REVIEW OF LITERATURE

The definition of fuzzy set was introduced by Zadeh in 1965. The important contributions to the study of fuzzy set theory were made by Zimmerman (1996), Yager (1980), Chang (1968), Lowen (1976), Gougen (1967), Gottwald (1993) and many others.

In 1968, based on fuzzy sets Chang introduced the concept of fuzzy topological spaces and defined some basic concepts such as open fuzzy sets, closed fuzzy sets, neighbourhood of a fuzzy set, interior of a fuzzy set, fuzzy continuity and fuzzy compactness.

In order to study deeper into the structure of fuzzy topological spaces, in 1976 Lowen modified the concept of fuzzy topological spaces of Chang. Also, the author introduced two functors $\widetilde{\omega}$ and $\tilde{\imath}$ to establish the connection between fuzzy topological spaces and topological spaces.

In 1994, Zhang introduced the notion of bipolar fuzziness. In 2019, Kim et al., introduced the concept of bipolar fuzzy topology and defined some basic concepts such as bipolar fuzzy point, bipolar fuzzy base, bipolar fuzzy subbase, bipolar fuzzy subspace, bipolar fuzzy quotient space, bipolar fuzzy neighbourhood, bipolar fuzzy initial topology, bipolar fuzzy continuity and bipolar fuzzy compactness and obtained some basic properties of each concepts.

In 1975, Zadeh introduced the concept of fuzzy set of type 2 (second order fuzzy set) as an extension of a fuzzy set. A detailed study of second order fuzzy sets was done by Mizumuto and Tanaka (1976, 1981). Norwich and Turksen (1981,1984) have used the concept of second order fuzzy sets in their stochastic fuzzy model.

In 2007, Kalaichelvi introduced the concept of second order fuzzy topological spaces using fuzzy sets of type 2 defined by Zadeh (1975) and studied some second order fuzzy structures.

In this thesis the following concepts are introduced and analysed with reference to second order bipolar fuzzy structures.

(i) Continuity

(ii) Product

(iii) Hausdorffness

(iv) Gradation of Openness

(v) Compactness

(vi) Matrix

The relevant literature related to these concepts are given.

Continuity:-

In 1978, Warren established that fuzzy continuous function can be characterized by fuzzy closure, subbasis of a fuzzy topology and fuzzy neighbourhoods.

In 1981a, Azad studied fuzzy topological spaces with special attention to the weaker forms of fuzzy continuity and introduced some basic concepts such as fuzzy semi open, fuzzy semi closed, fuzzy regular open and fuzzy regular closed sets. The generalizations of semi continuous mapping, semi open mapping, semi closed mapping, almost continuous mapping and weakly continuous mapping in fuzzy setting were considered. It was observed that fuzzy continuity ⇒ fuzzy almost continuity ⇒ fuzzy weakly continuity (none is reversible)

In 1985, Ganguly and Saha introduced and generalised the concepts of T_i – continuous functions in a fuzzy setting and studied them in connection with the fuzzy continuity and fuzzy separation axioms. Also some fuzzy topological properties under these functions were investigated.

In 1990, Mukherjee and Ghosh introduced and studied the concepts of certain classes of functions between fuzzy topological spaces. Each of these functions presents a stronger form of fuzzy continuous function. In 1997, Balasubramanian and Sundaram introduced and studied various generalizations of fuzzy continuous, fuzzy generalized connectedness, generalized fuzzy extremely disconnectedness and fuzzy generalized compactness.

In 2004, Ekici proved some characterizations and properties of fuzzy continuous functions and its weaker and stronger forms including fuzzy weakly continuous, fuzzy θ-continuous, fuzzy strongly θ-continuous, fuzzy almost strongly θ-continuous, fuzzy weakly θ-continuous, fuzzy almost continuous, fuzzy super continuous, fuzzy δ-continuous functions.

In 2015, Jeyaraman et al., obtained decomposition of fuzzy continuity in fuzzy topological spaces by using g‴- continuity.

In 2017, Cahit Tasdemir proved some theorems about fuzzy continuous functions using the definitions and properties related to fuzzy continuity, fuzzy membership functions and fuzzy continuous functions.

In 2019, Sobana et al., introduced fuzzy e - continuity, fuzzy e – open and fuzzy e – closed mapping and also investigated relationship between fuzzy continuity, fuzzy semi continuity, fuzzy δ – semi continuity, fuzzy δ – pre continuity and fuzzy γ – continuity.

In 2019, Tajayaying et al., introduced the concepts of G – fuzzy sequential closedness and G- fuzzy sequential continuity in a fuzzy topological space. Results and some characterization theorems are obtained.

In 2019, Kim et al., defined bipolar fuzzy continuity and obtained some basic properties. In 2020, Anita Shanthi and Saranya defined bipolar fuzzy rough topology by means of lower and upper approximations on bipolar fuzzy sets. In 2022, Zarife Zararsiz and Muhammad Riaz introduced the concept of bipolar metric space and studied its topological and functional properties. In 2022, Hami MALOC and Banu PAZAR VAROL, introduced bipolar fuzzy supra topological space and studied the concepts of bipolar fuzzy supra continuity and S* bipolar fuzzy supra continuity.

In 2011a, Kalaichelvi introduced second order fuzzy continuity and established that the different associations between first order fuzzy topological spaces and second order fuzzy topological spaces are functorial.

Product: -

In 1973, Goguen introduced L-set which is defined as a map on a set X to a complete lattice L. Using L-sets, L- topological space is defined. Also, arbitrary product of L-topological spaces is defined and proved fuzzy Tychonoff theorem. In 1974a, Wong introduced product fuzzy topology and quotient fuzzy topology for a family of fuzzy topological spaces. Product theorems were proved for a fuzzy topological space consisting of countable bases denoted as $\mathbb{C}_{11}$ and compact spaces.

In 1978, Gantner et al., proposed a new definition of fuzzy space compactness. It is observed that there is possible to have a degree of compactness called $\alpha -$ compactness ($\alpha -$ a member of a designated lattice) and for an arbitrary product of α-compact fuzzy spaces and a 1-point compactification, a Tychonoff Theorem is derived.

In 1980, Bruce Hutton introduced a novel definition of the product, which allows to show a "Tychonoff product theorem" in both directions, which is not possible with the other definitions.

In 1991, Dib and Nabil. L.Youssef established a new approach to Cartesian product, relations and functions in fuzzy set theory. Using a suitable lattice, the concept of fuzzy Cartesian product is introduced and then a fuzzy relation is defined as a subset of the fuzzy Cartesian product analogous to crisp case.

In 1998, Pratualananda Das introduced the notion of product fuzzy topology on fuzzy sets and the product invariance of fuzzy Hausdorffness, compactness and connectedness are studied. Using product fuzzy topology, fuzzy group topology on a fuzzy subgroup of a group G is defined and some properties of fuzzy topological groups are obtained.

In 2022, Singh et al., introduced the concepts of countably compact, second countable and Lindelof bipolar fuzzy topological spaces and defined ε – partition of a given cover of bipolar fuzzy topological spaces. The product bipolar fuzzy topological spaces for a family of bipolar fuzzy topological spaces are defined and studied some of its properties.

In 2012, Kalaichelvi introduced second order fuzzy product topology and theorems related to this concept were proved.

Hausdorffness:-

The first important article on fuzzy separation axioms entitles "Normality in fuzzy topological spaces" was published by Hutton in 1975.

In 1978, Gantner introduced fuzzy Hausdorff axioms which is used to prove results in fuzzy compact spaces. In 1980, Rodabough proposed a new Hausdorff axiom for fuzzy topological spaces namely the $\alpha -$ Hausdorff axiom and obtained characterizations of the $\alpha -$ Hausdorff subspaces of the fuzzy unit interval, the fuzzy open unit interval and the fuzzy real line.

In 1981, Sarkar studied few separation properties and some aspects of subspace fuzzy topology, where both the crisp and the fuzzy elements have been taken into consideration. A new more natural definition of proper compactness is given and few properties resulting from this are established.

In 1981 and in 1984, Srivastava et al., introduced the notion of fuzzy Hausdorff topological space by extending the idea of a Hausdorff space (where distinct points have disjoint neighborhoods) to the realm of fuzzy sets, allowing for degrees of membership rather than strict binary classifications within a topological space.

In 1981, Katsaras introduced the concept of ordered fuzzy topological spaces by combining order and fuzzy topological spaces. The notion of fuzzy Hausdorff space is also introduced and obtained some properties connecting these two spaces.

In 1981b, Azad introduced fuzzy Hausdorff space using the concept of λ - diagonals and a generalization of the theorem that the graph of a mapping on a topological space to a Hausdorff space is closed was obtained in the fuzzy setting. In 1983, Wuyts and Lowen introduced and studied certain number of separation properties of fuzzy topological spaces, fuzzy neighbourhood spaces and fuzzy uniform spaces which are extensions of T_0, T_1 and T_2 in topological separation axioms

In 1983, Rodabaugh carried out a detailed study of the fuzzy real line with special reference to fuzzy separation axioms. It is proved that the fuzzy real line satisfies the higher order separation axioms of Hutton and Mira Sarkar

In 1986, Mashhour et al., extended the $\alpha -$Hausdorff property of Rodabaugh to the concepts - $(\alpha - T_i)$, $i = 0,3,4$ and $(\alpha - T_i')$, $i = 0,1,2,3,4$.

In 1987, using the concept of quasi-coincidence, Ganguly and Saha introduced a new definition of separation axioms and developed a good theory by extending many of the classical theorems to fuzzy situation.

In 1984, Srivastava et al., pointed out that the definition of a fuzzy T_1 – Space suggested retains in some features. It is somewhat more tangible to work with, as it is described in terms of crisp rather than fuzzy points.

The separation axioms include T_0, T_1, T_2, R_0 and regular here it is proved that the separation properties T_0, T_1, T_2 and R_0 are productive and hereditary. Further, a Characterization of regularity by Ali in 1990.

Hausdorff axioms in first order fuzzy topological spaces introduced by Gantner et al. (1978), Srivastava et al. (1981) and Katsaras (1981) were extended to second order fuzzy topological spaces by Kalaichelvi (2000, 2011b, 2013) and denoted them as W- Hausdorff, S-Hausdorff and K-Hausdorff axioms respectively.

Gradation of openness:-

In 1985, Alexander. P. Sostak introduced the new fuzzy topology in the name of category of fuzzy topology which is a mapping from $\tau: I^X \to I$.

In 1994, Parvathi, A. extended the concept of gradation of openness to Boolean algebras and obtained interesting results.

In 1994, Rekha Srivastava introduced separation axioms and some allied notions such as base, subbase, etc…, in gradation spaces.

In 1999, Valentin Gregori and Anna vidal introduced a gradation of openness for the open sets of Chang fuzzy topological spaces by means of a map $\sigma: I^X \to I$, which is at the same time of a fuzzy topology on X in Shostak's sense.

In 1992, Hazra et al., introduced new definition of fuzzy topology using the concept of gradation of openness of fuzzy subsets and studied fuzzy continuity. In order to make the concept more appropriate, in 1992 Chattopadhyay et al., modified the definition of gradation function and studied subspace of fuzzy topological spaces and gradation preserving maps.

In 1992 and in 1993, Chattopadhyay et al., developed a theory on gradation of openness. Every gradation of openness has a Chang fuzzy topology associated with it and vice versa. Also introduced the definition of gradation preserving maps. These articles examine the ideas of fuzzy connectedness and fuzzy compactness in terms of gradation.

In 2011, Thakur et al., introduced a definition of gradation of continuity in graded fuzzy topological spaces and studied its various characteristic properties. Concept of gradation is also introduced in openness, closedness, Homeomorphic properties of mappings and T_2 – separation axioms. Effects of the grades interrelated with N- compactness, T_2 - separation and homeomorphism of mappings are studied.

In 2020, Subhadip Roy et al., introduced the definition of bipolar gradation of openness of bipolar fuzzy subsets of X by means of lower and upper approximation and given a new definition of bipolar fuzzy topological spaces. A bipolar gradation preserving map is also introduced and a decomposition theorem involving bipolar fuzzy topology and bipolar gradation of openness is proved.

In 2000, Kalaichelvi introduced second order gradation of openness and studied the connections between first order gradation of openness and second order gradation of openness. Also proved that for every first order gradation of openness there corresponds a second order gradation of openness and every second order gradation of openness induces a second order fuzzy topology.

Compactness :-

In 1968, Chang defined more basic concepts such as fuzzy open set, fuzzy closed set, interior of a fuzzy set, neighbourhood of a fuzzy set, fuzzy continuity and fuzzy compactness. Related theorems were proved under fuzzy continuity and fuzzy compactness. In 1976, Lowen introduced a new definition of fuzzy compactness and obtained some results analogous to fuzzy compactness. In 1978, Gantner and Steinlage proposed a new definition of fuzzy compactness that is, $\alpha -$ compactness and obtained Tychonoff theorem for an arbitrary product of $\alpha -$ compact fuzzy spaces.

In 1978, Lowen studied various compactness notions such as quasi-fuzzy compact, $\alpha -$ compact, $\alpha^* -$ compact, strong fuzzy compact, ultra fuzzy compact and weakly fuzzy compact. Further it was analysed that which of the above concepts were good extensions and the implications between different notion were also studied. In 1988, Artico and Moresco proved that the unit interval is α^* compact for special values of α.

In 1982 and in 1983, Lowen and Wuyts introduced the notion of precompactness and completeness in fuzzy uniform spaces and obtained some characterizations of compactness in terms of precompactness and completeness.

In 2008, Park and Min introduced several types of r-semi generalized fuzzy compactness and fuzzy r- compactness in fuzzy topological spaces and investigated the relations between these compactness.

In 2009, Fu-gushi introduced a new definition of almost fuzzy compactness in L- topological spaces in terms of both open L-sets and closed L-sets. In 2011, Fu-gushi introduced a new notion of L-fuzzy compactness in L-fuzzy topological spaces which is a generalization of Lowen fuzzy compactness in L-topological spaces. Tychonoff theorem for L-fuzzy compactness is proved.

In 2019, Kim et al., introduced the notion of bipolar fuzzy compactness and obtained some of its properties. In 2022, Hami MALKOC and Banu PAZAR VAROL introduced bipolar fuzzy supra topological space as a generalization of fuzzy supra topological space and investigated some basic properties. The concept of compactness on bipolar fuzzy supra topology is also defined.

In (2011), Kalaichelvi introduced five different types of second order fuzzy compactness.

Fuzzy Matrix:-

In 1977, Thomason introduced the concept of fuzzy matrix which plays a vital role in scientific development and studied the convergence of powers of fuzzy matrix. In 1994, Ragab et al. analyzed some properties on determinant and adjoint of square fuzzy matrix.

Bipolar Fuzzy Matrix:-

In 2019, bipolar fuzzy matrix was proposed by M.Pal and Sanjib Mondal. Several findings about transitive closure and power-convergence of BFMs were examined.

In classical MCDM methods, the attribute values and weights are determined precisely. In 2000, Chen introduced the fuzzy version of TOPSIS to deal with problems consisting of incomplete and vague information. In 2002, Chung and Chu presented fuzzy TOPSIS method under group decision for facility location selection problem. In 2014, Hadi et al. proposed the fuzzy inferior ratio method for multiple attribute decision making problems. In 2022, Natthinee Deetae proposed method a new methodology BFS-TOPSIS, which considers the positive and negative decisions of the decision-maker. The BFS-TOPSIS is a combination of bipolar value sets with the fuzzy TOPSIS method and uses a score function to select the best alternative.