# CHAPTER I

## DESIGNING AND ANALYZING CONSUMER PROTECTION OF GENERALIZED TIGHTENED TWO LEVEL CONTINUOUS SAMPLING PLAN

This chapter proposes the generalized tightened two level continuous sampling plan having general clearance numbers and general sampling rates of inspection. Operating procedure of the plan and derivation of performance measures by simplified Markov-chain approach are given. Designing of generalized tightened two level continuous sampling plan using the indices (AQL, AOQL) and (LQL, AOQL) are given. The analysis of the consumer protection is carried out for the given LQL with consumer risk of 0.10 and $f$. Examples are presented to illustrate the selection of plans. The method of construction of tables is also indicated.

In a manufacturing process, producing units in a conveyor belt or other straight line system with units flowing one after another in progressive assembly such that it is neither practical nor convenient to group the products, Dodge (1943) introduced the concept of continuous sampling plan and designated as CSP-1. Following this various single level continuous sampling plans such as CSP-2, CSP-3, CSP-V are considered as variations and extensions to the basic procedure of CSP-1.

The conditions for application of continuous sampling plans are

(i) there is a continuous flow of units in the order of production process

(ii) the units offered for inspection are one by one in the order of production

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(iii) the process quality level is stable

(iv) the inspection is relatively easy and quick

Lieberman and Solomon (1955) considered multilevel plan which allows for smoother transition between sampling inspection and screening inspection as an extension of continuous sampling plan, CSP-1. Derman, Littauer and Solomon (1957) presented three generalizations to the plan of Lieberman and Solomon (1955). The tightest of the three plans is designated as MLP-T. This plan provides for an infinite number of sampling inspection levels with only same clearance number and geometrical reduction in sampling rates.

The newly proposed generalized tightened two level continuous sampling plan carry simultaneous reduction of sampling rates and clearance number while switching from lower level to a next higher level of sampling inspection. This two level plan has two levels of sampling inspection with general clearance numbers and general sampling rates. This plan is designated as tightened MLP-2(G).

The proposed plan (i) introduces rapid reduction in inspection when the quality is superior and to require screening only when quality is poor (ii) permits a faster release of the screening crew to return for the production duties.

Consumer protection has always been a prime factor in the construction of acceptance sampling plans. For continuous sampling plans the average outgoing quality limit (AOQL) is widely used as primary performance index. This motivated the investigator to estimate the consumer protection for a given LQL with $P_a \leq 0.10$ of the selected plan in terms of $i$, the AOQL requirement and the average fraction inspected to guarantee minimum inspection cost.

MIL-STD-1235C (1988), US Military standard on continuous sampling plans provides the tables and procedures for applying the continuous sampling plans without consumer protection. The analysis of consumer protection assists in the future to incorporate the aspect of consumer protection in US Military standard table.

**Operating procedure of tightened MLP-2(G) plan**

The operating procedure of generalized tightened two level continuous sampling plan has the following steps, when the units offered for inspection in the order of production

Step 1 : At the outset, start with the screening inspection

Step 2 : After $i$ consecutive units in succession are found to be conforming, switch to sampling inspection at rate $f_1$

Step 3 : If $i_1$ units inspected consecutively at rate $f_1$ are found to be conforming switch to second level of sampling inspection at rate $f_2$ and continue sampling inspection in the same level

Step 4 : On finding a nonconforming unit at any sampling level revert to screening inspection

During sampling inspection, sample units are selected at random one at a time from flow of products so as to assure an unbiased sample. All nonconforming units found are replaced with conforming units. Tightened MLP-2(G) plan is characterized by the four parameters $i, i_1, f_1$ and $f_2$ where $i$ and $i_1$ are integers $f_1$ and $f_2$ are fractions.

The **chief features** of tightened MLP-2(G) plan are

i. The number of sampling levels is fixed at two
ii. The sampling rates $f_1$ and $f_2$ satisfy the relation $f_2 < f_1$
iii. The clearance numbers $i$ and $i_1$ satisfy the relation $i_1 < i$ and
iv. The reversion of screening inspection on finding a nonconforming unit at any level of sampling inspection

**Derivation of performance measures**

The derivation of performance measures using the simplified Markov chain approach due to Brugger (1972) consists of defining various phases, developing the transitional probability matrix, identifying the expected phase lengths and forming the working table.

The various phases of tightened MLP-2(G) plan are

SC : Screening inspection phase

SA1 : First sampling inspection phase at rate $f_1$

SA2 : Second sampling inspection phase at rate $f_2$

The transitional probability matrix for tightened MLP-2(G) plan is

| | | To | | |
|---|---|---|---|---|
| | | SC | SA1 | SA2 |
| From | SC | 0 | 1 | 0 |
| | SA1 | $1-q^{i_1}$ | 0 | $q^{i_1}$ |
| | SA2 | 1 | 0 | 0 |

The steady state probabilities in terms of any one state ($P''_{SA1}$) are

$$P''_{SC} = P''_{SA1}$$

$$P''_{SA1} = P''_{SA1}$$

$$P''_{SA2} = q^{i_1} P''_{SA1}$$

The expected lengths for each phase are

| State | Expected length |
|---|---|
| SC | $(1-q^i)/pq^i$ |
| SA1 | $(1-q^i)/f_1 p$ |
| SA2 | $1/f_2 p$ |

**Working table for Tightened MLP-2(G) plan**

| Phase | Expected Length | Simplification of column 2 | Coefficient in terms of $P''_{SA1}$ | AFI | |
|---|---|---|---|---|---|
| | | | | Denominator | Numerator |
| $SC$ | $(1-q^i)/pq^i$ | $f_1f_2(1-q^i)$ | 1 | $f_1f_2(1-q^i)$ | $f_1f_2(1-q^i)$ |
| $SA1$ | $(1-q^i)/pq^i$ | $f_2q^i(1-q^{i_1})$ | 1 | $f_2q^i(1-q^{i_1})$ | $f_1f_2q^i(1-q^{i_1})$ |
| $SA2$ | $1/f_2p$ | $f_1q^i$ | $q^{i_1}$ | $f_1q^{i+i_1}$ | $f_1f_2q^{i+i_1}$ |

From the working table the performance measures of tightened MLP-2(G) may be derived.

**Performance measures**

The five most common performance measures to evaluate the plan are

1. The average number of units inspected under the screening inspection is

$$U = (1-q^i)/pq^i \tag{1.1}$$

2. The average number of units passed under the sampling inspection is

$$V = (f_1q^{i_1} + f_2(1-q^{i_1}))/f_1f_2p \tag{1.2}$$

3. The average fraction of total produced units inspected (AFI) in the long run is

$$F = f_1f_2/D \tag{1.3}$$

4. The average outgoing quality when nonconforming units are replaced with conforming unit is

$$AOQ = p(f_2q^i(1-f_1) + q^{i+i_1}(f_1-f_2))/D \tag{1.4}$$

5. The average fraction of total production accepted on sampling basis (the OC function) is

$$P_a = q^i(f_2 + q^{i_1}(f_1 - f_2)/D \tag{1.5}$$

where $$D = f_1 f_2\left(1 - q^i\right) + q^i(f_2 + q^{i_1}(f_1 - f_2)) \tag{1.6}$$

**Designing of plans**

Tables 1.1 and 1.2 are constructed to enable the selection of tightened MLP-2(G) plan under the following specified the conditions

(i) Given AOQL and AQL with $P_a = 0.95$
(ii) Given AOQL and LQL with $P_a = 0.10$

Numerical valued in these tables reveal that

i. increase in AQL decreases $i$ and $f$
ii. increase in LQL decreases $i$ and $f$

Tables 1.1 and 1.2 may be used respectively to select generalized tightened two level continuous sampling plan for given (AQL, AOQL) and (LQL, AOQL).

Following procedure may be followed to design the desired tightened two level continuous sampling plan.

(i) To choose the appropriate table specify the level of sampling inspection plan, the primary index AOQL and any one of the secondary indices (AQL or LQL). Select the table based on the specifications.
(ii) Compute the ratio AOQL/AQL or LQL/AOQL depending on the specified secondary index.
(iii) If AQL is specified, from the suitable table choose the tabular ratio which is less than or equal to the computed ratio. If LQL is specified select the tabular ratio which is greater than or equal to the computed ratio in the suitable table.

(iv) Analogous to the selected tabular ratio and the specified value of secondary index in the selected table, find $i$ and $1/f$ values. This pair represents the basic parameters of the required plan.

**Selection of plans for given AQL and AOQL**

(i) Suppose that one requires the generalized tightened two level plan satisfying the conditions AQL =0.4% with $P_a = 0.95$ and AOQL = 0.8%. Then compute the ratio AOQL/AQL = 2. Locate under the column AOQL/AQL, the ratio which is less than or equal to the computed ratio. Corresponding to AQL=0.4% and tabulated ratio 2, table 1.1 gives $i$ = 216, $f$ = 1/22. Therefore the required tightened MLP-2(G) plan $(i, i_1, f_1, f_2)$ is (216, 108, 1/22,1/44).

(ii) Suppose that one desires to have the generalized tightened two level plan satisfying the conditions AQL = 0.25% with $P_a = 0.95$ and AOQL = 0.35%. Then compute the ratio AOQL/AQL =1.4. Locate under the column AOQL/AQL, the ratio which is less than or equal to the computed ratio. From the table 1.1 one gets $i$ = 820, $f$ = 1/122. Therefore, the required generalized tightened two level continuous sampling plan $(i, i_1, f_1, f_2)$ is (820, 410, 1/122, 1/244).

**Selection of plans for given LQL and AOQL**

(i) Suppose that one needs the generalized tightened two level plan satisfying the conditions LQL = 0.25% with $P_a = 0.10$ and AOQL = 0.06%. Compute the ratio LQL/AOQL = 4.16. Under the column captioned with LQL/AOQL in table 1.2 locate the ratio which is greater than or equal to the computed ratio. The tabulated ratio is 4.16. Associated parameters are $i$ =1670, $f$ = 1/6. Therefore the required tightened MLP-2(G) plan $(i, i_1, f_1, f_2)$ is (1670, 835, 1/6, 1/12).

(ii) Suppose that one wishes the generalized tightened two level plan satisfying the conditions LQL = 1.5% with $P_a = 0.10$ and AOQL = 0.3%. Compute the ratio LQL/AOQL = 5. Under the column captioned with LQL/AOQL in table 1.2 locate the ratio which is greater than or equal to the computed ratio. The tabulated ratio is

5. Associated parameters are $i$=242, $f$ = 1/3. Therefore the required plan $(i, i_1, f_1, f_2)$ is (242, 121, 1/3, 1/6).

**Consumer protection in tightened MLP-2(G) plan**

The analysis of protection provided to the consumer by tightened MLP-2(G) plan is carried out numerically. Tables of i, AOQL and AFI values at the incoming quality corresponding to AOQL indexed by $f$ and LQL with consumer's risk of 0.10 are constructed. These consumer oriented tables are prepared to incorporate in the US Military standard MIL-STD-1235C (1988).

Tables 1.3 and 1.4 are constructed to enable the selection of generalized tightened two level continuous sampling plan with the consumer orientation on the basis of LQL and $f$, corresponding to $i_1 = i/2$ and $i_1 = i$

Table values reveal that

(i) increase in LQL decreases $i$ and increases AOQL

(ii) decrease in $f$ increases $i$ and increases AOQL

**Illustration**

Suppose that one wishes to have the tightened MLP-2(G) plan, for a given LQL of 1.3% and sampling rate $f$ =1/4 with $i_1 = i/2$. Table 1.3 gives $i$=280, AOQL=0.28% and AFI= 0.5331 for the plan (280, 140, 1/4, 1/8). The application of the above plan is as follows

Start inspecting each unit submitted in the order of production. If 280 consecutive units are found to be conforming then switch to first level of sampling inspection at rate $f_1 = 1/4$. If 140 consecutive units selected at rate $f_1$ are found to be conforming then switch to second level of sampling inspection at rate $f_2 = 1/8$. Continue with the second level of sampling inspection. On finding a defective unit, revert to screening inspection at any level of inspection.

Using the location method the following plans may be designed for the specified specifications.

(i) For a given LQL of 6.5% and sampling rate $f = 1/7$ having $i_1 = i/2$. one gets $i$ = 62, AOQL = 1.71% and AFI = 0.4311 from table 1.3

(ii) For a given LQL of 12.5% and sampling rate $f = 1/200$ with $i_1 = i/2$ table 1.3 gives $i$ = 56, AOQL = 5.56% and AFI = 0.2159

(iii) For a given LQL of 2% and sampling rate $f = 1/3$ with $i_1 = i$ table 1.4, gives $i$ = 166, AOQL = 0.33% and AFI = 0.5761

**(iv)** For a given LQL of 10% and sampling rate $f = 1/50$ having $i_1 = i$, table 1.4 gives $i$ = 58, AOQL = 3.62% and AFI = 0.3152

**Construction of Tables 1.1 to 1.2**

Tables 1.1 and 1.2 are constructed to obtain the parameters of the tightened MLP-2(G) plan for a given (AQL, AOQL) and (LQL, AOQL) respectively.

The operating characteristic (OC) function representing the average fraction of total production accepted on a sampling basis for the plan is given in (1.5). By taking $f_1 = f$, $f_2 = f/2$ and rewriting (1.5) to express $f$ in terms of $i, i_1$ and $P_a$ as

$$f = ((1 - P_a)q^i(1 + q^{i_1}))/P_a(1 - q^i) \tag{1.7}$$

(1.5) and (1.7) may also be expressed in the form

$$P_a = A/(A + fB) \tag{1.8}$$

$$f = A(1 - P_a)/(BPa) \tag{1.9}$$

with $A = q^i(1 + q^{i_1})$

and $B = (1 - q^i)$

In the construction of Tables 1.1 and 1.2 for tightened MLP-2(G) plan $i_1 = i/2$ is assumed. The following **six step procedure** is used to construct the tables.

(i) If the AQL condition is specified take $p =$ AQL and $P_a = 0.95$; if the LQL condition is specified take $p =$ LQL and $P_a = 0.10$

(ii) Find $f$ using (1.9)

(iii) Take $n$ = int $(1/f+0.9999)$ if AQL is specified, $n$ = int $(1/f)$ if LQL is specified

(iv) Take the revised $f$ as $f = 1/n$ (this makes $f$ an integer)

(v) Using iterative procedure find AOQL($p_L$), which is the maximum of AOQ given in (1.3) with $p$ takes values from 0 to 1 in steps of 0.0001

(vi) For the desired AOQL, find $i$ and $f$ for which Difference = desired AOQL – $p_L$ which is non negative and minimum. These $i$ and $1/f$ values are tabulated.

**Construction of Tables 1.3 and 1.4**

Tables 1.3 and 1.4 are constructed to derive the values of $i$, AOQL and AFI of the tightened MLP-2(G) plan for the selected LQL with $P_a = 0.10$ and $f$ values by assuming $i_1 = i/2$. LQL values are taken from consumer oriented plans of CSP-1 by Stephens (1981), and $f$ values from MIL-STD-1235 C (1988).

(i) For generalized tightened two level continuous sampling plan, the average fraction of total units accepted on a sampling basis, $P_a$ is given in (1.5). By using the equation and following search procedure with $i$ varying between (2, 2000), the values of $i$ which is closest to the 10% probability of acceptance for a given LQL and $f$ is determined.

(ii) With these values of $i$ and $f$, using iterative procedure AOQL, the maximum of AOQ given in (1.4) is computed for tightened MLP-2(G) sampling plan when $p$ takes the values from 0 to 1 in steps of 0.0001.

(iii) Average fraction inspected in the long run (AFI) for generalized tightened two level continuous plan is given in (1.3). With the values of $i, f,$ AOQL and $p$ at which AOQL is attained, the value of AFI is computed and presented.

Table 1.1 Tightened MLP-2(G) plan indexed by AQL at $P_a = 0.95$ and AOQL / AQL

| AOQL / AQL | AQL IN PERCENT | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.1 | | 0.15 | | 0.25 | | 0.4 | | 0.65 | | 1 | | 1.5 | | 2.5 | | 4 | | 6.5 | |
| | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ |
| 3 | 358 | 7 | 200 | 5 | 132 | 6 | 76 | 5 | 46 | 5 | 30 | 5 | 22 | 5 | 10 | 4 | 6 | 4 | 4 | 5 |
| 2.6 | 488 | 10 | 310 | 9 | 176 | 8 | 116 | 9 | 72 | 9 | 48 | 10 | 32 | 10 | 18 | 9 | 12 | 10 | 6 | 8 |
| 2.2 | 700 | 16 | 456 | 15 | 280 | 16 | 170 | 15 | 110 | 17 | 68 | 15 | 46 | 16 | 28 | 16 | 16 | 14 | 10 | 15 |
| 2 | 880 | 23 | 578 | 22 | 346 | 22 | 216 | 22 | 130 | 21 | 86 | 22 | 56 | 21 | 34 | 22 | 20 | 20 | 12 | 20 |
| 1.9 | 992 | 28 | 662 | 28 | 386 | 26 | 244 | 27 | 148 | 26 | 96 | 26 | 64 | 27 | 38 | 26 | 22 | 23 | 14 | 25 |
| 1.8 | 1104 | 33 | 750 | 35 | 450 | 35 | 278 | 34 | 172 | 34 | 112 | 35 | 74 | 34 | 42 | 31 | 26 | 31 | 16 | 32 |
| 1.6 | 1474 | 58 | 982 | 58 | 586 | 57 | 366 | 57 | 226 | 56 | 146 | 57 | 96 | 56 | 58 | 57 | 34 | 51 | 20 | 48 |
| 1.56 | 1560 | 65 | 1040 | 65 | 624 | 65 | 390 | 65 | 240 | 66 | 154 | 64 | 104 | 66 | 62 | 66 | 38 | 64 | 22 | 58 |
| 1.54 | 1622 | 71 | 1080 | 71 | 648 | 71 | 404 | 71 | 246 | 69 | 160 | 69 | 106 | 69 | 62 | 66 | 38 | 64 | 22 | 58 |
| 1.52 | 1668 | 75 | 1080 | 71 | 648 | 76 | 414 | 74 | 254 | 74 | 164 | 73 | 110 | 75 | 66 | 75 | 40 | 72 | 24 | 70 |
| 1.5 | 1718 | 80 | 1146 | 80 | 692 | 82 | 430 | 81 | 262 | 79 | 170 | 79 | 114 | 81 | 66 | 75 | 42 | 80 | 24 | 70 |
| 1.46 | 1842 | 94 | 1224 | 93 | 738 | 95 | 458 | 93 | 282 | 94 | 182 | 92 | 122 | 94 | 72 | 92 | 44 | 89 | 26 | 83 |
| 1.44 | 1902 | 101 | 1272 | 102 | 762 | 102 | 476 | 102 | 292 | 102 | 190 | 102 | 126 | 102 | 74 | 98 | 46 | 98 | 28 | 99 |
| 1.4 | 2052 | 122 | 1368 | 122 | 820 | 122 | 512 | 122 | 314 | 121 | 202 | 118 | 134 | 118 | 80 | 118 | 50 | 120 | 30 | 116 |
| 1.39 | 2092 | 128 | 1394 | 128 | 836 | 128 | 522 | 128 | 320 | 127 | 206 | 124 | 138 | 127 | 82 | 125 | 50 | 120 | 30 | 116 |
| 1.38 | 2128 | 133 | 1420 | 134 | 850 | 133 | 532 | 134 | 326 | 133 | 210 | 130 | 140 | 131 | 82 | 125 | 50 | 120 | 30 | 116 |
| 1.36 | 2212 | 147 | 1476 | 148 | 882 | 146 | 552 | 147 | 340 | 148 | 218 | 143 | 146 | 146 | 86 | 141 | 52 | 132 | 32 | 137 |
| 1.34 | 2294 | 162 | 1538 | 165 | 918 | 162 | 572 | 161 | 352 | 162 | 228 | 161 | 152 | 162 | 90 | 159 | 56 | 160 | 34 | 160 |
| 1.32 | 2396 | 182 | 1600 | 183 | 956 | 181 | 596 | 180 | 366 | 180 | 238 | 181 | 158 | 180 | 94 | 179 | 58 | 176 | 34 | 160 |
| 1.31 | 2448 | 193 | 1626 | 191 | 978 | 193 | 610 | 192 | 374 | 191 | 242 | 189 | 160 | 186 | 96 | 189 | 58 | 176 | 36 | 187 |

Table 1.2 Tightened MLP-2(G) plan indexed by LQL at $P_a$ = 0.10 and LQL / AOQL

| LQL / AOQL | LQL IN PERCENT | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0.15 | | 0.25 | | 0.4 | | 0.65 | | 1 | | 1.5 | | 2.5 | | 4 | | 6.5 | | 10 | |
| | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ | $i$ | $1/f$ |
| 5.6 | 2084 | 2 | 1250 | 2 | 800 | 2 | 560 | 3 | 364 | 3 | 242 | 3 | 146 | 3 | 78 | 2 | 48 | 2 | 30 | 2 |
| 5.2 | 2500 | 4 | 1400 | 3 | 876 | 3 | 560 | 3 | 364 | 3 | 242 | 3 | 146 | 3 | 88 | 3 | 54 | 3 | 34 | 3 |
| 5 | 2748 | 6 | 1574 | 4 | 876 | 3 | 606 | 4 | 364 | 3 | 242 | 3 | 146 | 3 | 88 | 3 | 54 | 3 | 34 | 3 |
| 4.8 | 2964 | 8 | 1574 | 4 | 944 | 4 | 606 | 4 | 378 | 4 | 252 | 4 | 152 | 4 | 94 | 4 | 58 | 4 | 34 | 3 |
| 4.4 | 3164 | 11 | 1618 | 5 | 1012 | 5 | 622 | 5 | 404 | 5 | 270 | 5 | 162 | 5 | 100 | 5 | 60 | 5 | 35 | 4 |
| 4.2 | 3264 | 13 | 1670 | 6 | 1044 | 6 | 642 | 6 | 418 | 6 | 278 | 6 | 168 | 6 | 104 | 6 | 605 | 5 | 38 | 5 |
| 4 | 3396 | 16 | 1720 | 7 | 1076 | 7 | 662 | 7 | 430 | 7 | 286 | 7 | 172 | 7 | 104 | 6 | 62 | 6 | 40 | 6 |
| 3.8 | 3486 | 19 | 1788 | 8 | 1118 | 8 | 688 | 8 | 448 | 8 | 298 | 8 | 174 | 8 | 108 | 8 | 64 | 7 | 40 | 6 |
| 3.6 | 3628 | 24 | 1904 | 11 | 1192 | 11 | 708 | 10 | 460 | 10 | 306 | 10 | 182 | 10 | 114 | 10 | 68 | 9 | 42 | 8 |
| 3.4 | 3812 | 32 | 1990 | 14 | 1244 | 14 | 746 | 13 | 484 | 13 | 322 | 13 | 194 | 13 | 120 | 13 | 72 | 12 | 46 | 12 |
| 3.2 | 4056 | 46 | 2064 | 18 | 1290 | 18 | 794 | 18 | 516 | 18 | 344 | 18 | 206 | 18 | 126 | 17 | 76 | 16 | 46 | 12 |
| 3 | 4300 | 67 | 2232 | 27 | 1378 | 26 | 848 | 26 | 552 | 26 | 364 | 25 | 218 | 25 | 134 | 24 | 80 | 22 | 50 | 20 |
| 2.9 | 4428 | 82 | 2304 | 33 | 1442 | 33 | 878 | 32 | 570 | 32 | 380 | 32 | 226 | 31 | 140 | 31 | 84 | 29 | 52 | 24 |
| 2.8 | 4568 | 102 | 2384 | 41 | 1490 | 41 | 912 | 40 | 592 | 40 | 394 | 40 | 234 | 39 | 144 | 37 | 86 | 33 | 54 | 30 |
| 2.7 | 4740 | 132 | 2484 | 53 | 1552 | 53 | 954 | 53 | 620 | 53 | 408 | 50 | 242 | 48 | 150 | 48 | 90 | 44 | 56 | 38 |
| 2.6 | 4906 | 171 | 2606 | 72 | 1624 | 71 | 998 | 71 | 648 | 71 | 430 | 70 | 254 | 66 | 156 | 62 | 94 | 58 | 58 | 47 |

Table 1.3 $i$ and performance measures of tightened MLP-2(G) plan indexed by LQL at $P_a = 0.10$ and $f$ for $i_1 = i/2$

| $f$ | | LQL | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.005 | 0.008 | 0.013 | 0.020 | 0.032 | 0.050 | 0.065 | 0.080 | 0.100 | 0.125 | 0.150 | 0.200 |
| 1/2 | $i$ | 622 | 388 | 238 | 154 | 96 | 60 | 46 | 38 | 28 | 24 | 20 | 14 |
| | AOQL | 0.0007 | 0.0012 | 0.0019 | 0.0030 | 0.0048 | 0.0076 | 0.0099 | 0.0120 | 0.0162 | 0.0189 | 0.0226 | 0.0318 |
| | AFI | 0.6262 | 0.6025 | 0.6025 | 0.6238 | 0.6294 | 0.6345 | 0.6303 | 0.6235 | 0.6305 | 0.6220 | 0.6239 | 0.6212 |
| 1/3 | $i$ | 678 | 425 | 260 | 170 | 106 | 68 | 52 | 42 | 34 | 26 | 22 | 16 |
| | AOQL | 0.0009 | 0.0015 | 0.0024 | 0.0038 | 0.0060 | 0.0094 | 0.0123 | 0.0151 | 0.0186 | 0.0242 | 0.0285 | 0.0388 |
| | AFI | 0.5245 | 0.5009 | 0.5863 | 0.5782 | 0.5657 | 0.5504 | 0.5604 | 0.5533 | 0.5551 | 0.5564 | 0.5539 | 0.5491 |
| 1/4 | $i$ | 732 | 456 | 280 | 188 | 114 | 72 | 56 | 44 | 36 | 28 | 22 | 16 |
| | AOQL | 0.0011 | 0.0017 | 0.0028 | 0.0042 | 0.0069 | 0.0108 | 0.1386 | 0.176 | 0.0214 | 0.0273 | 0.0345 | 0.0468 |
| | AFI | 0.4679 | 0.5746 | 0.5331 | 0.5374 | 0.5101 | 0.5084 | 0.5051 | 0.5119 | 0.5138 | 0.5120 | 0.5069 | 0.5018 |
| 1/5 | $i$ | 770 | 482 | 296 | 192 | 120 | 76 | 58 | 46 | 36 | 30 | 24 | 18 |
| | AOQL | 0.0012 | 0.0018 | 0.0030 | 0.0047 | 0.0074 | 0.0117 | 0.0153 | 0.0192 | 0.0244 | 0.0311 | 0.0361 | 0.0476 |
| | AFI | 0.4252 | 0.5390 | 0.4952 | 0.4819 | 0.4690 | 0.4914 | 0.4734 | 0.4814 | 0.4811 | 0.4722 | 0.4761 | 0.4765 |
| 1/7 | $i$ | 834 | 522 | 322 | 206 | 130 | 82 | 62 | 52 | 40 | 32 | 26 | 18 |
| | AOQL | 0.0013 | 0.0020 | 0.0033 | 0.0052 | 0.0082 | 0.0129 | 0.0171 | 0.0202 | 0.0262 | 0.0326 | 0.0399 | 0.0567 |
| | AFI | 0.3701 | 0.4893 | 0.4451 | 0.4237 | 0.4528 | 0.4367 | 0.4311 | 0.4362 | 0.4417 | 0.4475 | 0.4386 | 0.4334 |
| 1/10 | $i$ | 902 | 562 | 346 | 224 | 140 | 88 | 68 | 54 | 42 | 34 | 28 | 20 |
| | AOQL | 0.0014 | 0.0023 | 0.0036 | 0.0056 | 0.0090 | 0.0143 | 0.0184 | 0.0231 | 0.0295 | 0.0362 | 0.0437 | 0.0603 |
| | AFI | 0.3179 | 0.4372 | 0.3903 | 0.3749 | 0.3995 | 0.4054 | 0.4064 | 0.3929 | 0.3979 | 0.3961 | 0.4012 | 0.3971 |

Continuation of Table 1.3

| $f$ | | LQL | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.005 | 0.008 | 0.013 | 0.020 | 0.032 | 0.050 | 0.065 | 0.080 | 0.100 | 0.125 | 0.150 | 0.200 |
| 1/15 | $i$ | 978 | 612 | 376 | 244 | 152 | 96 | 74 | 60 | 48 | 38 | 30 | 22 |
| | AOQL | 0.0015 | 0.0025 | 0.0040 | 0.0061 | 0.0098 | 0.0154 | 0.0206 | 0.0246 | 0.0306 | 0.0384 | 0.0482 | 0.0648 |
| | AFI | 0.2652 | 0.3872 | 0.2041 | 0.3870 | 0.3869 | 0.3556 | 0.3750 | 0.3698 | 0.3763 | 0.3605 | 0.3573 | 0.3582 |
| 1/25 | $i$ | 1076 | 674 | 414 | 268 | 166 | 106 | 82 | 66 | 52 | 40 | 34 | 24 |
| | AOQL | 0.0016 | 0.0027 | 0.0043 | 0.0067 | 0.0108 | 0.0169 | 0.0218 | 0.0269 | 0.0340 | 0.0439 | 0.0513 | 0.0715 |
| | AFI | 0.4667 | 0.3283 | 0.3804 | 0.3264 | 0.3225 | 0.3240 | 0.3200 | 0.3266 | 0.3200 | 0.3252 | 0.3249 | 0.3189 |
| 1/50 | $i$ | 1210 | 756 | 466 | 304 | 190 | 118 | 92 | 74 | 58 | 46 | 38 | 28 |
| | AOQL | 0.0018 | 0.0029 | 0.0048 | 0.0074 | 0.0118 | 0.0189 | 0.0242 | 0.0299 | 0.0379 | 0.0475 | 0.0571 | 0.0764 |
| | AFI | 0.3989 | 0.2565 | 0.3101 | 0.2618 | 0.3065 | 0.3001 | 0.2894 | 0.2877 | 0.2840 | 0.2802 | 0.2772 | 0.2794 |
| 1/100 | $i$ | 1346 | 842 | 518 | 334 | 208 | 136 | 102 | 82 | 64 | 52 | 42 | 30 |
| | AOQL | 0.0020 | 0.0032 | 0.0052 | 0.0082 | 0.0130 | 0.0198 | 0.0263 | 0.0326 | 0.0415 | 0.0508 | 0.0624 | 0.0859 |
| | AFI | 0.3370 | 0.1991 | 0.2482 | 0.2595 | 0.2340 | 0.2653 | 0.2472 | 0.2413 | 0.2446 | 0.2417 | 0.2483 | 0.2400 |
| 1/200 | $i$ | 1482 | 926 | 570 | 370 | 228 | 146 | 112 | 92 | 72 | 56 | 46 | 34 |
| | AOQL | 0.0025 | 0.0034 | 0.0056 | 0.0086 | 0.0141 | 0.0218 | 0.0283 | 0.0351 | 0.0436 | 0.0556 | 0.0672 | 0.0896 |
| | AFI | 0.2801 | 0.1508 | 0.1952 | 0.2103 | 0.2190 | 0.2199 | 0.2127 | 0.2370 | 0.2204 | 0.2159 | 0.2181 | 0.2143 |

Table 1.4 $i$ and performance measures of tightened MLP-2(G) plan indexed by LQL at $P_a = 0.10$ and $f$ for $i_1 = i$

| $f$ | | LQL | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.005 | 0.008 | 0.013 | 0.020 | 0.032 | 0.050 | 0.065 | 0.080 | 0.100 | 0.125 | 0.150 | 0.200 | 0.320 |
| 1/2 | $i$ | 596 | 372 | 228 | 148 | 92 | 59 | 45 | 36 | 29 | 22 | 18 | 13 | 8 |
| | AOQL | 0.0006 | 0.0010 | 0.0017 | 0.0026 | 0.0042 | 0.0066 | 0.0086 | 0.0107 | 0.0133 | 0.0174 | 0.0211 | 0.0289 | 0.04587 |
| | AFI | 0.6724 | 0.6490 | 0.6569 | 0.6711 | 0.6467 | 0.6341 | 0.6261 | 0.6422 | 0.6412 | 0.6376 | 0.6354 | 0.6427 | 0.6300 |
| 1/3 | $i$ | 671 | 418 | 257 | 166 | 103 | 65 | 50 | 40 | 31 | 25 | 21 | 15 | 9 |
| | AOQL | 0.0008 | 0.0013 | 0.0021 | 0.0033 | 0.0054 | 0.0086 | 0.0111 | 0.138 | 0.0178 | 0.0220 | 0.0260 | 0.0360 | 0.0584 |
| | AFI | 0.5793 | 0.5515 | 0.5619 | 0.5761 | 0.5806 | 0.5698 | 0.5711 | 0.5789 | 0.5751 | 0.5849 | 0.5725 | 0.5107 | 0.5607 |
| 1/4 | $i$ | 725 | 452 | 277 | 179 | 111 | 70 | 54 | 43 | 34 | 27 | 22 | 16 | 9 |
| | AOQL | 0.0009 | 0.0015 | 0.0024 | 0.0039 | 0.0062 | 0.0098 | 0.0127 | 0.0159 | 0.0200 | 0.0251 | 0.0306 | 0.0416 | 0.0715 |
| | AFI | 0.5203 | 0.6262 | 0.5007 | 0.5160 | 0.5206 | 0.5308 | 0.5284 | 0.5313 | 0.5335 | 0.5261 | 0.5286 | 0.5272 | 0.5172 |
| 1/5 | $i$ | 767 | 479 | 294 | 190 | 118 | 75 | 57 | 46 | 36 | 29 | 24 | 17 | 10 |
| | AOQL | 0.0010 | 0.0016 | 0.0027 | 0.0042 | 0.0067 | 0.0106 | 0.0139 | 0.0172 | 0.0218 | 0.0270 | 0.0323 | 0.0451 | 0.0744 |
| | AFI | 0.4778 | 0.5903 | 0.0027 | 0.4749 | 0.5162 | 0.5176 | 0.5028 | 0.5088 | 0.5031 | 0.5001 | 0.5241 | 0.4982 | 0.4902 |
| 1/7 | $i$ | 832 | 519 | 318 | 206 | 128 | 81 | 62 | 50 | 39 | 31 | 26 | 19 | 11 |
| | AOQL | 0.0011 | 0.0018 | 0.0030 | 0.0046 | 0.0075 | 0.119 | 0.155 | 0.0191 | 0.0244 | 0.0305 | 0.0362 | 0.0489 | 0.0818 |
| | AFI | 0.4194 | 0.5381 | 0.4909 | 0.4163 | 0.4595 | 0.4826 | 0.4652 | 0.4677 | 0.4687 | 0.4637 | 0.4510 | 0.4558 | 0.4509 |
| 1/10 | $I$ | 902 | 562 | 345 | 223 | 139 | 88 | 67 | 54 | 42 | 33 | 27 | 20 | 12 |
| | AOQL | 0.0012 | 0.0020 | 0.0033 | 0.0052 | 0.0083 | 0.0131 | 0.0171 | 0.0212 | 0.0271 | 0.0343 | 0.0416 | 0.0555 | 0.0970 |
| | AFI | 0.3637 | 0.4862 | 0.4375 | 0.4209 | 0.4054 | 0.4296 | 0.4277 | 0.4574 | 0.4346 | 0.4280 | 0.4213 | 0.4216 | 0.4217 |

Continuation of Table 1.4

| $f$ | | LQL | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.005 | 0.008 | 0.013 | 0.020 | 0.032 | 0.050 | 0.065 | 0.080 | 0.100 | 0.125 | 0.150 | 0.200 | 0.320 |
| 1/15 | $i$ | 981 | 612 | 346 | 243 | 151 | 95 | 73 | 59 | 46 | 36 | 30 | 22 | 13 |
| | AOQL | 0.0013 | 0.0022 | 0.0037 | 0.0057 | 0.0092 | 0.0145 | 0.0188 | 0.0232 | 0.0296 | 0.0376 | 0.0449 | 0.0604 | 0.0987 |
| | AFI | 0.3069 | 0.4320 | 0.3819 | 0.3649 | 0.3870 | 0.3936 | 0.3913 | 0.4040 | 03819 | 0.3827 | 0.3848 | 0.3772 | 0.3749 |
| 1/25 | $i$ | 1082 | 675 | 414 | 268 | 166 | 105 | 80 | 65 | 52 | 40 | 33 | 24 | 14 |
| | AOQL | 0.0015 | 0.0025 | 0.0041 | 0.0063 | 0.0102 | 0.0160 | 0.0210 | 0.0257 | 0.0320 | 0.0413 | 0.0498 | 0.0675 | 0.115 |
| | AFI | 0.2450 | 0.3682 | 0.3166 | 0.3652 | 0.3613 | 0.3566 | 0.3630 | 0.3554 | 0.3454 | 0.3431 | 0.3444 | 0.3381 | 0.3404 |
| 1/50 | $i$ | 1219 | 761 | 467 | 302 | 187 | 119 | 90 | 73 | 58 | 45 | 37 | 27 | 16 |
| | AOQL | 0.0016 | 0.0028 | 0.0046 | 0.0071 | 0.0114 | 0.0179 | 0.0236 | 0.0290 | 0.0362 | 0.0464 | 0.0560 | 0.0757 | 0.1230 |
| | AFI | 0.1176 | 0.2913 | 0.3432 | 0.2881 | 0.3261 | 0.3100 | 0.3051 | 0.3094 | 0.3152 | 0.3068 | 0.2990 | 0.2925 | 0.2848 |
| 1/100 | $i$ | 1357 | 847 | 520 | 336 | 209 | 132 | 101 | 81 | 64 | 51 | 41 | 31 | 18 |
| | AOQL | 0.0018 | 0.0031 | 0.0050 | 0.0078 | 0.125 | 0.0197 | 0.0257 | 0.0319 | 0.0402 | 0.0501 | 0.0618 | 0.0806 | 0.1333 |
| | AFI | 0.3694 | 0.2255 | 0.2752 | 0.2885 | 0.3055 | 0.2672 | 0.2642 | 0.2567 | 0.2552 | 0.2629 | 0.2552 | 0.2533 | 0.2547 |
| 1/200 | $i$ | 1495 | 933 | 572 | 371 | 230 | 146 | 111 | 89 | 71 | 56 | 46 | 34 | 19 |
| | AOQL | 0.0020 | 0.0033 | 0.0054 | 0.0084 | 0.0136 | 0.0213 | 0.0279 | 0.0346 | 0.0432 | 0.0544 | 0.0656 | 0.0875 | 0.1496 |
| | AFI | 0.3073 | 0.1712 | 0.3298 | 0.2304 | 0.2440 | 0.2382 | 0.2243 | 0.2294 | 0.2282 | 0.2341 | 0.264 | 0.2256 | 0.2169 |