



Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)

Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B

Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination – March 2021 I Semester

Class : I UG
Major : Mathematics

Time : 3 Hours
Max. Marks: 100

18BMAC01 Analytical Geometry

Part A

10 x 1 = 10

Choose the Correct Answer

1. A system of circles in which every pair of circles has the same radical axis is called a _____.
a. Coaxal system
b. Radical system
c. Polar system
d. None of the above
2. The directrix of equation $y^2 = 4ax$ is
a. $x - a = 0$
b. $x + a = 0$
c. $xa = 0$
d. $x + 2a = 0$
3. The chord of a rectangular hyperbola which subtend a right angle at a focus touch a _____.
a. fixed parabola
b. locus
c. fixed ellipse
d. directrix
4. The polar equation of a general conic is
a. $\frac{l}{r} = 1 + \cos\theta$
b. $\frac{l}{r} = 1 + e\cos\theta$
c. $\frac{l}{r} = 1 - \cos\theta$
d. $\frac{l}{r} = 1 - e\cos\theta$
5. Which one of following is a centre of the sphere: $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + e = 0$?
a. (u, v, w)
b. $(-u, -v, -w)$
c. $(2u, 2v, 2w)$
d. $(-2u, -2v, -2w)$
6. The intersection of a sphere and a plane is a _____.
a. semi-circle
b. circle
c. hemi-sphere
d. none
7. A cone whose equation is of second degree is called a
a. Conicoids
b. Quadric cone
c. Cylinder
d. None of the above
8. A function $f(x, y, z)$ which is such that $f(tx, ty, tz) = t^n f(x, y, z)$ is said to be _____ function.
a. homogeneous
b. linear
c. identity
d. non-homogeneous
9. The equation of tangent plane to the surface $3x^2 + y^2 + z^2 = 21$ at the point $(2, 3, 0)$ is
a. $2x + y = 7$
b. $3x + y = 5$
c. $2x - y = 7$
d. $x + 2y = 7$
10. The line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ is one of a set of three mutually perpendicular generators of the cone is _____.
a. $3xy - 5yz + 8zx = 0$
b. $xy - yz + zx = 0$
c. $3xy + 5yz - zx = 0$
d. $3xy - 5yz - 8zx = 0$

Part B**5 x 6 = 30****Answer ALL questions****Each answer should not exceed 400 words or two pages**

11.a. Prove that the tangent drawn from any point of a fixed circle of a coaxal system to two other fixed circles of the system are in a constant ratio.

(or)

11.b. Find the focus, vertex and the directrix of the parabola $y^2 - 2x - 6y + 5 = 0$.

12.a. Transform the Cartesian form of two perpendicular straight lines into polar form.

(or)

12.b. Derive the polar equation of a conic.

13.a. Determine the shortest distance between the lines.

$$l_1: \frac{x-5}{3} = \frac{y-6}{-4} = z-9, \quad l_2: 2x-2y+z-3=0 = 2x-y+2z-9.$$

(or)

13.b. Find the centre and radius of the circle of intersection of the plane $2x - 2y + z = 27$ and the sphere $x^2 + y^2 + z^2 = 225$.

14.a. Find the equation of the right circular cone whose vertex is the origin, axis is the line

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} \text{ and the semivertical angle is } 30^\circ.$$

(or)

14.b. Find the equation of the quadric cone through the x , y , z axes and the lines through the origin with d.r.'s 2, 4, 1; 3, 3, 1.

15.a. Show that $x^2 - 2y^2 + z^2 + 4yz + 2zx + 6xy - 12x - 10y - 10z + 21 = 0$ represents a cone. Find its vertex.

(or)

15.b. Derive the equation of the cone whose vertex is the point $A(\alpha, \beta, \gamma)$ and whose generating lines pass through the conic $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$.

Part C**5 x 12 = 60****Answer ALL questions****Each answer should not exceed 800 words or four pages**

16.a. Find the equation to the circle whose diameter is the common chord of the two circles

$$(x-a)^2 + y^2 = a^2 \text{ and } x^2 + (y-b)^2 = b^2.$$

Find also the length of the common chord.

(or)

16.b. A normal chord to an ellipse makes an angle of 45° with the axis. Prove that the square of its length is equal to $\frac{32a^4b^4}{(a^2+b^2)^3}$.

17.a. Show that $r^2 - kr \cos(\theta - \alpha) + kd = 0$ represents a system of coaxal circles for different values of k . Find the co-ordinates of the limiting points and the equation of the common radical axis.

(or)

17.b. Trace the conic $\frac{2}{r} = 1 + \cos\theta + \sin\theta$.

18.a. Find the length and the equations of the common perpendicular drawn to the lines

$$\frac{x+2}{2} = \frac{y+6}{3} = \frac{z-34}{-10}, \quad \frac{x+6}{4} = \frac{y-7}{-3} = \frac{z-7}{-2}$$

(or)

18.b. Show that the plane $2x + 2y - z = 12$ touches the sphere

$$x^2 + y^2 + z^2 - 2x - 4y - 6z + 5 = 0$$

Find the point of contact.

19.a. Determine the equations of the tangent planes which passes through the line

$$x + 9y - 3z = 0 = 3x - 3y + 6z - 5$$

and touches the conicoid $2x^2 - 6y^2 + 3z^2 = 5$.

(or)

19.b. Find the angle between the lines of intersection of the cone $2xy - 2yz + zx = 0$ and the plane $x + y - z = 0$.

20.a. Show that the lines of intersection of the plane $lx + my + nz = 0$ and the cone $xy + yz + zx = 0$ are perpendicular.

(or)

20.b. Show that the cone whose vertex is the origin and whose guiding curve is the circle of intersection of the sphere $x^2 + y^2 + z^2 = 3r^2$ and any plane at a distance r from the origin, three mutually perpendicular generators.
