(G_3) \\ &= G_3 \cap G_4^c \\ &= G_4^c. \end{aligned}$$

Then $f^{-1}(G_3^c) = \langle x, (0.4_a , 0.5_b), (0.2_a , 0.3_b)\rangle$ is an intuitionistic fuzzy regular closed set in $(X, \tau)$, as

$$\begin{aligned} cl(int(f^{-1}(G_3^c))) &= cl(G_2) \\ &= G_1^c \\ &= f^{-1}(G_3^c). \end{aligned}$$

Similarly $f^{-1}(G_4^c) = \langle x, (0.2_a, 0.3_b), (0.4_a, 0.5_b)\rangle$ is an intuitionistic fuzzy regular closed set in (X, τ), as

$$cl(int(f^{-1}(G_4^c))) = cl(G_1)$$

$$= G_2^c$$

$$= f^{-1}(G_4^c).$$

Therefore f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

**Proposition 4.2.3:**

A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy completely continuous mapping where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy regular closed set in X. Hence f is an intuitionistic completely continuous mapping.

**Proposition 4.2.4:**

A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping if and only if the inverse image of each intuitionistic fuzzy $b^{\#}$ open set in Y is an intuitionistic fuzzy regular open set in X.

**Proof:**

**Necessity:**

Let A be an intuitionistic fuzzy $b^{\#}$ open set in Y. Then $A^c$ is an intuitionistic fuzzy $b^{\#}$ closed set in Y. Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(A^c)$ is an intuitionistic fuzzy regular closed set in X.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$,

$f^{-1}(A)$ is an intuitionistic fuzzy regular open set in X.

**Sufficiency:**

Let A be an intuitionistic fuzzy $b^{\#}$ closed set in Y. Then $A^c$ is an intuitionistic fuzzy $b^{\#}$ open set in Y. By hypothesis, $f^{-1}(A^c)$ is an intuitionistic fuzzy regular open set in X.

Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X.

Hence f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

**Proposition 4.2.5:**

A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy $b^{\#}$ continuous mapping where X and Y are intuitionistic fuzzy $T_{b^{\#}}$ spaces.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y, as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy regular closed set in X.

Since every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, $f^{-1}(A)$ is an intuitionistic fuzzy closed set in X. Here $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X, as X is an intuitionistic fuzzy $T_{b^{\#}}$ space. Hence f is an intuitionistic fuzzy $b^{\#}$ continuous mapping.

**Proposition 4.2.6:**

A mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping where X and Y are intuitionistic fuzzy $T_{b^{\#}}$ spaces.

**Proof:**

Let B be an intuitionistic fuzzy regular closed set in Y. Since every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, B is an intuitionistic fuzzy closed set in Y. Since Y is an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy regular closed set in X. We know that every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. As X is an intuitionistic fuzzy $T_{b^{\#}}$ space,

$f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X.

Hence f is intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 4.2.7:**

If f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space, then for each intuitionistic fuzzy point $p_{(\alpha,\ \beta)} \in X$ and for every intuitionistic fuzzy neighbourhood A of $f(p_{(\alpha,\ \beta)})$, there exists an intuitionistic fuzzy regular open set B of X such that

$$p_{(\alpha,\ \beta)} \in B \text{ and } f(B) \subseteq A.$$

**Proof:**

Let $p_{(\alpha,\ \beta)}$ be an intuitionistic fuzzy point of X and let A be an intuitionistic fuzzy neighbourhood of $f(p_{(\alpha,\ \beta)})$ such that $f(p_{(\alpha,\ \beta)}) \in C \subseteq A$, where C is an intuitionistic fuzzy open set in X. Since every intuitionistic fuzzy open set is an intuitionistic fuzzy $b^{\#}$ open set in an intuitionistic fuzzy $T_{b^{\#}}$ space, C is an intuitionistic fuzzy $b^{\#}$ open set in Y as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Hence by hypothesis, $f^{-1}(C)$ is an intuitionistic fuzzy regular open set in X and $p_{(\alpha,\ \beta)} \in f^{-1}(C)$. Put $B = f^{-1}(C)$. Therefore

$$p_{(\alpha,\ \beta)} \in B = f^{-1}(C) \subseteq f^{-1}(A). \text{Thus}$$

$$f(B) \subseteq f(f^{-1}(A)) \subseteq A.$$

$$\text{That is } f(B) \subseteq A.$$

**Proposition 4.2.8:**

A mapping f: (X, τ) → (Y, σ) is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping then

$$cl(int(f^{-1}(cl(B)))) \supseteq f^{-1}(B)$$

for every intuitionistic fuzzy set B in Y where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B ⊆ Y be an intuitionistic fuzzy set. Then cl(B) is an intuitionistic fuzzy closed set in Y and hence an intuitionistic fuzzy $b^{\#}$ closed set in Y as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. By hypothesis, $f^{-1}(cl(B))$ is an intuitionistic fuzzy regular closed set in X.

Hence $cl(int(f^{-1}(cl(B)))) = f^{-1}(cl(B)) \supseteq f^{-1}(B)$.

**Proposition 4.2.9:**

Let f: (X, τ) → (Y, σ) be an mapping. Then the following are equivalent:

i. f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping
ii. $f^{-1}(V)$ is an intuitionistic fuzzy regular open set in X for every intuitionistic fuzzy $b^{\#}$ open set V in Y
iii. for every intuitionistic fuzzy point $p_{(\alpha,\ \beta)} \in X$ and for every intuitionistic fuzzy $b^{\#}$ open set B in Y such that $f(p_{(\alpha,\ \beta)}) \in B$ there exists an intuitionistic fuzzy regular open set in X such that $p_{(\alpha,\ \beta)} \in A$ and $f(A) \subseteq B$

**Proof:**

(i)⇒(ii): Let V be an intuitionistic fuzzy $b^{\#}$ open set in Y. Then $V^c$ is an intuitionistic fuzzy $b^{\#}$ closed set in Y. Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(V^c)$ is an intuitionistic fuzzy regular closed set in X.

Since $f^{-1}(V^c) = (f^{-1}(V))^c$, $f^{-1}(V)$ is an intuitionistic fuzzy regular open set in X.

(ii)⇒(iii): Let $p_{(\alpha,\ \beta)} \in X$ and B ⊆Y such that $f(p_{(\alpha,\ \beta)}) \in B$. This implies $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since B is an intuitionistic fuzzy $b^{\#}$ open set in Y, by hypothesis $f^{-1}(B)$ is an intuitionistic fuzzy regular open set in X. Let $A = f^{-1}(B)$.Then

$$p_{(\alpha,\ \beta)} \in f^{-1}(f(p_{(\alpha,\ \beta)})) \in f^{-1}(B) = A.$$

Therefore $p_{(\alpha,\ \beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.This implies $f(A) \subseteq B$.

(iii)$\Rightarrow$(ii): Let B $\subseteq$ Y be an intuitionistic fuzzy b# open set. Let $p_{(\alpha,\ \beta)} \in X$ and f($p_{(\alpha,\ \beta)}$) ∈ B. By hypothesis, there exists an intuitionistic fuzzy regular open set C in X such that $p_{(\alpha,\ \beta)} \in C$ and f(C) ⊆ B. This implies C ⊆ $f^{-1}$(f(C)) ⊆ $f^{-1}$(B).

Therefore $p_{(\alpha,\ \beta)} \in C \subseteq f^{-1}(B)$.

That is $f^{-1}(B) = \bigcup_{p(\alpha,\beta)\in f^{-1}(B)} p_{(\alpha,\beta)} \subseteq \bigcup_{p(\alpha,\beta)\in f^{-1}(B)} C \subseteq f^{-1}(B)$.

This implies $f^{-1}(B) = \bigcup_{p(\alpha,\beta)\in f^{-1}(B)} C$ .

Since the union intuitionistic fuzzy regular open sets is an intuitionistic fuzzy regular open set, $f^{-1}$(B) is an intuitionistic fuzzy regular open set in X.

Hence f is intuitionistic fuzzy completely b# continuous mapping.

**Proposition 4.2.10:**

A mapping f : X → Y is an intuitionistic fuzzy completely b# continuous mapping then the following are equivalent:

i. for any intuitionistic fuzzy b# open set A in Y and for any intuitionistic fuzzy point $p_{(\alpha,\ \beta)} \in X$, if f($p_{(\alpha,\ \beta)}$)$_q$ A, then $p_{(\alpha,\ \beta)q}$ int($f^{-1}$(A)).
ii. for any intuitionistic fuzzy b# open set A in Y and for any $p_{(\alpha,\ \beta)} \in X$, if f($p_{(\alpha,\ \beta)}$)$_q$ A , then there exists an intuitionistic fuzzy open set B such that $p_{(\alpha,\ \beta)q}$ B and f(B) ⊆ A.

**Proof:**

(i)$\Rightarrow$(ii): Let A ⊆ Y be an intuitionistic fuzzy b# open set and let $p_{(\alpha,\ \beta)} \in X$. Let f($p_{(\alpha,\ \beta)}$)$_q$ A. Then $p_{(\alpha,\ \beta)q}$$f^{-1}$(A) (i) implies that $p_{(\alpha,\ \beta)q}$ int($f^{-1}$( A)) where int($f^{-1}$( A)) is an intuitionistic fuzzy open set in X. Let B = int($f^{-1}$( A)). Since int($f^{-1}$( A)) ⊆ $f^{-1}$( A) , B ⊆ $f^{-1}$( A). Then f(B) ⊆ f($f^{-1}$( A)) ⊆ A.

(ii)$\Rightarrow$(i): Let A $\subseteq$ Y be an intuitionistic fuzzy b$^{\#}$ open set and let $p_{(\alpha,\ \beta)} \in X$. Suppose f $(p_{(\alpha,\ \beta)})_q$ A, then by (ii) there exists an intuitionistic fuzzy open set B in X such that $p_{(\alpha,\ \beta)q}$ B and f(B) $\subseteq$ A. Now B $\subseteq$ $f^{-1}$(f( B)) $\subseteq$ $f^{-1}$( A).

That is B = int(B) $\subseteq$ int($f^{-1}$( A)).

Therefore $p_{(\alpha,\ \beta)q}$ B implies $p_{(\alpha,\ \beta)q}$ int($f^{-1}$( A)).

**Proposition 4.2.11:**

Let $f_1$: (X, $\tau$) → (Y, $\sigma$) and $f_2$ : (X, $\tau$) → (Y, $\sigma$) be any two intuitionistic fuzzy completely b$^{\#}$ continuous mappings. Then the mapping ($f_1$,$f_2$) : (X, $\tau$) → $(Y \times Y,\ \sigma \times \sigma)$ is also an intuitionistic fuzzy completely b$^{\#}$ continuous mapping.

**Proof:**

Let $A \times B$ be an intuitionistic fuzzy b$^{\#}$ closed set of $Y \times Y$.

Then $(f_1, f_2)^{-1}(A \times B)(x) = (A \times B)(f_1(x), f_2(x))$

$$=\langle x, \min(\mu_A f_1(x), \mu_B f_2(x))), \max(\nu_A(f_1(x)), \nu_B(f_2(x)))\rangle$$

$$= \langle x, \min(f_1^{-1}(\mu_A)(x), f_2^{-1}(\mu_B)(x)), \max(f_1^{-1}(\nu_A)(x), f_2^{-1}(\nu_B)(x)\rangle$$

$$= f_1^{-1}(A) \cap f_2^{-1}(B)(x).$$

Since $f_1$ and $f_2$ are an intuitionistic fuzzy completely b$^{\#}$ continuous mapping, $f^{-1}$(A) and $f^{-1}$(B) are intuitionistic fuzzy regular open sets in X. Since the intersection of two intuitionistic fuzzy regular open sets is an intuitionistic fuzzy regular open set, $f_1^{-1}(A) \cap f_2^{-1}(B)$ is an intuitionistic fuzzy regular open set in X. Hence ($f_1$, $f_2$) is an intuitionistic fuzzy completely b$^{\#}$ continuous mapping.

**Proposition 4.2.12:**

Let f : X → Y be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy $b^{\#}$ irresolute mapping, where X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy $b^{\#}$ closed set in Y. Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy regular closed set in X. Since every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in X, as X is an intuitionistic fuzzy $T_{b^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X. Hence f is an intuitionistic fuzzy $b^{\#}$ irresolute mapping.

**Proposition 4.2.13:**

Let f : X → Y and g : Y → Z be any two mappings. If f and g are intuitionistic fuzzy completely $b^{\#}$ continuous mapping, then $g \circ f$ is also an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy $b^{\#}$ closed set in Z. Since g is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $g^{-1}$ (B) is an intuitionistic fuzzy regular closed set in Y. Since every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, $g^{-1}(B)$ is an intuitionistic fuzzy closed set in Y. As Y is an intuitionistic fuzzy $T_{b^{\#}}$ space, $g^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Now as f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping,

$f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}$ (B) is an intuitionistic fuzzy regular closed set in X.

Hence $g \circ f$ is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

**Proposition 4.2.14:**

Let f : X → Y and g : Y → Z be any two mappings. If f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping and g is an intuitionistic fuzzy $b^{\#}$ irresolute mapping then $g \circ f$ is also an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

**Proof:**

Let B be an intuitionistic fuzzy $b^{\#}$ closed set in Z. Since g is an intuitionistic fuzzy $b^{\#}$ irresolute mapping, $g^{-1}$ (B) is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Also, since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an intuitionistic fuzzy regular closed set in X.

Since $(g \circ f)^{-1}$ (B) = $f^{-1}(g^{-1}(B))$ , $g \circ f$ is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping.

**Proposition 4.2.15:**

Let f : X → Y and g : Y → Z be any two mappings. If f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping and g is an intuitionistic fuzzy $b^{\#}$ continuous mapping then $g \circ f$ is also an intuitionistic fuzzy completely continuous mapping.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Z. Since g is an intuitionistic fuzzy $b^{\#}$ continuous mapping, $g^{-1}$ (B) is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Also, since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an intuitionistic fuzzy regular closed set in X.

Since $(g \circ f)^{-1}$ (B) = $f^{-1}(g^{-1}(B))$ , $g \circ f$ is an intuitionistic fuzzy completely continuous mapping.

**Proposition 4.2.16:**

Let f : X → Y be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping and then f is an intuitionistic fuzzy completely continuous mapping if Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in Y, as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy regular closed set in X. Hence f is an intuitionistic fuzzy completely continuous mapping.

**Proposition 4.2.17:**

Let f : X → Y and g : Y → Z be any two mappings. If f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping and g is an intuitionistic fuzzy $b^{\#}$ continuous mapping then $g \circ f$ is also an intuitionistic fuzzy completely continuous mapping.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Z. Since g is an intuitionistic fuzzy $b^{\#}$ continuous mapping, $g^{-1}$ (B) is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Also, since f is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an intuitionistic fuzzy regular closed set in X.

Since $(g \circ f)^{-1}$ (B) = $f^{-1}(g^{-1}(B))$ , $g \circ f$ is an intuitionistic fuzzy completely continuous mapping.

**Proposition 4.2.18:**

Let f : X → Y and g : Y → Z be any two mappings. If f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping and g is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping then g ∘ f is also an intuitionistic fuzzy $b^{\#}$ irresolute mapping.

**Proof:**

Let B be an intuitionistic fuzzy $b^{\#}$ closed set in Z. Since g is an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, $g^{-1}$ (B) is an intuitionistic fuzzy regular closed set in Y.

Also, since f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, $f^{-1}(g^{-1}(B))$ is an intuitionistic fuzzy $b^{\#}$ closed set in X.

Since $(g \circ f)^{-1}$ (B) = $f^{-1}(g^{-1}(B))$ , g ∘ f is an intuitionistic fuzzy $b^{\#}$ irresolute mapping.

**Proposition 4.2.19:**

Every intuitionistic fuzzy completely $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy semi continuous mapping if Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, A is an intuitionistic fuzzy $b^{\#}$ closed set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X. Since every regular closed set in an intuitionistic fuzzy semi closed set, $f^{-1}(A)$ is an intuitionistic fuzzy semi closed set in X. Therefore f is an intuitionistic fuzzy semi continuous mapping.

**Proposition 4.2.20:**

Every intuitionistic fuzzy completely $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy pre continuous mapping if Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, A is an intuitionistic fuzzy $b^{\#}$ closed set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X. Since every regular closed set in an intuitionistic fuzzy pre closed set, $f^{-1}(A)$ is an intuitionistic fuzzy pre closed set in X. Therefore f is an intuitionistic fuzzy pre continuous mapping.

**Proposition 4.2.21:**

Every intuitionistic fuzzy completely $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy $\beta$ continuous mapping if Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, A is an intuitionistic fuzzy $b^{\#}$ closed set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X. Since every regular closed set in an intuitionistic fuzzy $\beta$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\beta$ closed set in X. Therefore f is an intuitionistic fuzzy $\beta$ continuous mapping.

**Proposition 4.2.22:**

Every intuitionistic fuzzy completely $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy $\alpha$ continuous mapping if Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, A is an intuitionistic fuzzy $b^{\#}$ closed set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X. Since every regular closed set in an intuitionistic fuzzy $\alpha$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\alpha$ closed set in X. Therefore f is an intuitionistic fuzzy $\alpha$ continuous mapping

**Proposition 4.2.23:**

Every intuitionistic fuzzy completely $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy $\gamma$ continuous mapping if Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, A is an intuitionistic fuzzy $b^{\#}$ closed set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X. Since every regular closed set in an intuitionistic fuzzy $\gamma$ closed set, $f^{-1}(A)$ is an intuitionistic fuzzy $\gamma$ closed set in X. Therefore f is an intuitionistic fuzzy $\gamma$ continuous mapping.

**Proposition 4.2.24:**

Every intuitionistic fuzzy completely $b^{\#}$ continuous mapping f : X → Y is an intuitionistic fuzzy semipre continuous mapping if Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let f : X → Y be an intuitionistic fuzzy completely $b^{\#}$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Let A be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, A is an intuitionistic fuzzy $b^{\#}$ closed set in Y. By hypothesis, $f^{-1}(A)$ is an intuitionistic fuzzy regular closed set in X. Since every regular closed set in an intuitionistic fuzzy semipre closed set, $f^{-1}(A)$ is an intuitionistic fuzzy semipre closed set in X. Therefore f is an intuitionistic fuzzy semipre continuous mapping.

## 4.3 Intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings

In this section we have introduced intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced continuous mappings and the already existing continuous mappings in intuitionistic fuzzy topological spaces.

**Definition 4.3.1:**

A mapping f: (X, $\tau$) → (Y, $\sigma$) is called an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping if $f^{-1}$ (V) is an intuitionistic fuzzy clopen set in (X, $\sigma$) for every intuitionistic fuzzy $b^{\#}$ closed set V of (Y, $\sigma$).

**Example 4.3.2:**

Let X = {a, b}, Y = {u, v}. Then $\tau = \{0_\sim, G_1, G_2\ 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4\ 1_\sim\}$ are intuitionistic fuzzy topologies on X and Y respectively, where,

$G_1 = \langle x, (0.2_a, 0.3_b), (0.4_a, 0.5_b)\rangle$,

$G_2 = \langle x, (0.4_a, 0.5_b), (0.2_a, 0.3_b)\rangle$,

$G_3 = \langle y, (0.2_u, 0.3_v), (0.4_u, 0.5_v)\rangle$ and

$G_4 = \langle y, (0.4_u, 0.5_v), (0.2_u, 0.3_v)\rangle$.

Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v.

Here $G_3^c = \langle y, (0.4_u, 0.5_v), (0.2_u, 0.3_v)\rangle$ is an intuitionistic fuzzy $b^\#$ closed set in $(Y, \tau)$, as

$$\begin{aligned} int(cl(G_3^c))\cap cl(int(G_3^c)) &= int(G_3^c)\cap cl(G_4) \\ &= G_4\cap G_3^c \\ &= G_3^c. \end{aligned}$$

Similarly $G_4^c = \langle y, (0.2_u, 0.3_v), (0.4_u, 0.5_v)\rangle$ is an intuitionistic fuzzy $b^\#$ closed set in $(Y, \tau)$, as

$$\begin{aligned} int(cl(G_4))\cap cl(int(G_4)) &= int(G_4^c)\cap cl(G_3) \\ &= G_3\cap G_4^c \\ &= G_4^c. \end{aligned}$$

Now $f^{-1}(G_3^c) = \langle x, (0.4_a, 0.5_b), (0.2_a, 0.3_b)\rangle$ is an intuitionistic fuzzy clopen set in $(X, \tau)$, as

$$\begin{aligned} cl(f^{-1}(G_3^c)) &= G_1^c \\ &= f^{-1}(G_3^c) \quad \text{and} \end{aligned}$$

$$\begin{aligned} int(f^{-1}(G_3^c)) &= G_2 \\ &= f^{-1}(G_3^c). \end{aligned}$$

Similarly $f^{-1}(G_4^c) = \langle x, (0.2_a, 0.3_b), (0.4_a, 0.5_b)\rangle$ is an intuitionistic fuzzy clopen set in $(X, \tau)$, as

$$cl(f^{-1}(G_4^c)) = G_2^c$$
$$= f^{-1}(G_4^c) \quad \text{and}$$
$$int(f^{-1}(G_4^c)) = G_1$$
$$= f^{-1}(G_4^c).$$

Therefore f is an intuitionistic fuzzy perfectly $b^\#$ continuous mapping.

**Proposition 4.3.3:**

A mapping $f:(X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy perfectly $b^\#$ continuous mapping if and only if the inverse image of each intuitionistic fuzzy $b^\#$ open set in Y is an intuitionistic fuzzy clopen in X.

**Proof:**

**Neccessity:**

Let a mapping $f:(X, \tau) \rightarrow (Y,\sigma)$ be an intuitionistic fuzzy perfectly $b^\#$ continuous mapping. Let A be an intuitionistic fuzzy $b^\#$ open set in Y. Then $A^c$ is an intuitionistic fuzzy $b^\#$ closed set in Y.

Since f is an intuitionistic fuzzy perfectly $b^\#$ continuous mapping and

$f^{-1}(A^c) = (f^{-1}(A))^c$ , we have $f^{-1}(A)$ is an intuitionistic fuzzy clopen set in X.

**Sufficiency:**

Let B be an intuitionistic fuzzy $b^\#$ closed set in Y. then $B^c$ is an intuitionistic fuzzy $b^\#$ open set in Y. By hypothesis, $f^{-1}(B^c)$ is an intuitionistic fuzzy clopen set in X. Which implies $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X, as

$$f^{-1}(B^c) = (f^{-1}(B))^c.$$

Therefore f is an intuitionistic fuzzy perfectly $b^\#$ continuous mapping.

**Proposition 4.3.4:**

A mapping f:(X, $\tau$) → (Y, $\sigma$) is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy continuous mapping where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y, as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X.Thus$f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Hence f is an intuitionistic fuzzy continuous mapping.

**Proposition 4.3.5:**

A mapping f:(X, $\tau$) → (Y, $\sigma$) is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping, where X and Y are intuitionistic fuzzy $T_{b^{\#}}$ spaces.

**Proof:**

Let B be an intuitionistic fuzzy regular closed set in Y. Since every intuitionistic fuzzy regular closed set is an intuitionistic fuzzy closed set, B is an intuitionistic fuzzy closed set in Y. Since Y is an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y.

Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Thus $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X.

Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X, as X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Hence f is an intuitionistic fuzzy almost $b^{\#}$ continuous mapping.

**Proposition 4.3.6:**

A mapping $f:(X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping and then f is an intuitionistic fuzzy $b^{\#}$ continuous mapping where X and Y are intuitionistic fuzzy $T_{b^{\#}}$ spaces.

**Proof**:

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y, as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X.

Thus $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, $f^{-1}(B)$ is an intuitionistic fuzzy $b^{\#}$ closed set in X, as X is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Hence f is an intuitionistic fuzzy $b^{\#}$ continuous mapping.

**Proposition 4.3.7:**

A mapping $f:(X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy semi continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X.

Thus $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy semi closed set, $f^{-1}(B)$ is an intuitionistic fuzzy semi closed set in X.

Hence f is an intuitionistic fuzzy semi continuous mapping.

**Proposition 4.3.8:**

A mapping $f :(X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, then f is an intuitionistic fuzzy $\alpha$ continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space, B is an intuitionistic fuzzy $b^{\#}$ closed set in Y, as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X. Thus $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X.

Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $\alpha$ closed set, $f^{-1}(B)$ is an intuitionistic fuzzy $\alpha$ closed set in X. Hence f is an intuitionistic fuzzy $\alpha$ continuous mapping.

**Proposition 4.3.9:**

A mapping f : $(X, \tau) \rightarrow (Y, \sigma)$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping then f is an intuitionistic fuzzy pre continuous mapping, where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space.

**Proof:**

Let B be an intuitionistic fuzzy closed set in Y. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy $b^{\#}$ closed set in an intuitionistic fuzzy $T_{b^{\#}}$ space. B is an intuitionistic fuzzy $b^{\#}$ closed set in Y as Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Since f is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping, $f^{-1}(B)$ is an intuitionistic fuzzy clopen set in X.

Thus $f^{-1}(B)$ is an intuitionistic fuzzy closed set in X. Since every intuitionistic fuzzy closed set is an intuitionistic fuzzy pre closed set, $f^{-1}(B)$ is an intuitionistic fuzzy pre closed set in X. Hence f is an intuitionistic fuzzy pre continuous mapping.

**Proposition 4.3.10:**

Let f : X → Y and g : Y → Z be any two intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings where Y is an intuitionistic fuzzy $T_{b^{\#}}$ space. Then their composition $g \circ f : X \rightarrow Z$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping.

**Proof:**

Let A be an intuitionistic fuzzy $b^{\#}$ closed set in Z. Then by hypothesis, $g^{-1}(A)$ is an intuitionistic fuzzy clopen set in Y. Since Y is an intuitionistic fuzzy $T_{b^{\#}}$ space, $g^{-1}(A)$ is an intuitionistic fuzzy $b^{\#}$ closed set in Y. Again by hypothesis, $f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy clopen set in X. Since $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$, $(g \circ f)^{-1}(A)$ is an intuitionistic fuzzy clopen set in X. Hence $g \circ f$ is an intuitionistic fuzzy perfectly $b^{\#}$ continuous mapping.

## Summary and conclusion

Throughout the research work, the concepts of intuitionistic fuzzy almost $b^{\#}$ continuous mappings, intuitionistic fuzzy almost contra $b^{\#}$ continuous mappings, intuitionistic fuzzy completely $b^{\#}$ continuous mappings and intuitionistic fuzzy perfectly $b^{\#}$ continuous mappings are introduced and studied. They are compared with the already existing intuitionistic fuzzy continuous mappings in intuitionistic fuzzy topological spaces. Also intuitionistic fuzzy $T_{b^{\#}}$ space and intuitionistic fuzzy $T_{cb^{\#}}$ space, are introduced and investigated to remove some difficulties arise at the time of comparison of our newly defined continuous mappings and already existing intuitionistic fuzzy continuous mappings. Many properties and characterizations of the newly defined continuous mappings are obtained and analyzed with the help of the newly introduced spaces.

The future research directions based on this research work may be extended as follows:

1. Various types of intuitionistic fuzzy $b^{\#}$ continuous mappings can be studied for homeomorphisms, compactness in intuitionistic fuzzy topological spaces.
2. Various types of intuitionistic fuzzy $b^{\#}$ closed mappings, intuitionistic fuzzy $b^{\#}$ open mappings can be introduced.
3. The notion of intuitionistic fuzzy $b^{\#}$ closed sets can be extended to bitopological spaces, supra topological spaces and nano topological spaces.

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# Publications

1. **Dhivya, S.,** and **Jayanthi, D.,** Almost $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces, International Organization of Scientific Research Journal of Mathematics, (to be appeared).
2. **Dhivya, S.,** and **Jayanthi, D.,** Completely $b^{\#}$ continuous mappings in intuitionistic fuzzy topological spaces, International Journal of Engineering Research and Technology, (to be appeared).