4

Various Types of β^{**} Generalized Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

4.1 Introduction

Continuity in intuitionistic fuzzy topological spaces was introduced by Gurcay et al. (1997). Here in this chapter we have introduced intuitionisitic fuzzy completely β^{**} generalized continuous mappings, intuitionistic fuzzy perfectly β^{**} generalized continuous mappings, intuitionistic fuzzy quasi β^{**} generalized continuous mappings. We have compared them with other existing intuitionistic fuzzy continuous mappings.

4.2 Completely β^{**} Generalized Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

In this section we have introduced intuitionistic fuzzy completely β^{**} generalized continuous mappings and studied some of their properties.

Definition 4.2.1 : A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be an intuitionistic fuzzy completely β^{**} generalized (IF completely β^{**}G) continuous mapping if $f^{-1}(V)$ is an IFRCS in X for every IFβ^{**}GCS V in Y.

Proposition 4.2.2 : Every IF completely β^{**}G continuous mapping is an IF continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF completely β^{**}G continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, B is an IFβ^{**}GCS in Y. Then $f^{-1}(B)$ is an IFRCS in X, by hypothesis. Since every IFRCS is an IFCS, $f^{-1}(B)$ is an IFCS in X. Hence f is an IF continuous mapping.

Example 4.2.3 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is an IF continuous mapping but not an IF completely β^{**}G continuous mapping, since G_2^c is an IFβ^{**}GCS in Y but $f^{-1}(G_2^c)$ is not an IFRCS in X, as $cl(int(f^{-1}(G_2^c))) = 0_\sim \neq f^{-1}(G_2^c)$.

Proposition 4.2.4 : Every IF completely β^{**}G continuous mapping is an IFS continuous mapping but not conversely in general.

Proof : Let f : (X, τ) → (Y, σ) be an IF completely β^{**}G continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, B is an IFβ^{**}GCS in Y. Then $f^{-1}(B)$ is an IFRCS in X, by hypothesis. Since every IFRCS is an IFSCS, $f^{-1}(B)$ is an IFSCS in X. Hence f is an IFS continuous mapping.

Example 4.2.5 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_2 = \langle y, (0.8_u, 0.6_v), (0.1_u, 0.1_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IFS continuous mapping but not an IF completely β^{**}G continuous mapping, since G_2^c is an IFβ^{**}GCS in Y but $f^{-1}(G_2^c)$ is not an IFRCS in X, as $cl(int(f^{-1}(G_2^c))) = 0_\sim \neq f^{-1}(G_2^c)$.

Proposition 4.2.6 : Every IF completely β^{**}G continuous mapping is an IFP continuous mapping but not conversely in general.

Proof : Let f : (X, τ) → (Y, σ) be an IF completely β^{**}G continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, B is an IFβ^{**}GCS in Y. Then $f^{-1}(B)$ is an IFRCS in X, by hypothesis. Since every IFRCS is an IFPCS, $f^{-1}(B)$ is an IFPCS in X. Hence f is an IFP continuous mapping.

Example 4.2.7 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_2 = \langle y, (0.8_u, 0.6_v), (0.1_u, 0.1_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IFP continuous mapping but not an IF completely β^{**}G continuous mapping, since G_2^c is an IFβ^{**}GCS in Y but $f^{-1}(G_2^c)$ is not an IFRCS in X, as $cl(int(f^{-1}(G_2^c))) = 0_\sim \neq f^{-1}(G_2^c)$.

Proposition 4.2.8 : Every IF completely β^{**}G continuous mapping is an IFα continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF completely β^{**}G continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, B is an IFβ^{**}GCS in Y. Then $f^{-1}(B)$ is an IFRCS in X, by hypothesis. Since every IFRCS is an IFαCS, $f^{-1}(B)$ is an IFαCS in X. Hence f is an IFα continuous mapping.

Example 4.2.9 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_2 = \langle y, (0.8_u, 0.6_v), (0.1_u, 0.1_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IFα continuous mapping but not an IF completely β^{**}G continuous mapping, since G_2^c is an IF β^{**}GCS in Y, but $f^{-1}(G_2^c)$ is not an IFRCS in X, as $cl(int(f^{-1}(G_2^c))) = 0_\sim \neq f^{-1}(G_2^c)$.

Proposition 4.2.10 : Every IF completely β^{**}G continuous mapping is an IFβ continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF completely β^{**}G continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, B is an IFβ^{**}GCS in Y. Then $f^{-1}(B)$ is an IFRCS in X, by hypothesis. Since every IFRCS is an IFβCS, $f^{-1}(B)$ is an IFβCS in X. Hence f is an IFβ continuous mapping.

Example 4.2.11 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_2 = \langle y, (0.8_u, 0.6_v), (0.1_u, 0.1_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IFβ continuous mapping but not an IF completely β^{**}G continuous mapping, since G_2^c is an IFβ^{**}GCS in Y, but $f^{-1}(G_2^c)$ is not an IFRCS in X, as $cl(int(f^{-1}(G_2^c))) = 0_\sim \neq f^{-1}(G_2^c)$.

Proposition 4.2.12 : Every IF completely β^{**}G continuous mapping is an IFG continuous mapping but not conversely in general.

Proof : Let f : (X, τ) → (Y, σ) be an IF completely β^{**}G continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, B is an IFβ^{**}GCS in Y. Then $f^{-1}(B)$ is an IFRCS in X, by hypothesis. Since every IFRCS is an IFGCS, $f^{-1}(B)$ is an IFGCS in X. Hence f is an IFG continuous mapping.

Example 4.2.13 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_2 = \langle y, (0.8_u, 0.6_v), (0.1_u, 0.1_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IFG continuous mapping but not an IF completely β^{**}G continuous mapping, since G_2^c is an IFβ^{**}GCS in Y, but $f^{-1}(G_2^c)$ is not an IFRCS in X, as $cl(int(f^{-1}(G_2^c))) = 0_\sim \neq f^{-1}(G_2^c)$.

Proposition 4.2.14 : Every IF completely β^{**}G continuous mapping is an IFβ^{**}G continuous mapping but not conversely in general.

Proof : Let f : (X, τ) → (Y, σ) be an IF completely β^{**}G continuous mapping. Let B be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, B is an IFβ^{**}GCS in Y. Then $f^{-1}(B)$ is an IFRCS in X, by hypothesis. Since every IFRCS is an IFβ^{**}GCS, $f^{-1}(B)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G continuous mapping.

Example 4.2.15 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_2 = \langle y, (0.8_u, 0.6_v), (0.1_u, 0.1_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IFβ^{**}G continuous mapping but not an IF completely β^{**}G continuous mapping, since G_2^c is an IFβ^{**}GCS in Y but $f^{-1}(G_2^c)$ is not an IFRCS in X, as $cl(int(f^{-1}(G_2^c))) = 0_\sim \neq f^{-1}(G_2^c)$.

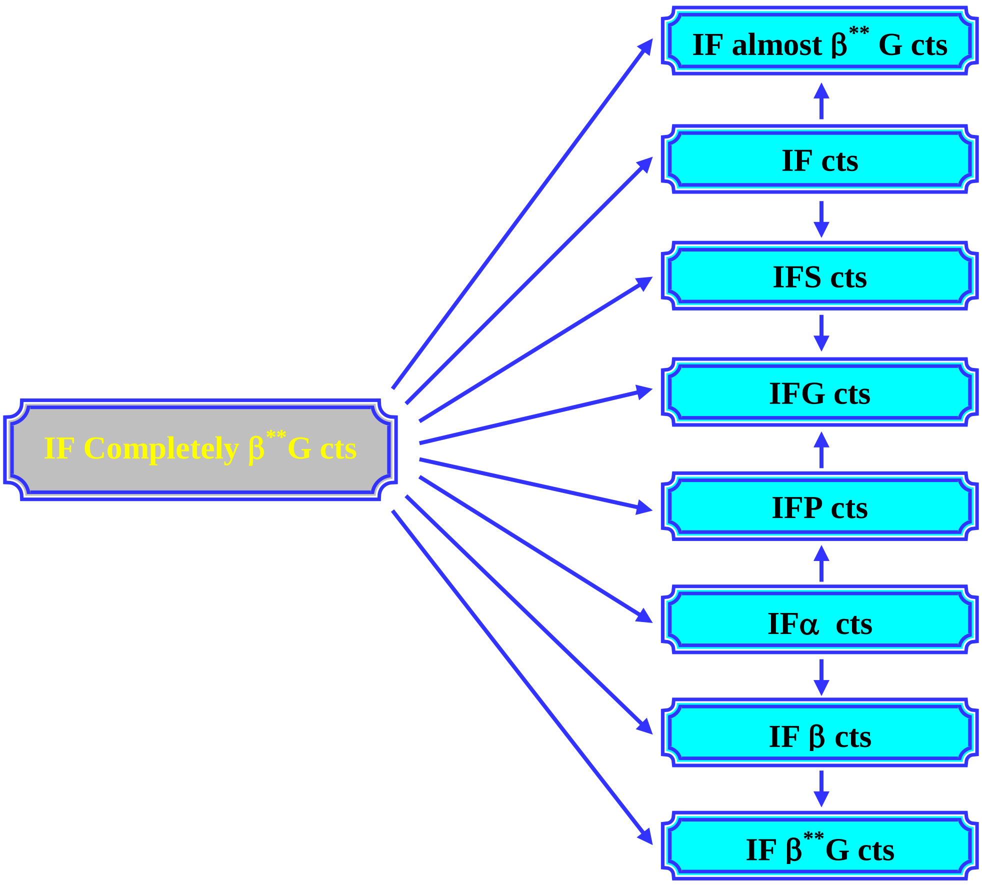
Proposition 4.2.16 : Every IF completely β^{**}G continuous mapping is an IF almost β^{**}G continuous mapping but not conversely in general.

Proof : Let f : (X, τ) → (Y, σ) be an IF completely β^{**}G continuous mapping. Let B be an IFRCS in Y. Since every IFRCS is an IFβ^{**}GCS, B is an IFβ^{**}GCS in Y.

Then $f^{-1}(B)$ is an IFRCS in X, by hypothesis. Since every IFRCS is an IFβ^{**}GCS, $f^{-1}(B)$ is an IFβ^{**}GCS in X. Hence f is an IF almost β^{**}G continuous mapping.

Example 4.2.17 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ and $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.4_v), (0.4_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is an IF almost β^{**}G continuous mapping but not an IF completely β^{**}G continuous mapping, since $G_3^c = \langle y, (0.4_u, 0.6_v), (0.6_u, 0.4_v)\rangle$ is an IFβ^{**}GCS in Y, but $f^{-1}(G_3^c)$ is not an IFRCS in X, as $cl(int(f^{-1}(G_3^c))) = G_1^c \neq f^{-1}(G_3^c)$.

The relation between various types of intuitionistic fuzzy continuity is given in the following diagram. In this diagram ‘cts’ means continuous mapping.



The reverse implications are not true in general in the above diagram.

Proposition 4.2.18 : If $f : (X, \tau) \to (Y, \sigma)$ is an IF completely β^{**}G continuous mapping, then $\beta cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every IFβOS $A \subseteq Y$.

Proof : Let A be an IFβOS in Y. Then cl(A) is an IFRCS in Y. Hence cl(A) is an IFβ^{**}GCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IFRCS in X and thus an IFβCS in X. Therefore $\beta cl(f^{-1}(A)) \subseteq \beta cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$.

Proposition 4.2.19 : Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. Then the following are equivalent :

(i) f is an IF completely β^{**}G continuous mapping,

(ii) $f^{-1}(V)$ is an IFROS in X for every IFβ^{**}GOS V in Y.

(iii) For every IFP $p_{(\alpha,\beta)} \in X$ and for every IFβ^{**}GOS B in Y such that $f(p_{(\alpha,\beta)}) \in B$ there exists an IFROS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

Proof : (i) $\Leftrightarrow$ (ii) is obvious as $f^{-1}(A^c) = (f^{-1}(A))^c$.

(ii) $\Rightarrow$ (iii) : Let $p_{(\alpha,\beta)} \in X$ and $B \subseteq Y$ such that $f(p_{(\alpha,\beta)}) \in B$. This implies $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since B is an IF$\beta^{**}$GOS in Y, by hypothesis $f^{-1}(B)$ is an IFROS in X. Let $A = f^{-1}(B)$. Then $p_{(\alpha,\beta)}) \in f^{-1}(f(p_{(\alpha,\beta)}) \in f^{-1}(B) = A$. Therefore $p_{(\alpha,\beta)}) \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(iii) $\Rightarrow$ (i) : Let $B \subseteq Y$ be an IFβ^{**}GOS. Let $p_{(\alpha,\beta)}) \in X$ and $f(p_{(\alpha,\beta)}) \in B$. By hypothesis, there exists an IFROS C in X such that $p_{(\alpha,\beta)}) \in C$ and $f(C) \subseteq B$. This implies $C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B)$. Therefore $p_{(\alpha,\beta)}) \in C \subseteq f^{-1}(B)$. That is $f^{-1}(B) = \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} \{p_{(\alpha\beta)}\} \subseteq \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} C \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} C$. Since the union of IFROSs is an IFROS, $f^{-1}(B)$ is an IFROS in X. Hence f is an IF completely β^{**}G continuous mapping.

Proposition 4.2.20 : Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be any two mappings. Then

(i) g o f is an IF completely β^{**}G continuous mapping, if f is an IF completely β^{**}G continuous mapping and g is an IFβ^{**}G irresolute mapping.

(ii) g o f is an IFβ^{**}G continuous mapping, if f is an IF completely β^{**}G continuous mapping and g is an IFβ^{**}G continuous mapping.

(iii) g o f is an IFβ^{**}G irresolute mapping, if f is an IF continuous mapping and g is an IF completely β^{**}G continuous mapping.

Proof : (i) Let B be an IFβ^{**}GOS in Z. Since g is an IFβ^{**}G irresolute mapping, $g^{-1}(B)$ is an IFβ^{**}GOS in Y. Also, since f is an IF completely β^{**}G continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFROS in X. Since $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, g o f is an IF completely β^{**}G continuous mapping.

(ii) Let B be an IFOS in Z. Since g is an IFβ^{**}G continuous mapping, $g^{-1}(B)$ is an IFβ^{**}GOS in Y. Also, since f is an IF completely β^{**}G continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFROS in X. Hence $f^{-1}(g^{-1}(B))$ is an IFβ^{**}GOS in X. As $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, g o f is an IF$\beta^{**}$G continuous mapping.

(iii) Let B be an IFβ^{**}GOS in Z. Since g is an IF completely β^{**}G continuous mapping, $g^{-1}(B)$ is an IFROS in Y. Since every IFROS is an IFOS, $g^{-1}(B)$ is an IFOS in Y. Also, since f is an IF continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFOS in X. Hence $f^{-1}(g^{-1}(B))$ is an IFβ^{**}GOS in X. As $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$, g o f is an IF$\beta^{**}$G irresolute mapping.

Proposition 4.2.21 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IF completely β^{**}G continuous mapping, then $int(cl(f^{-1}(int(B)))) \subseteq f^{-1}(B)$ for every IFS B in Y.

Proof : Let $B \subseteq Y$. Then int(B) is an IFOS in Y and hence it is an IFβ^{**}GOS in Y. By hypothesis, $f^{-1}(int(B))$ is an IFROS in X. Hence $int(cl(f^{-1}(int(B)))) = f^{-1}(int(B)) \subseteq f^{-1}(B)$.

Proposition 4.2.22 : If a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IF completely β^{**}G continuous mapping, then for every IFP $p_{(\alpha,\beta)} \in X$ and for every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFROS $B \subseteq X$ such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.

Proof : Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha,\beta)}) \in C \subseteq A$. Since every IFOS is an IFβ^{**}GOS, C is an IFβ^{**}GOS in Y. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \subseteq f^{-1}(C) \subseteq f^{-1}(A)$ and therefore $p_{(\alpha,\beta)}) \in f^{-1}(C)$. Now let $f^{-1}(C) = B$. Therefore $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.

Proposition 4.2.23 : If a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IF completely β^{**}G continuous mapping, then for every IFP $p_{(\alpha,\beta)} \in X$ and for every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFROS $B \subseteq X$ such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$.

Proof : Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha,\beta)}) \in C \subseteq A$. Since every IFOS is an IFβ^{**}GOS, C is an IFβ^{**}GOS in Y. Hence by hypothesis, $f^{-1}(C)$ is an IFROS in X and $p_{(\alpha,\beta)} \in f^{-1}(C)$. Now let $f^{-1}(C) = B$. Therefore $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$. Thus $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

Proposition 4.2.24 : The composition of any two IF completely β^{**}G continuous mapping is an IF completely β^{**}G continuous mapping.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \delta)$ be any two intuitionistic fuzzy completely β^{**}G continuous mappings. Let B be an IFβ^{**}GOS in Z. Since g is an IF completely β^{**}G continuous mapping, $g^{-1}(B)$ is an IFROS in Y. Since every IFROS is an IFβ^{**}GOS, $g^{-1}(B)$ is an IFβ^{**}GOS in Y. Since f is an IF completely β^{**}G continuous mapping, $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is an IFROS in X. Hence g o f is an IF completely β^{**}G continuous mapping.

4.3 Perfectly β^{**} Generalized Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

In this section we have introduced intuitionistic fuzzy perfectly β^{**} generalized continuous mappings and investigated some of their properties. Some interesting propositions are obtained and analyzed.

Definition 4.3.1 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an **intuitionistic fuzzy perfectly β^{**} generalized** (IF perfectly β^{**}G) **continuous mapping** if $f^{-1}(A)$ is a intuitionistic fuzzy clopen set in X for every IFβ^{**}GCS A of Y.

Proposition 4.3.2 : Every IF perfectly β^{**}G continuous mapping is an IF continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF perfectly β^{**}G continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, A is an IFβ^{**}GCS in Y. Then $f^{-1}(A)$ is an IF clopen set in X by hypothesis. Thus $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous mapping.

Example 4.3.3 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here the mapping f is an IF continuous mapping but not an IF perfectly β^{**}G continuous mapping, since $A = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFβ^{**}GCS in Y, and $f^{-1}(A)$ is not an IF clopen set in X, as $cl(f^{-1}(A)) = G_1^c \neq f^{-1}(A)$ and $int(f^{-1}(A)) = 0_\sim \neq f^{-1}(A)$.

Proposition 4.3.4 : Every IF perfectly β^{**}G continuous mapping is an IF semi continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF perfectly β^{**}G continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, A is an IFβ^{**}GCS in Y. Since f is an IF perfectly β^{**}G continuous mapping, $f^{-1}(A)$ is an IF clopen set in X. That is $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFSCS in X, f is an IF semi continuous mapping.

Example 4.3.5 : Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ and $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF semi continuous mapping, but not an IF perfectly β^{**}G continuous mapping, since $A = \langle y, (0.1_u, 0.2_v), (0.9_u, 0.8_v)\rangle$ is an IFβ^{**}GCS in Y, and $f^{-1}(A)$ is not an IF clopen set in X, as $cl(f^{-1}(A)) = G_1^c \neq f^{-1}(A)$ and $int(f^{-1}(A)) = 0_\sim \neq f^{-1}(A)$.

Proposition 4.3.6 : Every IF perfectly β^{**}G continuous mapping is an IF pre continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF perfectly β^{**}G continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, A is an IFβ^{**}GCS in Y. Since f is an IF perfectly β^{**}G continuous mapping, $f^{-1}(A)$ is an IF clopen set in X. That is $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFPCS, $f^{-1}(A)$ is an IFPCS in X. Hence f is an IF pre continuous mapping.

Example 4.3.7 : Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF pre continuous mapping, but not an IF perfectly β^{**}G continuous mapping.

Proposition 4.3.8 : Every IF perfectly β^{**}G continuous mapping is an IFα continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF perfectly β^{**}G continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, A is an IFβ^{**}GCS in Y. Since f is an IF perfectly β^{**}G continuous mapping, $f^{-1}(A)$ is an IF clopen set in X. That is $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFαCS, $f^{-1}(A)$ is an IFαCS in X. Hence f is an IFα continuous mapping.

Example 4.3.9 : Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IFα continuous mapping, but not an IF perfectly β^{**}G continuous mapping.

Proposition 4.3.10 : Every IF perfectly β^{**}G continuous mapping is an IFβ continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF perfectly β^{**}G continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, A is an IFβ^{**}GCS in Y. Since f is an IF perfectly β^{**}G continuous mapping, $f^{-1}(A)$ is an IF clopen set in X. That is $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFβCS, $f^{-1}(A)$ is an IFβCS in X. Hence f is an IFβ continuous mapping.

Example 4.3.11 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IFβ continuous mapping, but not an IF perfectly β^{**}G continuous mapping.

Proposition 4.3.12 : Every IF perfectly β^{**}G continuous mapping is an IFβ^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF perfectly β^{**}G continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, A is an IFβ^{**}GCS in Y. Since f is an IF perfectly β^{**}G continuous mapping, $f^{-1}(A)$ is an IF clopen set in X. That is $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFβ^{**}GCS, $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G continuous mapping.

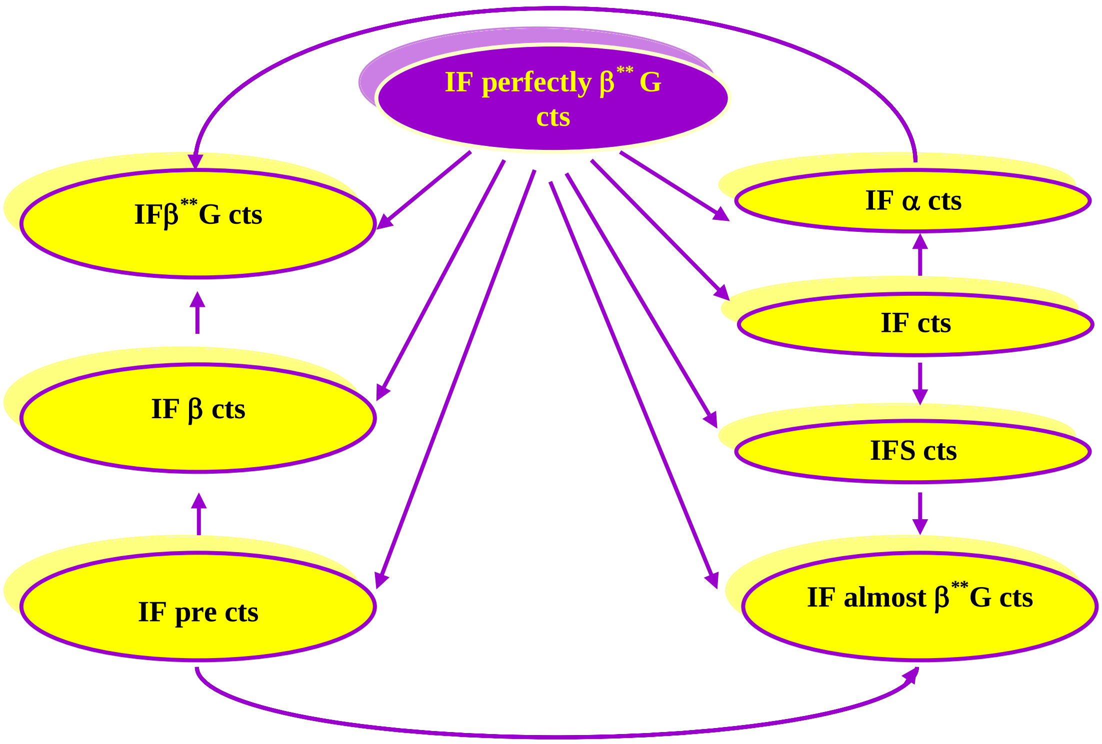
Example 4.3.13 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IFβ^{**}G continuous mapping, but not an IF perfectly β^{**}G continuous mapping.

Proposition 4.3.14 : Every IF perfectly β^{**}G continuous mapping is an IF almost β^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF perfectly β^{**}G continuous mapping. Let A be an IFRCS in Y. Since every IFRCS is an IFβ^{**}GCS, A is an IFβ^{**}GCS in Y. By hypothesis, $f^{-1}(A)$ is an IF clopen set in X. Thus $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFβ^{**}GCS, $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IF almost β^{**}G continuous mapping.

Example 4.3.15 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF almost β^{**}G continuous mapping but not an IF perfectly β^{**}G continuous mapping.

The relation between various types of intuitionistic fuzzy continuous mapping is given in the following diagram. In this diagram “cts” means continuous mapping.



The reverse implications are not true in general in the above diagram.

Proposition 4.3.16 : Every IF perfectly β^{**}G continuous mapping is an IF completely β^{**}G continuous mapping.

Proof : Let f : (X, τ) → (Y, σ) be an IF perfectly β^{**}G continuous mapping. Let A be an IFβ^{**}GCS in Y. By hypothesis, $f^{-1}(A)$ is an IF clopen set in X. Therefore $cl(f^{-1}(A)) = f^{-1}(A)$ and $int(f^{-1}(A)) = f^{-1}(A)$. Now $cl(int(f^{-1}(A))) = cl(f^{-1}(A)) = f^{-1}(A)$. Therefore $f^{-1}(A)$ is an IFRCS in X. Hence f is an IF completely β^{**}G continuous mapping.

Proposition 4.3.17 : A mapping f : (X, τ) → (Y, σ) is an IF perfectly β^{**}G continuous mapping if and only if the inverse image of each IFβ^{**}GOS A in Y is an IF clopen in X.

Proof : Necessity : Let a mapping f : (X, τ) → (Y, σ) be IF perfectly β^{**}G continuous mapping. Let A be an IFβ^{**}GOS in Y. Then A^c is an IFβ^{**}GCS in Y. Since f is an IF perfectly β^{**}G continuous mapping, $f^{-1}(A^c)$ is IF clopen in X. As $f^{-1}(A^c) = (f^{-1}(A))^c$, we have $f^{-1}(A)$ is IF clopen in X.

Sufficiency : Let B be an IFβ^{**}GCS in Y. This implies B^c is an IFβ^{**}GOS in Y. By hypothesis $f^{-1}(B^c)$ is IF clopen in X, which implies $f^{-1}(B)$ is IF clopen in X, as $f^{-1}(B^c) = (f^{-1}(B))^c$. Therefore f is an IF perfectly β^{**}G continuous mapping.

Proposition 4.3.18 : Let f : (X, τ) → (Y, σ) be an IF continuous mapping and g : (Y, σ) → (Z, δ) be an IF perfectly β^{**}G continuous mapping, then g o f : (X, τ) → (Z, δ) is an IF perfectly β^{**}G continuous mapping.

Proof : Let A be an IFβ^{**}GCS in Z. Since g is an IF perfectly β^{**}G continuous mapping, $g^{-1}(A)$ is an IF clopen set in Y. Since f is an IF continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFCS in X, as well as IFOS in X. Hence g o f is an IF perfectly β^{**}G continuous mapping.

Proposition 4.3.19 : The composition of two IF perfectly β^{**}G continuous mapping is an IF perfectly β**G continuous mapping in general.

Proof : Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, δ) be any two IF perfectly β^{**}G continuous mappings. Let A be an IFβ^{**}GCS in Z. By hypothesis, $g^{-1}(A)$ is IF clopen in Y and hence an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, $g^{-1}(A)$ is an IFβ^{**}GCS in Y. Further, since f is an IF perfectly β^{**}G continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is IF clopen in X. Hence g o f is an IF perfectly β^{**}G continuous mapping.

Proposition 4.3.20 : Let f : (X, τ) → (Y, σ) be an IF perfectly β^{**}G continuous mapping and g : (Y, σ) → (Z, δ) is an IFβ^{**}G irresolute mapping, then g o f : (X, τ) → (Y, σ) is an IF perfectly β^{**}G continuous mapping.

Proof : Let A be an IFβ^{**}GCS in Z. By hypothesis, $g^{-1}(A)$ is an IFβ^{**}GCS in Y. Since f is an IF perfectly β^{**}G continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is IF clopen in X. Hence g o f is an IF perfectly β^{**}G continuous mapping.

Proposition 4.3.21 : Let f : (X, τ) → (Y, σ) be an IF continuous mapping and g : (Y, σ) → (Z, δ) is IF perfectly β^{**}G continuous mapping, then g o f : (X, τ) → (Y, σ) is an IF continuous mapping.

Proof : Let A be an IFCS in Z. Then A is an IFβ^{**}GCS in Z. Since g is an IF perfectly β^{**}G continuous mapping, $g^{-1}(A)$ is an IF clopen set in Y. Thus $g^{-1}(A)$ is IFCS in Y. Since f is an IF continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFCS in X. Hence g o f is an IF continuous mapping.

4.4 Quasi β^{**} Generalized Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

In this section, we have introduced intuitionistic fuzzy quasi β^{**} generalized continuous mappings and studied some of their properties. Composition of these mappings with other continuous mappings are investigated and analyzed.

Definition 4.4.1 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is called **an intuitionistic fuzzy quasi β^{**} generalized** (IF quasi β^{**}G) **continuous mapping** if $f^{-1}(V)$ is an IFCS in X for every IFβ^{**}GCS V of Y.

Proposition 4.4.2 : Every IF quasi β^{**}G continuous mapping is an IF continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF quasi β^{**}G continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS in Y, A is an IFβ^{**}GCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous mapping.

Example 4.4.3 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ and $G_4 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v.

Here f is an IF continuous mapping but not an IF quasi β^{**}G continuous mapping, since $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ is an IFβ^{**}GCS in Y but $f^{-1}(G_3)$ is not an IFCS in X, as $cl(f^{-1}(G_3)) = G_2^c \neq f^{-1}(G_3)$.

Proposition 4.4.4 : Every IF quasi β^{**}G continuous mapping is an IF semi continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF quasi β^{**}G continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS in Y, A is an IFβ^{**}GCS in Y. Then by hypothesis, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFSCS, $f^{-1}(A)$ is an IFSCS in X. Hence f is an IF semi continuous mapping.

Example 4.4.5 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ and $G_4 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF semi continuous mapping, but not an IF quasi β^{**}G continuous mapping.

Proposition 4.4.6 : Every IF quasi β^{**}G continuous mapping is an IF pre continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF quasi β^{**}G continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS in Y, A is an IFβ^{**}GCS in Y. Then by hypothesis, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFPCS, $f^{-1}(A)$ is an IFPCS in X. Hence f is an IF pre continuous mapping.

Example 4.4.7 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ and $G_4 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF pre continuous mapping, but not an IF quasi β^{**}G continuous mapping.

Proposition 4.4.8 : Every IF quasi β^{**}G continuous mapping is an IFα continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF quasi β^{**}G continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS in Y, A is an IFβ^{**}GCS in Y. Then hypothesis $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFαCS, $f^{-1}(A)$ is an IFαCS in X. Hence f is an IFα continuous mapping.

Example 4.4.9 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ and $G_4 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IFα continuous mapping but not an IF quasi β^{**}G continuous mapping.

Proposition 4.4.10 : Every IF quasi β^{**}G continuous mapping is an IFβ continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF quasi β^{**}G continuous mapping. Let A be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS in Y, A is an IFβ^{**}GCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFβCS, $f^{-1}(A)$ is an IFβCS in X. Hence f is an IFβ continuous mapping.

Example 4.4.11 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ and $G_4 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ are IFTs on X and Y respectively. Then, f is an IFβ continuous mapping but not an IF quasi β^{**}G continuous mapping.

Proposition 4.4.12 : Every IF quasi β^{**}G continuous mapping is an IFβ^{**}G continuous mapping but not conversely in general.

Proof : Let A be an IFCS in Y. Since every IFCS is an IFβ^{**}GCS in Y, A is an IFβ^{**}GCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFβ^{**}GCS, $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G continuous mapping.

Example 4.4.13 : Let X = {a, b} and Y = {u, v}. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is an IFβ^{**}G continuous mapping but not an IF quasi β^{**}G continuous mapping, since $G_3^c = \langle y, (0.5_u, 0.6_v), (0.3_u, 0.4_v)\rangle$ is an IFβ^{**}GCS in Y but $f^{-1}(G_3^c)$ is not an IFCS in X, as $cl(f^{-1}(G_3^c)) = 1_\sim \neq f^{-1}(G_3^c)$.

Proposition 4.4.14 : Every IF quasi β^{**}G continuous mapping is an IF almost β^{**}G continuous mapping but not conversely in general.

Proof : Let A be an IFRCS in Y. Since every IFRCS is an IFβ^{**}GCS in Y, A is an IFβ^{**}GCS in Y. Then by hypothesis, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFβ^{**}GCS, $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IF almost β^{**}G continuous mapping.

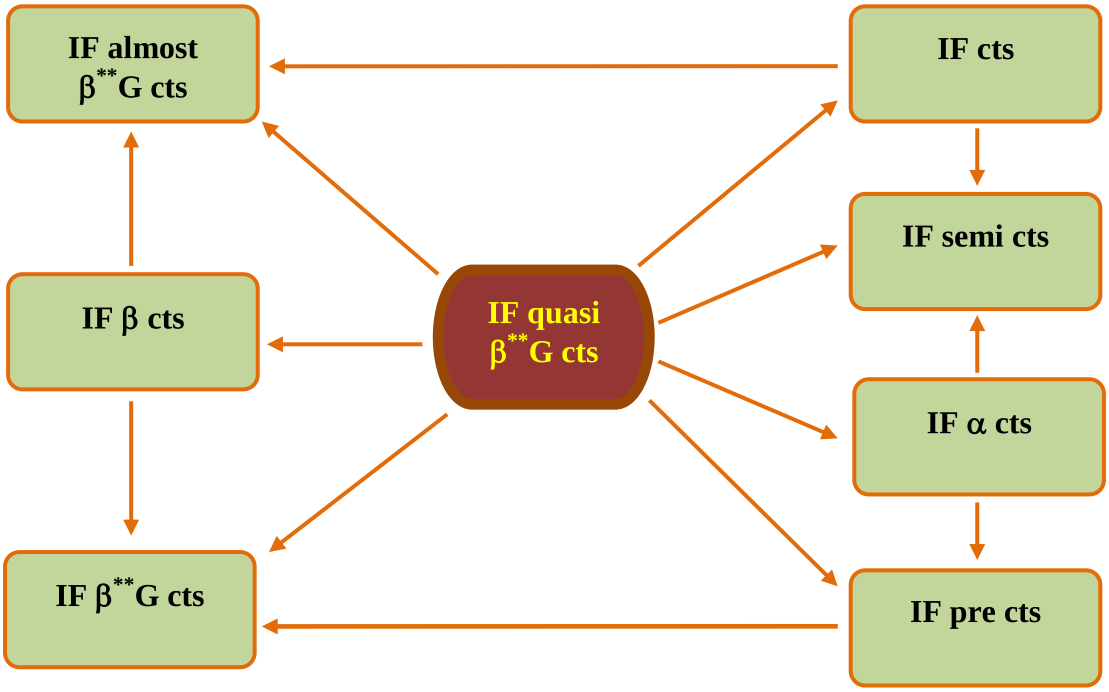
Example 4.4.15 : Let X = {a, b} and Y = {u, v}. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Here, f is an IF almost β^{**}G continuous mapping but not an IF quasi β^{**}G continuous mapping, since $G_3^c = \langle y, (0.5_u, 0.6_v), (0.3_u, 0.4_v)\rangle$ is an IFβ^{**}GCS in Y but $f^{-1}(G_3^c)$ is not an IFCS in X.

Proposition 4.4.16 : Every IF quasi β^{**}G continuous mapping is an IFβ^{**}G irresolute mapping but not conversely in general.

Proof : Let A be an IFβ^{**}GCS in Y. Then $f^{-1}(A)$ is an IFCS in X by hypothesis. Since every IFCS in an IFβ^{**}GCS, $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G irresolute mapping.

Example 4.4.17 : Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$, $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$ and $G_4 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, G_4, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IFβ^{**}G irresolute mapping but not an IF quasi β^{**}G continuous mapping, since $G_4^c = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$ is an IFβ^{**}GCS in Y but $f^{-1}(G_4^c)$ is not an IFCS in X, as $cl(f^{-1}(G_4^c)) = G_1^c \neq f^{-1}(G_4^c)$.

From the above propositions and examples we have the following diagram. However the reverse implications are not true in general. In this diagram 'cts' means continuous mapping.



The reverse implications are not true in general in the above diagram.

Proposition 4.4.18 : Let f : (X, τ) $\rightarrow$ (Y, σ) be an mapping. Then the following statements are equivalent :

(i) f is an IF quasi β^{**}G continuous mapping,

(ii) f^{-1}(A) is an IFOS in X for every IFβ^{**}GOS A in Y.

Proof : Proof is obvious as $f^{-1}(A^c) = (f^{-1}(A))^c$.

Proposition 4.4.19 : The composition of two IF quasi β^{**}G continuous mapping is an IF quasi β^{**}G continuous mapping.

Proof : Let f : (X, τ) $\rightarrow$ (Y, σ) and g : (Y, σ) $\rightarrow$ (Z, δ) be any two IF quasi β^{**}G continuous mappings. Let A be an IFβ^{**}GCS in Z. By hypothesis, g^{-1}(A) is an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, g^{-1}(A) is an IFβ^{**}GCS in Y. Further, since f is an IF quasi β^{**}G continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is an IFCS in X. Hence g o f is an IF quasi β^{**}G continuous mapping.

Proposition 4.4.20 : If f : (X, τ) $\rightarrow$ (Y, σ) be an IF quasi β^{**}G continuous mapping and g : (Y, σ) $\rightarrow$ (Z, δ) is an IFβ^{**}G irresolute mapping, then g o f : (X, τ) $\rightarrow$ (Z, δ) is an IF quasi β^{**}G continuous mapping.

Proof : Let B be an IFβ^{**}GCS in Z. Since g is an IFβ^{**}G irresolute mapping, g^{-1}(A) is an IFβ^{**}GCS in Y. Since f is an IF quasi β^{**}G continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is IFCS in X. Hence g o f is an IF quasi β^{**}G continuous mapping.

Proposition 4.4.21 : Let f : (X, τ) $\rightarrow$ (Y, σ) be an IF continuous mapping and g : (Y, σ) $\rightarrow$ (Z, δ) an IF quasi β^{**}G continuous mapping, then g o f : (X, τ) $\rightarrow$ (Z, δ) is an IF quasi β^{**}G continuous mapping.

Proof : Let A be an IFβ^{**}GCS in Z. By hypothesis, g^{-1}(A) is an IFCS in Y. Since f is an IF continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is an IFCS in X. Hence g o f is an IF quasi β^{**}G continuous mapping.

Proposition 4.4.22 : Let f : (X, τ) → (Y, σ) be an IF quasi β^{**}G continuous mapping and g : (Y, σ) → (Z, δ) is an IF continuous mapping, then g o f : (X, τ) → (Z, δ) is an IF continuous mapping.

Proof : Let A be an IFCS in Z. By hypothesis, $g^{-1}(A)$ is an IFCS in Y. Since every IFCS is an IFβ^{**}GCS, $g^{-1}(A)$ is an IFβ^{**}GCS in Y. Then $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is an IFCS in X, by hypothesis. Hence g o f is an IF continuous mapping.

Proposition 4.4.23 : Let f : (X, τ) → (Y, σ) be an IF quasi β^{**}G continuous mapping and g : (Y, σ) → (Z, δ) is an IFβ^{**}G continuous mapping, then g o f : (X, τ) → (Z, δ) is an IF continuous mapping.

Proof : Let A be an IFCS in Z. By hypothesis, $g^{-1}(A)$ is an IFβ^{**}GCS in Y. Since f is an IF quasi β^{**}G continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is an IFCS in X. Hence g o f is an IF continuous mapping.