Mathematical Approaches to Entrepreneurship: Fuzzy Multi Criteria Decision Support for Open Innovation Partnership

By

BRINDHASRI G B

(23PMA020)

Supervisor

Dr. C ANTONY CRISPIN SWEETY

Thesis submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore – 641 043

In Partial Fulfilment of The Requirements for the Degree of

Master of Science in Mathematics

April 2025

Mathematical Approaches to Entrepreneurship: Fuzzy Multi Criteria Decision Support for Open Innovation Partnership

By

Brindhasri G B

(23PMA020)

Supervisor

Dr. C Antony Crispin Sweety

Thesis submitted to
Avinashilingam Institute for Home Science and Higher Education for Women
Coimbatore – 641 043

In Partial Fulfilment of the Requirements for the Degree of
Master of Science in Mathematics

April 2025

Signature of the Director

28/04/25
Signature of the Supervisor

DECLARATION

DECLARATION

I declare that the thesis **Mathematical Approaches to Entrepreneurship: Fuzzy Multi Criteria Decision Support for Open Innovation Partnership**" submitted by me for the degree of **Master of Science (M. Sc)** is the record of work carried out during the period of December 2024 to April 2025 under the guidance of **Dr. C. Antony Crispin Sweety B.Ed., M.Sc., M.Phil., Ph.D.,** Assistant Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and has not formed the basis for the award of any Degree, Diploma, Associateship, Titles in this institute or any other University or other similar institution of Higher Learning.

G. B. Brindhasri
28.04.2025
Signature of the Candidate

ACKNOWLEDGEMENT

ACKNOWLEDGEMENT

I humbly thank the **GOD ALMIGHTY** who has showered his abundant grace on me and endowed me with wisdom, mental courage and good health throughout the period of my research work.

My foremost thanks to our **Rev. AYYA** and **AMMA AVARGAL** for the blessings pouring on us.

I express my heartfelt thanks to **Dr. T. S. K. MEENAKSHISUNDARAM,** Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for the moral support for the conduct of my research work.

I express my sincere gratitude to **Dr. V. BHARATHI HARISHANKAR,** Vice-Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her constant encouragement throughout the research work.

I would express my sincere thanks to **Dr. H. INDHU,** Registrar, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her constant support.

I express my warm gratitude to **Dr. S. RAJA**, Director - SF Programmes, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for the guidance and support in carrying out my research work.

I express my sincere thanks to **Dr. V. SAVITHA,** Assistant Director - SF Programmes, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her support, encouragement and guidance during the course of the investigation.

I express my sincere thanks to **Dr. G. PADMAVATHI,** Former Dean, School of Physical Sciences and Computational Sciences, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her support, encouragement, and guidance during the course of the investigation.

I would like to thank **DR. V. RADHA**, Dean, School of Physical Sciences and Computational Sciences, Avinashilingam Institute for Home Science and Higher Education for Women. Coimbatore, for her constant support and encouragement throughout the course of investigation.

I would like to express my deep and sincere gratitude to my research supervisor **Dr. C. ANTONY CRISPIN SWEETY,** Assistant Professor, Department of Mathematics (SF), Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her constructive comments, encouragements and invaluable support throughout my research work. Her wide knowledge and logical way of thinking have been of great help for me.

I would also like to extend my heartfelt thanks to **RAUNAQUE MUJEEB QUAISER**, Department of Marketing and Strategy, Indian Institute of Management Rohtak, and **PRAVEEN RANJAN SRIVASTAVA**, Department of IT and Systems, Indian Institute of Management Rohtak, for generously sharing their original data, which greatly enriched the quality and depth of this research.

I am thankful to all the **Staff Members of the Department of MATHEMATICS,** who rendered their help whenever required.

I owe my special thanks to my **Beloved Parents and My Dear Friends** for their kind support and motivation to complete my thesis work successful.

CONTENTS

CONTENTS

ABSTARCT

This thesis explores the integration of mathematical approaches into entrepreneurship, with a specific focus on decision support mechanisms for outbound open innovation partnerships. Recognizing the inherent complexity and uncertainty in entrepreneurial decision-making, the study employs Fuzzy Multi - Criteria Decision Making techniques - namely the Best-Worst Method and Multi-Objective Optimization on the Basis of Ratio Analysis. These methods enable systematic evaluation of strategic criteria using expert input expressed through linguistic terms and triangular fuzzy numbers. The research identifies and prioritizes four critical factors influencing innovation partnerships: Organizational Commitment, Innovation Ecosystem, Knowledge Transfer, and Technology Relevance. Through a hybrid fuzzy - BWM and MOORA model, the study quantifies expert preferences and ranks strategic alternatives, offering a structured, transparent, and adaptable framework for decision-making in innovation-driven environments. The results underscore the significance of Technology Relevance and Innovation Ecosystem in successful collaborations, while also highlighting the dynamic role of knowledge flow and organizational support. This mathematical framework contributes to entrepreneurship theory by demonstrating how structured quantitative tools can enhance strategic planning, resource alignment, and innovation outcomes in real-world scenarios.

CHAPTER 1

Chapter 1

INTRODUCTION

Mathematics plays a crucial role in theory development across disciplines such as physics, engineering and economics, but its application in organizational studies has been limited. This is unfortunate because mathematics offers a structured, precise and logical framework to describe phenomena, build new theories, and refine existing ones. Models are constructed from existing theories that serve as the basis for the model's assumptions. Such models can be inclusive of many independent and dependent variables as well as functional forms and still be simple and precise. They also have the ability to define strong evidential criteria that may provide, and be used to justify, unexpected results. Having these advantages makes a strong case for the use of mathematics in theory formulation in organizational analysis. Mathematics has always played a foundational role in analyzing and solving complex real-world problems. In the field of entrepreneurship, mathematical approaches offer a rigorous and systematic framework to evaluate uncertainties, optimize strategies, and support decision-making under multiple conflicting criteria. As entrepreneurial ventures grow in complexity - intertwined with innovation ecosystems, technological disruptions, and strategic collaborations - the need for mathematical models to guide rational choices becomes increasingly critical.

In the field of entrepreneurship, there are many instances of problem situations or their components, in which the dynamic characteristics of the controlled decision - making system and the decision - makers goal can be captured quantitatively. These include the use of mathematical model by

1. Estimating the thresholds of knowledge accumulation for making decisions regarding the cessation of information search about the opportunity,
2. Determining how various surrounding conditions modify the timing of entry to the market by a new firm depending on the learning by participation and observation, and
3. Forming the scope of the decision as a new firm maturity of level. After the firm's profitability is subjected to risk and environmental impact, it is the being observed.

Mathematics is a tool for modeling from which one can identify patterns, and the resulting mathematical model and its implications lead to theory.

Innovation is the process of developing new ideas, products, services, or processes that create value and drive change. Open innovation extends this concept by encouraging organizations to seek external ideas, knowledge, and technologies, collaborating with partners outside the firm to enhance their own innovation processes. Within open innovation, inbound innovation refers to the integration of external ideas and technologies into the organization, while outbound innovation involves sharing or licensing internal innovations to external partners, enabling the organization to profit from its intellectual property and ideas.

Open innovation is a business model and strategy that emphasizes the use of external and internal ideas, knowledge, and resources to accelerate innovation and drive business growth. In contrast to the traditional closed innovation model, where organizations rely solely on internal resources and capabilities, open innovation encourages collaboration and knowledge exchange with external partners such as other firms, universities, research institutions, customers, and even competitors. This collaborative approach enables organizations to leverage diverse perspectives, expertise, and technologies, leading to the development of innovative solutions that may not be achievable through internal efforts alone.

Multi-Criteria Decision-Making (MCDM) is a critical tool in entrepreneurship, particularly when applied to open innovation, because it helps entrepreneurs and organizations make informed decisions amidst complex situations that involve multiple, often conflicting, criteria. In the context of open innovation, entrepreneurs need to evaluate a variety of options - such as different technologies, partnerships, business models, or market strategies - each with various factors that need to be considered, like cost, risk, potential returns, and alignment with organizational goals.

Indeed, there are numerous Multi-Criteria Decision-Making (MCDM) techniques, each offering a unique way to handle complex decision-making situations where multiple conflicting criteria need to be considered. These techniques help decision-makers evaluate alternatives in a systematic and structured way, ensuring that the final decision aligns with the overall goals and priorities. Here’s an overview of some common MCDM techniques

Among these, the Best-Worst Method (BWM) has emerged as a powerful tool for deriving reliable weights for decision criteria through pairwise comparisons. In BWM, decision-makers identify the most and least important criteria, and then express preferences between them and the others. The use of fuzzy logic enhances BWM by allowing for linguistic inputs, better capturing uncertainty and human judgment. This is especially valuable in entrepreneurial settings where different roles - such as R&D leaders, managers, or investors - bring diverse perspectives to the decision process.

Once criteria weights are established, MOORA (Multi-Objective Optimization on the basis of Ratio Analysis) is applied to rank and prioritize alternative open innovation partnerships. MOORA enables evaluation across multiple dimensions by normalizing data, incorporating the calculated weights, and optimizing based on both beneficial and non-beneficial criteria. The method is known for its simplicity, robustness, and ability to handle conflicting objectives - an essential quality in entrepreneurial contexts that aim to balance innovation potential with organizational fit and resource constraints.

Together, Fuzzy BWM and MOORA form a hybrid decision support system that aligns with the mathematical modeling. They allow entrepreneurial researchers and practitioners to formalize subjective judgments, compare strategic alternatives, and support theory - building through repeatable, testable structures.

ABBREVIATIONS

- BWM – Best Worst Method
- MOORA - Multi-Objective Optimization on the basis of Ratio Analysis
- OOI – Outbound Open innovation
- OI – Open Innovation
- MCDM – Multi Criteria Decision Making
- OC – Organization Commitment
- IE – Innovation Ecosystem
- KT – Knowledge Transfer
- TR – Technology Relevance
- OR – Operational Research
- SEM - Structural Equation Modeling
- AHP – Analytic Hierarchy Process
- ANP – Analytic Network Process

1.1 LITERATURE REVIEW

In (1978) and (1989) Dubin and Whetten, robust theoretical model is indistinguishable from a theory itself, reinforcing that mathematical formulation are not only models but also theoretical contributions.

In (2000) Boekema, F., K. Morgan, S. Bakkers, and R. Rutten discussed “Knowledge Innovation, and Economic Growth,” highlighting the critical role of knowledge and innovation in entrepreneurship, which ties into the operational research and mathematical skills needed by entrepreneurs.

In (2002) Levesque and Shephred demonstrated the utility of such models by exploring the curvilinear relationship between competitive rivalry and entry timing, and illustrating how mathematical reasoning can yield insightful findings. In (2001) Minniti and Bygrave explored entrepreneurial learning.

In (2004) Levesque emphasizes the underutilized the role of mathematics in theory development within organizational studies, particularly in the field of entrepreneurship in (1978) Dubin and Whetten reinforces the notion that a model is indistinguishable from a theory when it clearly articulates that what, how, why, and contextual boundaries of a phenomena.

In (2004) Bogenhold explored “Multiple Meaning and Consequences in Entrepreneurship,” offering insights into the various dimensions of entrepreneurship, especially its relationship with innovation and risk - taking, both of which require mathematical modelling and strategic planning

In (2008) Finkle, T. A., and A. Thomas work on entrepreneurship in international markets is relevant for understanding in global dimension of entrepreneurial decision- making, where mathematical analysis become essential in market and financial analysis.

Innovation ecosystem, which refers the broader environment that supports and nurtures innovative activities. In (2013) Nambisan and Baron identified the innovation ecosystem are essential for startup development, and shared knowledge.

Yadav, S.R., and Malik, A.K. (2014) In their book Operations Research, they highlight the mathematical tools and methodologies that can be applied to the decision-making processes of entrepreneurs, especially in cost calculation and optimization.

In (2014) Almirall et al. emphasized the importance of integrated ecosystem that align community and competition for collective growth. In (2022) Cirule and Uvarova viewed these ecosystems as vital for driving technological innovation in startups. In (2017) Olaisen and Revang noted the value creation potential when knowledge is effectively shared.

In (2015) Weiblen and Chesbrough highlighted how the openness of big firms, especially in technology sharing and incubation models, empowers startups by providing access to essential resources.

Technology relevance is frequently cited as a key determinant of innovation success in startup collaborations. In (2015) Weiblen and Chesbrough explained that big firms enable faster market entry for startups through access to advanced technologies. Knowledge transfer emerged as another critical factor, in (2015) Brunswicker and Vanhaverbeke showing how SMEs benefit from external knowledge sourcing.

Openness and commitment as fundamental to fostering innovation across organizational boundaries. In (2018) Bogers et al. emphasized that openness is critical in establishing collaborative innovation frameworks, where mutual trust and knowledge exchange enable in co -creation.

In (2019) Prashantham and Kumar stated the importance of outbound knowledge transfer by multinational companies to enable startups in building disruptive solutions.

In (2020) Barbosa et al. focused on the importance of coordination and communication to enhance innovation performance through knowledge flow.

In (2020) Castillo – Vergara linked technological capacity to overall innovation performance. In (2021) Wang et al. stated that relevant technologies enhance network openness and innovation outcomes. In (2022) Dall – Orsoletta et al. supported these findings by highlighting that the exchange of new technologies helps minimize the resource required for innovation.

In (2022) Fallah also contributed by presenting a model for conceptualizing open innovation ecosystem in startups

In (2022) Borges and silva explained that successful incubation programs rely heavily on the startups ability to absorb knowledge.

In (2023) Giglio et al. further argued that commitment between organizations is a driving force behind cultural transformation and innovation success. In (2019) added that stronger inter – organizational relationships enhance the value of open innovation initiatives.

In (2015) Rezaei introduced the Best Worst Method (BWM) for solving multi criteria decision making problems. And Fuzzy BWM was developed by Guo and Zhao. This method identifies the most and least important criteria for a decision problem. In (2016) this paper advanced by the original BWM by proposing a linear programming model to solve the optimization problem, making it more applicable in practice.

Brauers, W. K. M., and Zavadskas, E. K. (2006) Introduced the MOORA method, which is effective in handling both benefit and cost criteria using simple ratio analysis. It is well-known for its computational simplicity and applicability in engineering, economics, and decision sciences.

1.2 OUTLINE OF THE THESIS

Chapter 1 provides the key concepts essential for understanding the application of mathematical approaches to entrepreneurship and open innovation. It defines fuzzy sets and explaining how they handle uncertainty in decision-making.

Chapter 2 focuses on how mathematical models and analytical methods can be applied to support, analyze, and optimize decisions throughout the entrepreneurial process.

Chapter 3 focuses on the application of the Best Worst Method (BWM), a robust pairwise comparison technique used to derive the weights of key decision criteria influencing outbound open innovation. The chapter explains the step-by-step methodology of BWM, including the selection of criteria, identification of the best and worst elements, and the formulation of a linear programming model to compute optimal weights. It incorporates fuzzy logic to handle the inherent vagueness in expert judgments, thus improving the reliability of the weight assignments. The application is contextualized with real-world data from expert surveys, focusing on four criteria. The results of the BWM analysis identify the relative importance of each criterion, revealing which factors are most critical in fostering successful innovation collaborations.

Chapter 4 introduces the Multi-Objective Optimization on the Basis of Ratio Analysis (MOORA) as a method for ranking alternatives based on the weighted criteria obtained from BWM. This chapter details the process of constructing the decision matrix, normalizing values, and separating beneficial from non-beneficial attributes to compute the final assessment scores. MOORA is applied to prioritize the effectiveness of different innovation strategies or partnership alternatives, offering a clear, data-driven decision-making structure. To enhance the robustness of the evaluation, the study integrates an Ensemble method, combining MOORA with other ranking techniques to produce a more comprehensive prioritization. The comparative analysis of MOORA and Ensemble outcomes provides valuable insights into the effectiveness and reliability of each method, enabling a balanced incorporation of both subjective expert judgments and objective performance data

1.3 BASIC CONCEPTS

This chapter provides an introduction to our study, highlighting the necessity of mathematical modeling.

Definition 1.3.1

Let X be a nonempty set. A fuzzy set A drawn from X is defined as

$$A = \{\langle x, \mu_A(x)\rangle | x \in X\}$$

Where $\mu: X \to [0,1]$ is the membership function of the fuzzy set A.

Definition 1.3.2

The support of Ã is the crisp set (or nonfuzzy set) of all x ∈ X, such that $\mu_{\tilde{A}}(x) > 0$ and is denoted by S (Ã)or Sup (Ã).

Definition 1.3.3

A fuzzy singleton (or fuzzy point) x_α (or $\tilde{x}$) is a fuzzy set whose support is a single point x ∈ X, with membership function:

$$x_\alpha(y) = \begin{cases} \alpha, & if\ x = y \\ 0, & if\ x \neq y \end{cases}$$

Definition 1.3.4

The height of a fuzzy set Ã (denoted by hgt (Ã)) is the supremum of $\mu_{\tilde{A}}(x)$ over all x ∈X. If hgt (Ã) = 1, then Ã is normal, otherwise it is subnormal, and a fuzzy set may be always normalized by defining the scaled membership function:

$$\mu^*_{\tilde{A}}(x) = \frac{\mu_{\tilde{A}}(x)}{Sup\ \mu_{\tilde{A}}(x)}, \forall\, x \in X$$

Definition 1.3.5

The crossover points of a fuzzy set Ã is that point in X, whose grade of membership in Ã is 0.5.

Definition 1.3.6

$\tilde{A} = \tilde{B}$ if and only if, $\mu_{\tilde{A}}(x) = \mu_{\tilde{B}}(x) \forall x \in X$.

Definition 1.3.7

$\tilde{A} \subseteq \tilde{B}$ if and only if $\mu_{\tilde{A}}(x) \leq \mu_{\tilde{B}}(x)$, $\forall x \in X$.

Definition 1.3.8

$\tilde{A}^C$ is the complement of $\tilde{A}$ with membership function $\mu_{\tilde{A}^C}(x) = 1 - \mu_{\tilde{A}}(x)$,$\forall$ x ∈ X.

Definition 1.3.9

The empty fuzzy set $\tilde{\phi}$ and the universal set X, have the membership function $\mu_{\tilde{A}}(x) = 0$ and $\mu_{\tilde{A}}(x) = 1$, respectively, $\forall$ x ∈ X.

Definition 1.3.10

$\tilde{C} = \tilde{A} \cap \tilde{B}$ is a fuzzy set with membership function:

$\mu_{\tilde{C}}(x) = min\,\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \forall\, x \in X$

More generally, for any index set J, then $\bigcap_{j \in J} \tilde{A}_j$ is also a fuzzy set of X with membership function:

$$\mu_{\bigcap_{j \in J} \tilde{A}_j}(x) = \underset{i \in J}{inf}\ \mu_{\tilde{A}_j}(x), \forall\, x \in X$$

Definition 1.3.11

$\widetilde{D} = \tilde{A} \cup \tilde{B}$is a fuzzy set with membership function:

$\mu_{\widetilde{D}}(x) = max\,\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \forall\, x\ \in\ X$

More generally, for any index set J, then $\bigcup_{j\epsilon J} \tilde{A}_j$ is also a fuzzy set of X with membership function:

$$\mu_{\bigcup_{j\epsilon J} \tilde{A}_j}(x) = \underset{i\,\in J}{sup}\ \mu_{\tilde{A}_j}(x), \forall\, x \in X$$

Definition 1.3.12

A fuzzy subset $\tilde{A}$ of $\mathbb{R}$ is said to be convex if:

$$\mu_{\tilde{A}}(\lambda x_1\ +\ (1-\lambda)x_2\,) \geq\ min\,\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}$$

for all $x_1, x_2 \in \mathbb{R}$, and all $\lambda\ \in\ [0,1]$.

Definition 1.3.13

Fuzzy number is a generalization of a regular, real number, it refers to a connected set of possible values, where each possible values has its own weight between 0 and 1. A fuzzy number is thus a special case of convex, normalized fuzzy set of the real line.

Definition 1.3.14

A fuzzy number $\tilde{A}$ = (a, b, c) is called triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a\,; \\ \dfrac{x-a}{b-a}, & a \leq x\ \leq\ b\,; \\ \dfrac{c-x}{c-b}, & b \leq x\ \leq\ c\,; \\ 0, & x > c, \end{cases}$$

Chapter 2

Chapter 2

2.1 MATHEMATICAL APPROCHES TO ENTREPRENEURSHIP

Mathematical approaches to entrepreneurship refer to the use of mathematical models and methods to analyze (study and compare options), support (help with making decisions), and optimize decisions in entrepreneurial processes.

FINANCIAL MODELING VIA MATHEMATICAL FINANCE

Entrepreneurial ventures are inherently financial undertakings. Entrepreneurs continuously face financial decisions regarding investment timing, capital structuring, risk management, and valuation. Traditional entrepreneurial theories often treat finance as a static backdrop, yet modern entrepreneurship demands a dynamic, probabilistic approach to financial modelling.

Mathematical Finance provides powerful tools to represent and analyze such financial decisions under uncertainty, particularly by incorporating techniques from stochastic processes, real options theory, and portfolio management. These tools not only allow us to value entrepreneurial opportunities, but also help in strategically managing risk and formulating investment strategies.

THE CONCEPT OF REAL OPTIONS THEORY

In traditional finance, an option gives the holder the right, but not the obligation, to buy or sell something at a certain price within a specific time. In real option theory to value real world entrepreneurial decision such as

1. Expanding production
2. Abandoning a project
3. Launching a new project

An entrepreneurial opportunity is thus viewed as an option – its value lies inflexibility and the ability to defer or modify decisions based on new information.

Strategic Flexibility

A startup might build a prototype not to launch immediately, but to have the option to scale if market feedback is positive. Entering a partnership or new market can be treated as an option - test first, scale later. They might invest in R&D projects as real options - small investment today, with the possibility to expand later if results are promising. Holding patents or technology licenses also acts like options - they may or may not use them depending on future needs.

Real Option in Business Strategy

In real option theory, where companies treat investment decision like financial options.

Option To Expand: A company opens as mall outlet to test a new market. If it works, they expand.

Option To Abandon: If a new product fails in early trials, the company can pull out with minimal loss.

Option To Delay: Waiting to invest until uncertainty is reduced (e.g., waiting for clearer market trends before launching a product).

Investment Decision – Making Under Uncertainity

Entrepreneurship must decide when and how much to invest in innovation, market entry, or new ventures. Using Real Options Theory, entrepreneurial actions can be framed as financial options - granting the right but not the obligation to act.

Risk Management

Option reduces risk by providing choices. Similarly, entrepreneurs can reduce risk by designing business models with built in flexibility. So, they can pivot, scale or exit when needed.

Mathematical finance provides a robust toolkit for developing entrepreneurship theory that accounts for uncertainty, risk, and dynamic strategic behavior. By integrating real options, valuation techniques, we can create models that better represent entrepreneurial realities, offering valuable insights for both theory and practice.

Monte Carlo Simulation in Entrepreneurship

Monte Carlo Simulation is a computational technique that uses random sampling to model uncertainty in complex systems or calculations. In entrepreneurial finance, it's used to simulate possible future outcomes of a startup’s valuation, cash flows, or risk exposure - especially when inputs are uncertain.

- Startups deal with high uncertainty in revenue, costs, market response, etc.
- Monte Carlo Simulation provides a range of outcomes rather than a single-point estimate.
- It Helps in risk-aware decisions like funding, expansion, pricing, partnerships).

ROLE OF STATISTICS IN ENTREPRENEURSHIP THEORY DEVELOPMENT

Entrepreneurship is a complex and dynamic field, characterized by uncertainty, diversity of behavior, and multifactorial influences. To develop robust and testable theories, researchers increasingly rely on statistics to move from anecdotal evidence to empirical generalizations. Statistics provides the tools to measure, analyze, and validate theoretical constructs, helping convert abstract entrepreneurial concepts into evidence-based theory.

Why statistics matters in Entrepreneurship theory

Entrepreneurship involves question like,

1.What factors influence startup success?

2. How does innovation startup success?

3. What is the relationship between leadership style and entrepreneurial performance?

4. How do ecosystems impact entrepreneurial outcomes?

Answering such questions requires quantitative evidence. Statistics enables us to,

Describe patterns in entrepreneurial data,

Infer causal relationships,

Test hypothesis, and

Predict outcomes.

Inferential Statistics in Entrepreneurship Theory Development

Inferential statistics is fundamental to advancing entrepreneurship theory because it allows researchers to make generalizations from a limited sample of entrepreneurs, ventures, or ecosystems to broader populations. This approach provides the empirical validation needed to support or reject theoretical propositions and hypotheses about entrepreneurial behavior, performance, and systems. In entrepreneurship research, theories are often tested using hypothesis-driven models, where inferential techniques such as regression, analysis of variance, and probability-based testing are applied to determine whether observed patterns are statistically significant or due to random chance.

Descriptive Statistics

It helps in organize data on entrepreneurial phenomena, such as

1. Startup survival rates,
2. Revenue growth,
3. Founder demographics

It helps to frame initial insights and shape theoretical assumption.

Inferential Statistics

Inferential statistics enable generalization from samples to populations. Tools like,

1. t-tests and ANOVA help test differences between groups (e.g., male vs. female entrepreneurs).
2. Regression analysis identifies relationships between variables (e.g., risk-taking and business performance).
3. Structural Equation Modeling (SEM) is widely used to validate complex conceptual models (e.g., linking innovation, networking, and success).

These techniques help operationalize constructs like innovation capability, entrepreneurial orientation, and ecosystem support.

From Data to theory: The Role of Hypothesis Testing

Statistics enables hypothesis-driven research. A typical progression includes,

1. Theory development (e.g., entrepreneurial self-efficacy influences innovation)

2.Operationalization of variables

3. Data collection (e.g., surveys, panel data)

4. Hypothesis testing using statistical techniques

5.Model refinement and theory validation

This cycle of theory data refinement is essential for advancing entrepreneurship as a scientific discipline.

Statistics is a cornerstone of entrepreneurship theory development. It provides the methodological foundation to transform abstract ideas into measurable constructs, test complex relationships, and validate new theoretical frameworks. By grounding theory in data, statistics ensures that entrepreneurship research is not only rigorous but also relevant, replicable, and capable of guiding practice and policy.

OPERATIONAL RESEARCH IN THE DEVELOPMENT OF ENTREPRENEURSHIP THEORY

Entrepreneurship, by nature, involves complex decision-making under uncertainty, resource limitations, and dynamic market conditions. Entrepreneurs face questions such as: What product to launch first? How to allocate limited capital? Which opportunity provides the best trade-off between risk and reward?

Operational Research is the scientific approach to problem-solving and decision-making, primarily through the use of mathematical modelling, statistical analysis, and optimization. In entrepreneurship, it supports rational decision-making across areas such as:

1.Resource Allocation

2.Product Development Planning

3.Risk Analysis

4.Scaling and Operational Efficiency

5.Strategic Decision-Making

Multi Criteria Decision Making

Entrepreneurs face qualitative and quantitative trade – offs. MCDM technique such as

1. Analytic Hierarchy Process
2. Analytic Network Process
3. Best-Worst Method
4. MOORA or TOPSIS

help prioritize options like market entry strategies, funding alternatives, or innovation pathways.

Operational Research as Theoretical Infrastructure

Entrepreneurship theory can be enhanced by framing entrepreneurial behavior as a set of optimization problems. Operational research introduces structure and measurability to otherwise subjective or heuristic entrepreneurial practices.

Entrepreneur as a Decision Optimizer

Under the Operation Research lens, the entrepreneur:

1. Identifies decision variables (e.g., capital allocation, product pricing)
2. Operates under constraints (resources, time, competition)
3. Seeks optimal or near-optimal outcomes (maximizing utility or growth)

The integration of operational research into entrepreneurship not only improves decision-making but also contributes to a more unified and analytical theory of entrepreneurial behavior.

Entrepreneurial Opportunity as a Model

Each opportunity can be modelled as problem - solving space defined by,

1. Objectives (maximize revenue, minimize failure)
2. Decision Variables (investment, hiring, production)
3. Uncertainity (market demand, cost)
4. Constraints (funding, technology, regulation)

GAME THEORY IN THE DEVELOPMENT OF ENTREPRENEURSHIP THEORY

Game theory is a beast approach to model strategic interactions among various agents in entrepreneurship, particularly when making decisions under uncertainty or competition. In the context of your thesis on mathematical approaches to entrepreneurship, integrating game theory can help analyze the dynamics of decision-making between entrepreneurs, investors, competitors, and partners

Competitive Strategies:

- Entrepreneurs often face competitive pressures. Game theory can be used to model strategic behavior between competing firms, focusing on pricing, product development, and market entry strategies.
- Nash Equilibrium can help determine the optimal strategies where no entrepreneur would want to unilaterally change their decision, given the decisions of their competitors.

Nash Equilibrium can be a powerful tool for analyzing strategic decision-making between entrepreneurs, investors, competitors, or partners. Game theory, and particularly Nash Equilibrium, helps model situations where multiple agents (firms, individuals, etc.) interact and make decisions that affect each other's outcomes.

Pricing Strategies in Competitive Markets:

Entrepreneurs in competitive markets often face decisions about pricing their products or services. Nash Equilibrium helps determine the optimal pricing strategy, where no entrepreneur can unilaterally change their price to improve their profit, given the pricing strategies of competitors.

Example: Two startups in the same industry decide on their product pricing. If both startups are competing for market share, they will consider what the other will charge. If both settle on the same price, neither can raise or lower the price without losing customers or reducing their profit. The equilibrium occurs when both firms have no incentive to change their prices.

Product Differentiation and Innovation:

Entrepreneurs often need to decide how much to differentiate their product or innovate to capture market share. If competitors are already offering a similar product, the Nash Equilibrium helps identify the optimal level of differentiation or innovation, where neither firm can improve their position by changing their approach.

Example: Two competing firms in the tech industry must decide whether to introduce a new feature to their product. The Nash Equilibrium would occur when both firms have chosen the level of differentiation that balances the costs of innovation with the expected return, and neither firm can improve their payoff by altering its strategy alone.

Collaboration vs. Competition:

Entrepreneurs often face a dilemma between cooperating with other firms or competing against them. Cooperative strategies might involve joint ventures, R&D collaborations, or co-marketing efforts, while competitive strategies involve trying to outperform others in product quality or market share.

Example: Two firms in the same industry might decide whether to cooperate by sharing technology or to compete by developing their own independent innovations. Nash Equilibrium would show the point where neither firm can improve its outcome by changing its strategy from cooperating to competing or vice versa.

Cooperative Game Theory: When firms collaborate in open innovation, the Nash Equilibrium can show how they can share the benefits (e.g., intellectual property or joint revenues) optimally without one partner exploiting the other.

These applications could help develop a more robust framework for entrepreneurship that factors in strategic behavior, competitive dynamics, and decision-making processes, providing valuable insights into entrepreneurship development and innovation.

CHAPTER 3

Chapter 3

DETERMINING THE MOST IMPORTANT INNOVATION FACTORS WITH BEST WORST METHOD

Entrepreneurship thrives on decision-making, especially in dynamic environments like open innovation partnerships, where firms collaborate to accelerate innovation and share resources. In these partnerships, entrepreneurs must evaluate potential collaborators, assess risks, and make decisions about resource allocation and innovation strategies. These decisions often involve balancing multiple conflicting criteria, such as technological compatibility, market potential, financial stability, and cultural alignment, all under conditions of uncertainty.

The sample for this study was purposefully selected to include experts with substantial experience and knowledge in startup-corporate innovation collaborations. Participants were drawn from diverse yet relevant backgrounds, including academia, startup founders, innovation managers in large enterprises, and professionals specializing in open innovation. In total, 16 respondents contributed to the study, providing valuable insights grounded in both practical experience and domain expertise.

ANALYTICAL TOOLS AND TECHNIQUES EMPLOYED

It employs fuzzy MCDM techniques to evaluate open innovation partnerships:

- Fuzzy Best-Worst Method (BWM): Determines the weights of evaluation criteria by comparing the most and least important factors, using fuzzy logic to capture expert uncertainty.
- Multi Objective Optimization by Ratio Analysis (MOORA) Method: Ranks partnership alternatives based on the weighted criteria, optimizing both beneficial and non-beneficial outcomes.
- Triangular Fuzzy Numbers (TFNs): Convert linguistic expert inputs into quantifiable data for precise analysis under uncertainty.

3.1 FACTORS OF OUTBOUND PROCESS

In outbound open innovation, organization actively shares their knowledge, technology or intellectual properties with external stakeholders to drive innovation and create value.

Four critical factors for effective outbound processes, significantly shape the success of outbound process include,

(1) Organization Commitment
(2) Innovation Ecosystem
(3) Knowledge Flow
(4) Technology Relevance

Each of these plays a vital role in fostering strategic partnerships, enhancing innovation outcomes, and ensuring sustainable entrepreneurial development.

FACTOR STUDY

To understand each four factors, and how it is helping to enable more innovations between big organization and startups.

(1) Organization Commitment

A strong commitment ensures sustained efforts, proper allocation of resources, and a culture that encourages risk-taking and external cooperation—essential for successful innovation partnerships.

(2) Innovation Ecosystem

A vibrant innovation ecosystem enables organizations to access diverse competencies, rapidly test ideas, and scale innovations by leveraging external capabilities and markets.

(3) Knowledge Flow

Effective knowledge transfer mechanisms are essential to ensure that the innovation being shared is understood, utilized, and further developed by external partners, thus maximizing impact.

(4) Technology relevance

High relevance enhances the attractiveness of a technology for external adoption, increasing the potential for successful commercialization and collaborative innovation outcomes.

These four factors - Organizational Commitment, Innovation Ecosystem, Knowledge Transfer, and Technology Relevance - are essential for the success of outbound open innovation. Together, they create a strong foundation for organizations to effectively share knowledge, build strategic partnerships, and drive entrepreneurial growth in a competitive and dynamic environment.

3.2 MULTI CRITERIA DECISION MAKING

The practice of decision making is an integral element of nearly all human activities, whether executing daily to daily actions or any professional task. Although comparatively more straightforward decision has less influence on day-to-day work, some decision with more complexity can have unpleasant effects. Real- world issues are often non - trivial, requiring the search for the best solution after considering several conflicting criteria. In industries, optimal decision not only minimize different risks but also enhance the quality of products. The Multi Criteria Decision Making (MCDM) method provides quality decisions based on assessing the situation, evaluating and ranking the available alternatives, and managing the finite performance criteria for a specified application. It helps in decision makers provide the exact information concerning performance rankings and respective criteria weights.

In this combining Fuzzy Multi Criteria Decision Making (MCDM), Best Worst Method (BWM) and Multi Objective Optimization by Ratio Analysis (MOORA) to support strategic decision in both large organization and startups. The Fuzzy MCDM framework allows for the incorporation of uncertain and imprecise data, while BWM enables the identification of the most critical criteria. MOORA is then used to evaluate and rank alternatives based on multiple criteria. This approach is applied to real - world and demonstrating its effectiveness in addressing complex decision - making problems in diverse organizational settings.

MOORA method is applied to support strategic decision - making for both startups and big organization. The evaluation criteria were weighted using the Best -Worst Method (BWM), an advanced pairwise comparison technique that ensures consistent and reliable derivation of weights from expert judgments.

In this research, BEST WORST METHOD (BWM) and Multi Objective Optimization by Ratio Analysis (MOORA) are employed to evaluate and rank 16 sub-criteria (questions) grouped into four major criteria influencing outbound open innovation (OOI) partnership. These are:

ORGANIZATIONAL COMMITMENT (OC)

OC1: Participation in product design and development

OC2: Provision of training for faster time-to-market

OC3: Support in solving issues with innovative approaches

OC4: Ethical commitment during product development

INNOVATION ECOSYSTEM (IE)

IE1: Organizing knowledge-sharing events

IE2: Access to a product innovation ecosystem through co-working

IE3: Provision of working infrastructure (labs, instruments)

IE4: Access to technology experts

KNOWLEDGE TRANSFER (KT)

KT1: Help in product definition lifecycle

KT2: Facilitation of collaboration opportunities

KT3: Training in new technologies

KT4: Co-working to foster innovative products

TECHNOLOGY RELEVANCE (TR)

TR1: Relevance of technology to market needs

TR2: Enablement of disruptive innovation

TR3: Market traction of the product under development

TR4: Gaining a competitive edge through collaboration.

3.2 BEST WORST METHOD (BWM)

In Strategic decision - making in modern organization whether startups or big organization enterprises require careful evaluation of multiple criteria. In such environments, subjective judgements from experts and decision makers play a crucial role. To capture and structure these judgments effectively, Multi Criteria Decision Making (MCDM) methods are increasingly adopted.

BWM is one of the newest methods for solving multi-criteria decision-making problems. In this method, first, the best (e.g., strongly agree, agree) and the worst criteria (e.g., strongly disagree, disagree) are chosen by the decision-maker. Then pairwise comparisons should be performed between each of these two criteria (best and worst) and the other criteria. For determining the weights of decision-making criteria, a maximum problem is formulated. In order to check the reliability of the comparisons a consistency ratio is proposed for the BWM. In our proposed approach, this method was employed in order to weight the decision-making criteria, considering each one's advantages in comparison with other existing MCDM methods, being the requirement of less comparison data in conjunction with more consistent comparisons and more reliable results.

In real world scenarios, decision makers often use imprecise terms such as "more important" or "less important" instead of exact numerical values. There is a need to incorporate fuzziness to enhance the flexibility and reliability of the decision- making process.

BEST WORST METHODOLOGY

In this part, the steps of BWM for deriving the weights of the criteria are described

STEP 1.

Define a set of criteria

In this step the decision- maker should define a set of criteria ($\{c_1, c_2, c_3, \ldots, c_n\}$) that is used to make a decision about alternatives.

STEP 2.

Define the best and the worst criterion

At this point, we need to make decision to determine the best and the worst criterion regarded to their importance.

STEP 3.

Define the preferences of the best criterion over the other criteria

In this step the decision-maker determines a vector called Best-to-Other (BO) which is as below:

$$A_B = a_{B1}, a_{B2}, a_{B3}, \ldots, a_{Bn}, \tag{1}$$

Where a_{Bj}, is the preference of the best criterion B over the criterion j.

STEP 4.

Define the preferences of all criteria (j) over the worst criterion W

In this step, the decision-maker determines a vector called Other-to-Worst (OW) which is similar to the following vector:

$$A_W = (,a_{1W}, a_{2W}, a_{3W}, \ldots, a_{nW},)^T, \tag{2}$$

Where a_{jW} is the preference of the criterion j over the worst criterion j over the worst criterion W.

STEP 5.

Search for the optimal solution

In this step we must find the optimal weights of the criteria (vector W). It should be mentioned that the optimal weight for the criteria is the one where for each pair of $\frac{W_B}{W_j}$ and $\frac{W_B}{W_j}$, we have $\frac{W_B}{W_j} = a_{Bj}$ and $\frac{W_B}{W_j} = a_{jW}$. .Considering the non-negativity and sum condition for the weights, the following problem results:

$$\text{Min } \varepsilon$$

$$\left|\frac{W_B}{Wj} - a_{Bj}\right| \leq \varepsilon, for\ all\ j$$

$$\left|\frac{W_j}{Ww} - a_{jw}\right| \leq \varepsilon, for\ all\ j$$

$$\Sigma W_j = 1$$

$$W_j \geq 0,, for\ all\ j. \tag{3}$$

TABLE 3.2.1 Linguistic Terms and Fuzzy Numbers

LINGUISTIC TERMS	**FUZZY VALUES**
Strongly agree	(3.5,4,4.5)
Agree	(2.5,3,3.5)
Neutral	(1.5,1,1.5)
Disagree	(0.5,1,1.5)
Strongly disagree	(1,1,1)

TABLE 3.2.2. Evaluation Criteria from Experts

CRITERIA	STRONGLY AGREE	AGREE	NEUTRAL	DISAGREE	STRONGLY DISAGREE
OC1	7	7	2	1	1
OC2	5	4	4	1	2
OC3	4	9	2	0	1
OC4	8	7	4	1	0
IE1	4	7	4	1	0
IE2	5	9	1	0	1
IE3	7	6	3	0	0
IE4	3	7	4	2	0
KT1	1	6	7	1	1
KT2	5	5	5	1	0
KT3	2	9	3	1	1
KT4	6	6	3	1	0
TR1	8	7	1	0	0
TR2	7	7	1	1	0
TR3	2	11	3	0	0
TR4	7	3	5	1	0

TABLE 3.2.3

CRITERIA	LOW	MIDDLE	HIGH
Organization Commitment	2.6562	3.1562	3.6562
Innovation Ecosystem	2.4921	2.9843	3.4765
Knowledge Transfer	2.2656	2.75	3.2343
Technology Relevance	2.6562	3.1562	3.6562

TABLE 3.2.4. Weight for Each Criteria from BWM

CRITERIA	**L**	**M**	**U**	**DEFUZZIFED VALUE**
OC1	2.5937	3.0625	3.5312	3.1562
OC2	2.25	2.6875	3.125	
OC3	2.5312	3	3.4687	
OC4	3.25	3.875	4.5	
IE1	2.375	2.875	3.375	2.9843
IE2	2.6562	3.125	3.5937	
IE3	2.75	3.25	3.75	
IE4	2.1875	2.6875	3.1875	
KT1	1.9062	2.375	2.8437	2.75
KT2	2.375	2.875	3.375	
KT3	2.2187	2.6875	3.1562	
KT4	2.5625	3.0625	3.5625	
TR1	2.9375	3.4375	3.9375	3.1562
TR2	2.75	3.25	3.75	
TR3	2.4375	2.9375	3.4375	
TR4	2.5	3	3.5	
TOTAL				12.0468

TABLE 3.2.5. Comparison Between Technology Relevance to Others

COMPARISON	TECHNOLOGY RELAVANCE VALUE	OTHER VALUES	DIFFERENCE
TR VS OC	3.1562	3.1562	TR = OC
TR VS IE	3.1562	2.9843	TR Slightly > IE
TR VS KT	3.1562	2.75	TR Strongly > KT

FUZZY BEST TO OTHERS

CRITERIA	FUZZY VALUES	LINGUISTIC TERMS
ORGANIZATION COMMITMENT	(1,1,1)	Equal
INNOVATION ECOSYSTEM	(1.5,2,2.5)	Fairly strong
KNOWLEDGE TRANSFER	(2.5,3,3.5)	Very Strong
TECHNOLOGY TRANSFER	(1,1,1)	Equal

TABLE 3.2.6. Comparison Between Others to Knowledge Transfer

COMPARISON	OTHER SCORE	KNOWLEDGE TRANSFER	DIFFERENCE
OC VS KT	3.016	2.75	OC Strongly > KT
IE VS KT	2.9843	2.75	IE Slightly > KT
TR VS KT	3.1562	2.75	TR Strongly > KT
KT VS KT	2.75	2.75	EQUAL

FUZZY OTHERS TO WORST

CRITERIA	FUZZY VALUES	LINGUISTIC TERMS
ORGANIZATION COMMITMENT	(1.5,2,2.5)	Fairly strong
INNOVATION ECOSYSTEM	(1.5,2,2.5)	Fairly strong
KNOWLEDGE TRANSFER	(2.5,3,3.5)	Very Strong
TECHNOLOGY TRANSFER	(1,1,1)	Equal

TABLE 3.2.7. Final Normalized Weights

CRITERIA	WEIGHT
ORGANIZATION COMMITMENT	0.2619
INNOVATION ECOSYSTEM	0.2477
KNOWLEDGE TRANSFER	0.2282
TECHNOLOGY RELEVANCE	0.2619

FIGURE 2.1: BEST WORST METHOD GRAPH

In this, **Technology Relevance** and **Organizational Commitment** emerged as the most critical factors for successful outbound open innovation partnerships. This highlights the importance of aligning technological capabilities with current and future market needs, as well as the necessity of strong institutional support. When startups gain access to relevant technologies and receive consistent backing from established organizations, it creates a foundation for faster product development, innovation scaling, and mutual strategic alignment.

Conversely, **Knowledge Transfer** was consistently ranked as **the least important** factor among the criteria. While it remains a valuable aspect of collaboration, experts perceived it as less directly influential in decision-making compared to technology and commitment. This may suggest that knowledge exchange, although beneficial, is often viewed as a secondary outcome rather than a driving force in forming successful innovation partnerships.

CHAPTER 4

CHAPTER 4

4.1 MULTI - OBJECTIVE OPTIMIZATION ON THE BASIS OF RATIO ANALYSIS (MOORA)

In complex decision - making environments, especially those involving multiple conflicting criteria, traditional decision approaches often fall short in providing objective and justifiable outcomes. This challenge has led to the widespread adoption of Multi Criteria Decision Making (MCDM) methods, which support structured and rational analysis of alternatives across diverse fields.

Among the many MCDM techniques, the Multi Objective Optimization on the Basis of Ratio Analysis (MOORA) method has emerged as a reliable and efficient tool for decision analysis. MOORA is known for its simplicity, ease of computation, and capability to deliver consistent results even in complex multi criteria scenarios. It assists decision makers in evaluating alternatives by considering both beneficial and non – beneficial criteria through a process of normalization, weighting and aggregation. MOORA method to analyze and prioritize alternatives based on multiple decision criteria. The approach involves the normalization of the decision matrix, application of criteria weights derived from expert input and overall performance index for each alternative. The outcome offers a ranked list of alternatives, enabling stakeholders to make transparent decisions.

By combining the strength of BWM in weighting criteria with the ranking efficiency of MOORA. This provides valuable insight that can guide both emerging startups and established organization in making informed strategic choices aligned with long term innovation and growth.

MOORA METHODOLOGY

In this part, MOORA method starts with a decision matrix showing the performance of different alternatives with respect to various attributes.

STEP 1.

The first step is to determine the objective, and to identify the pertinent evaluation attributes.

STEP 2.

The next step is to represent all the information available for the attributes in the form of a decision matrix. The data given in eq. (1) are represented as matrix X_{mxn}. Where x_{ij} is the performance measure of i^{th} alternative on j^{th} attribute, m is the number of alternatives, and n is the number of attributes. Then the ratio system is developed in which each performance of an alternative on an attribute is compared to a denominator which is a representative for all the alternatives concerning that attribute.

$$x = \begin{bmatrix} x_{11} & x_{12} & . & x_{1n} \\ x_{21} & x_{21} & . & x_{2n} \\ . & . & . & . \\ x_{m1} & x_{m2} & . & x_{mn} \end{bmatrix} \tag{1}$$

STEP 3.

Brauers et al. (2008) concluded that for this denominator, the best choice is the square root of the sum of squares of each alternative per attribute. This ratio can be expressed as

$$x_{ij}^{*} = x_{ij} \Big/ \sqrt{\left[\sum_{i=1}^{m} x_{ij}^{2}\right]}$$

j = (1, 2, … ,n)

(2)

Where x_{ij} is a dimensionless number which belongs to the interval [0, 1] representing the normalized performance of i^{th} alternative on j^{th} attribute.

STEP 4.

For multi – objective optimization, these normalized performances are added in case of maximization (for beneficial attributes) and subtracted in case of minimization (for non - beneficial attributes). Then the optimization problem becomes,

$$y_i = \sum\nolimits_{j=1}^{g} x_{ij}^{*} - \sum\nolimits_{j=g+1}^{n} x_{ij}^{*}$$

(3)

Where g is the number of attributes to be maximized, (n – g) is the number of attributes to be minimized, and y_i is the normalized assessment value of i^{th} alternative with respect to all the attributes. In some cases, it is often observed that some attributes are more important than the others. In order to give more importance to an attribute, it could be multiplied with its corresponding weight (significance co-efficient). When these attributes weights are taken into consideration, Eq. 3 becomes as follows,

$$y_i = \sum_{j=1}^{g} w_j \, x_{ij}^{*} - \sum_{j=g+1}^{n} w_j \, x_{ij}^{*}$$

$$(j = 1, 2, \ldots, n) \tag{4}$$

Where w_j is the weight of the j^{th} attribute, which can be determined applying Best Worst method.

STEP 5.

The y_i value can be positive or negative depending of the totals of its maxima (beneficial attributes) and minima (non - beneficial attributes) in the decision matrix. An ordinal ranking of y_i shows the final preference. Thus, the best alternative has the highest y_i value, while the worst alternative has the lowest y_i value.

Table 4.1.1. Main Criteria Evaluation by the Experts

CRITERIA	STRONGLY AGREE	AGREE	NEUTRAL	DISAGREE	STRONGLY DISAGREE
ORGANIZATION COMMITMENT	24	25	9	2	4
INNOVATION ECOSYSTEM	19	29	12	3	1
KNOWLEDGE TRANSFER	14	26	18	4	2
TECHNOLOGY RELEVANCE	24	28	10	2	0

Table 4.1.2 Normalized Decision Matrix x_{ij}

CRITERIA	STRONGLY AGREE	AGREE	NEUTRAL	DISAGREE	STRONGLY DISAGREE
ORGANIZATION COMMITMENT	0.5805	0.4621	0.3532	0.3481	0.8729
INNOVATION ECOSYSTEM	0.4590	0.5361	0.4710	0.5222	0.2182
KNOWLEDGE TRANSFER	0.3386	0.4806	0.7065	0.6963	0.4364
TECHNOLOGY RELEVANCE	0.5805	0.5176	0.3925	0.3481	0

TABLE 4.1.3. Estimation Of y_i Value, For Beneficial and Non - Beneficial Attributes

CRITERIA	MOORA VALUE
ORGANIZATION COMMITMENT	0.1748
INNOVATION ECOSYSTEM	0.7262
KNOWLEDGE TRANSFER	0.3930
TECHNOLOGY RELEVANCE	1.1425

TABLE 4.1.4. Weight Obtained from Best Worst Method

CRITERIA	WEIGHT
ORGANIZATION COMMITMENT	0.2619
INNOVATION ECOSYSTEM	0.2477
KNOWLEDGE TRANSFER	0.2282
TECHNOLOGY RELEVANCE	0.2619

TABLE 4.1.5 Weight Estimation of Assessment Value (y_i)

CRITERIA	MOORA VALUE	BWM WEIGHT	FINAL VALUE
ORGANIZATION COMMITMENT	0.1748	0.2619	0.0458
INNOVATION ECOSYSTEM	0.7262	0.2477	0.1799
KNOWLEDGE TRANSFER	0.3930	0.2282	0.0897
TECHNOLOGY RELEVANCE	1.1425	0.2619	0.2993

TABLE 4.1.6. Final Ranking

RANK	CRITERIA	FINAL VALUE
I.	TECHNOLOGY RELEVANCE	0.2993
II.	INNOVATION ECOSYSTEM	0.1799
III.	KNOWLEDGE TRANSFER	0.0897
IV.	ORGANIZATION COMMITMENT	0.0458

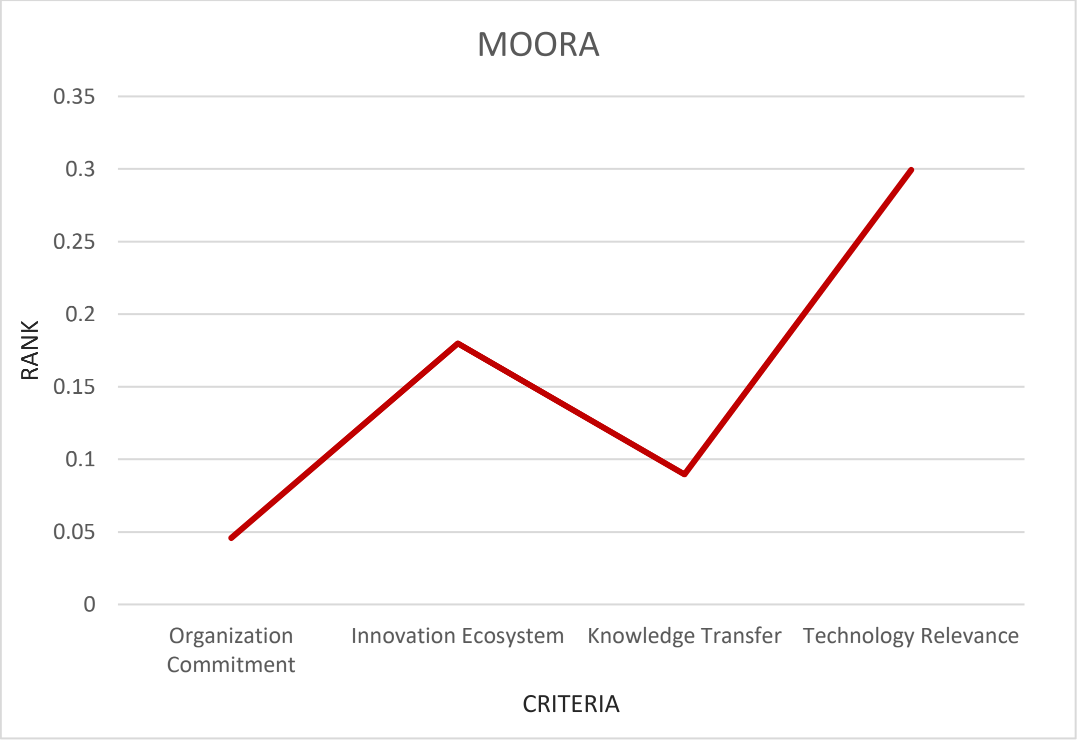

FIGURE 3.1: MOORA GRAPH

Based on the results of the MOORA evaluation, **Technology Relevance** emerged as the **most critical factor** in strategic decision-making within outbound open innovation partnerships. This emphasizes that organizations, particularly startups, prioritize technologies that are aligned with current market demands and have the potential to drive disruptive innovation. Access to relevant technology not only enhances the competitiveness of startups but also accelerates time-to-market and strengthens the overall impact of collaborative efforts. This reinforces the idea that technological fit plays a central role in determining the success and sustainability of innovation initiatives.

Meanwhile, Innovation Ecosystem and Knowledge Transfer were also considered important, but to a slightly lesser degree. These factors support collaboration by fostering environments where shared knowledge and innovation can thrive. However, Organizational Commitment was ranked as the least influential criterion in the MOORA analysis. This suggests that while commitment from large organizations is necessary, it may not be perceived as the main driver of successful partnerships in comparison to more tangible assets like technology and infrastructure. The findings highlight a practical focus on resources and capabilities over relational or ethical aspects in strategic decision-making processes.

Comparative Study OF MCDM Techniques

This study builds upon existing research that measured the effectiveness of outbound open innovation between large organizations and startups using fuzzy multi-criteria decision-making (MCDM) approaches. While prior studies have offered valuable insights using various fuzzy MCDM techniques, this research contributes a novel perspective by applying the Best-Worst Method (BWM) and MOORA to the same dataset. This dual-method approach enhances the robustness and reliability of the evaluation, offering a more structured and prioritized decision-making framework.

The comparison among the FUZZY AHP, FUZZY DEMATEL, FUZZY AND, BEST WORST METHOD MOORA is illustrated.

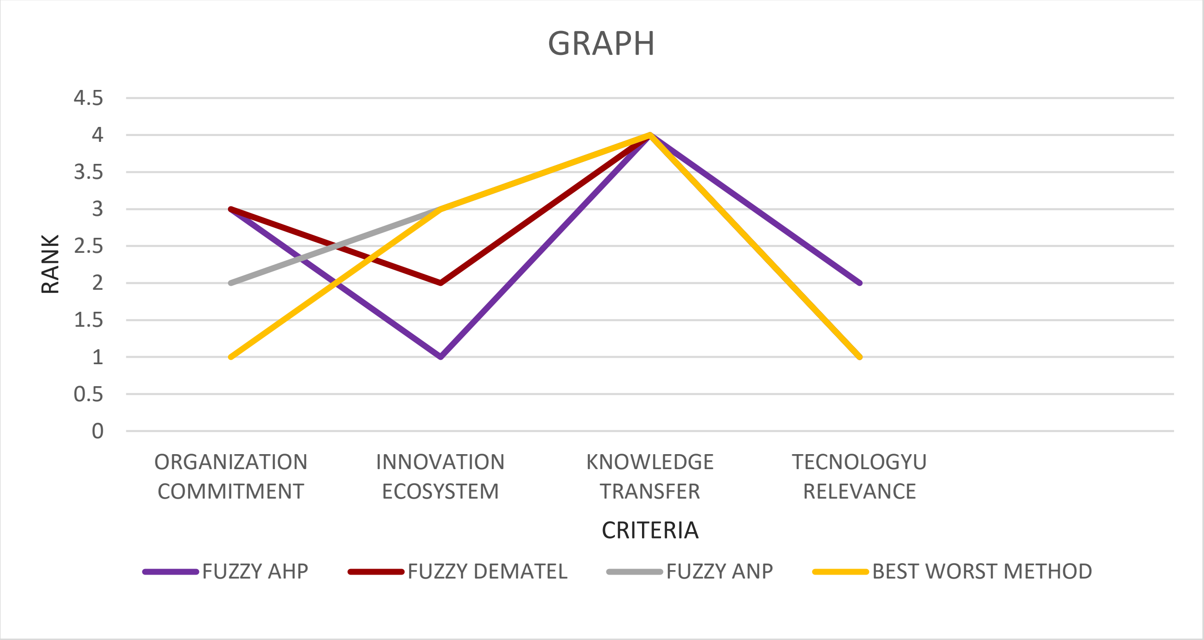

FIGURE 3.3: COMPARISION OF RESULTS AMONG F- AHP, F- DEMATEL, F- ANP, BEST WORST METHOD MOORA.

Table 4.1.7. COMPARISON TABLE

CRITERIA	FUZZY AHP	FUZZY DEMATEL	FUZZY ANP	BEST WORST METHOD
ORGANIZATION COMMITMENT	3	3	2	1
INNOVATION ECOSYSTEM	1	2	3	3
KNOWLEDGE TRANSFER	4	4	4	4
TECHNOLOGY RELEVANCE	2	1	1	1

The comparative analysis of five MCDM methods FUZZY AHP, FUZZY DEMATEL, FUZZY ANP, and BEST WORST METHOD provides diverse perspectives on the effectiveness of the outbound open innovation between established organization and startups.

Organizational Commitment consistently ranks high across all methods, and reaching top rank in BEST WORST METHOD, and maintaining a mid to high position in other techniques.

Innovation Ecosystem ranks highest in FUZZY AHP, but it ranks lowest in BEST WORST METHOD. This indicates that while innovation is crucial, it perceived importance can be method sensitive.

Knowledge Transfer was consistently ranked lowest or near lowest across the majority methods. Despite its theoretical importance in the entrepreneurial ecosystem, it appears to be perceived as less impactful in practice based on the evaluation results.

Technology Relevance consistently achieved the top rank across all methods except FUZZY AHP, where it ranked second. This suggests a broad consensus regarding the critical importance of aligning technology with entrepreneurial goals.

Cross Method Ranking Analysis of Key Innovation Factors

Table 4.1.7. COMPARISON BETWEEN ENSEMBLE METHOD AND MOORA

CRITERIA	ENSEMBLE	MOORA
ORGANIZATION COMMITMENT	3	4
INNOVATION ECOSYSTEM	2	2
KNOWLEDGE TRANSFER	4	3
TECHNOLOGY RELEVANCE	1	1

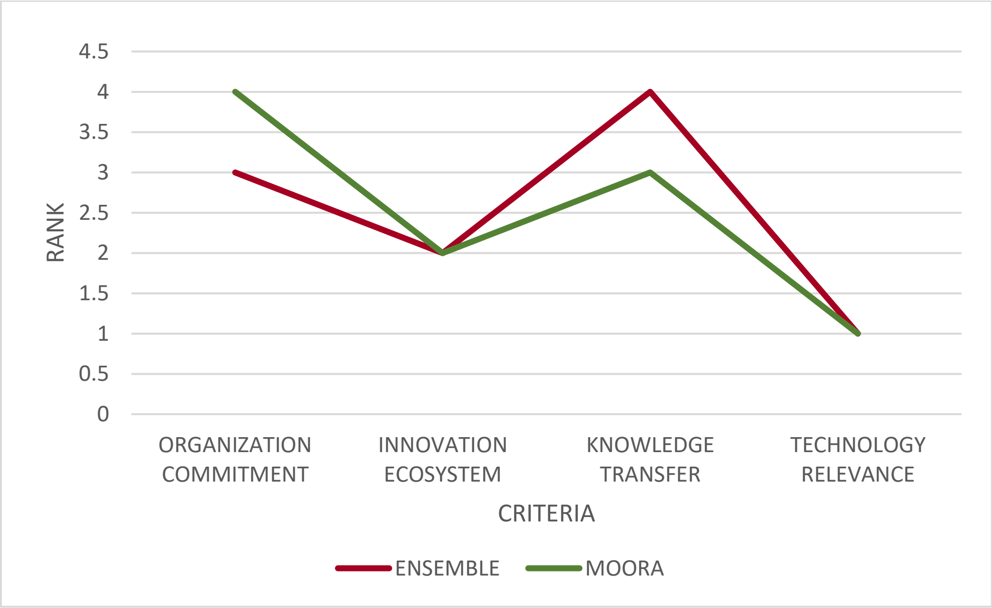

FIGURE 3.4: COMPARISON BETWEEN ENSEMBLE METHOD AND MOORA

The table compares the overall rankings of four criteria using two methods: Ensemble and MOORA.

The results indicate that Technology Relevance is the most critical factor across both methods, consistently ranking first due to its significant impact on the success of innovation initiatives. Innovation Ecosystem holds the second position in both Ensemble and MOORA rankings, underscoring the importance of fostering an environment conducive to collaborative innovation.

While Organization Commitment and Knowledge Transfer show slight variations in their rankings across the two methods, they remain essential contributors to achieving effective innovation outcomes. Organization Commitment is ranked third using the Ensemble approach, reflecting its strategic importance in aligning internal resources toward innovation goals. Conversely, Knowledge Transfer attains a higher ranking in the MOORA method, emphasizing its role in facilitating the dissemination of knowledge across organizational boundaries.

The alignment of results across both methodologies reinforces the robustness of the analysis, ensuring a comprehensive evaluation of the criteria influencing open innovation partnership. These findings provide valuable insights for decision-makers aiming to prioritize resources and strategies for enhancing organizational innovation performance.

CONCLUSION

SUMMARY AND CONCLUSION

This research provides the application of mathematical theory approaches to entrepreneurship, specifically within the context of open innovation partnerships. Mathematical model provides a structured and objective framework for assessing complex, multi-dimensional criteria that influence partnership success.

By fuzzy Multi-Criteria Decision-Making (MCDM) techniques such as Best-Worst Method (BWM) and Multi-Objective Optimization on the basis of Ratio Analysis (MOORA), have been successfully prioritized critical criteria that influence the success of open innovation. The use of expert-derived data for factors such as Organization Commitment, Innovation Ecosystem, Technology Relevance, and Knowledge Transfer has enabled a robust and nuanced analysis of the key elements that drive innovation outcomes.

The integration of BWM and MOORA methods has proven highly effective in identifying the relative importance of these criteria. Particularly, Technology Relevance and Innovation Ecosystem emerged as central factors, consistently ranking at the top across both methods. This emphasizes their pivotal role in fostering innovation and highlights their strategic importance in open innovation partnerships. On the other hand, minor variations observed in the rankings of Organization Commitment and Knowledge Transfer suggest that these factors, while still important, may exhibit dynamic roles that vary depending on the specific context of the partnership.

The comparison between the Ensemble and MOORA methods reveals important insights into the robustness and sensitivity of multi-criteria decision-making approaches. While both methods generally agree on the top priorities - notably ranking Technology Relevance and Innovation Ecosystem consistently - slight differences in the rankings of Organization Commitment and Knowledge Transfer reflect their methodological distinctions. The Ensemble method offers greater stability and generalization, making it suitable for complex decision-making environments, whereas MOORA offers clarity and precision, making it effective for straightforward optimization problems. Together, their comparison strengthens the validity of the final prioritization and enhances confidence in strategic decision - making.

In conclusion, this research demonstrates that the application of mathematical theory, particularly through fuzzy MCDM techniques and ensemble methods, is highly beneficial for guiding entrepreneurship in open innovation partnerships. The approach not only enhances the reliability and accuracy of decision-making but also supports more strategic, informed, and adaptable innovation processes.

REFERENCE

REFERENCE

1) Almirall, E., Lee, M. and Majchrzak, A. (2014), “Open innovation requires integrated competition community ecosystems: lessons learned from civic open innovation”, Business Horizons, Vol. 57 No. 3, pp. 391 - 400.

2) Audretsch, D.B., Belitski, M., Caiazza, R. and Siegel, D. (2023), “Effects of open innovation in startups: theory and evidence”, Technological Forecasting and Social Change, Vol. 194, 122694.

3) Bacon, E., Williams, M.D. and Davies, G.H. (2019), “Recipes for success: conditions for knowledge transfer across open innovation ecosystems”, International Journal of Information Management, Vol. 49, pp. 377 - 388.

4) Boekema, F., K. Morgan, S. Bakkers, and R. Rutten (eds) (2000). Knowledge, Innovation and Economic Growth. Cheltenham: Edward Elgar.

5) Bogenhold, D (2004). Entrepreneurship: Multiple Meaning and Consequences, International Journal of Entrepreneurship and Innovation Management, 4 (1): 3 - 10.

6) Brauers, W. K. M., & Zavadskas, E. K. (2006). The MOORA method and its application to privatization in a transition economy. Control and Cybernetics, 35(2), 445 - 46

7) Brauers, W.K.M., and Zavadskas, E. K., (2009). Robustness of the multi objective MOORA method with a test for the facilities sector, Technological and Economic Development of Economy: Baltic Journal on Sustainability, 15(2), pp 352 - 375.

8) Bull, I., Willard, G.E., 1993. Towards a theory of entrepreneurship. J. Bus. Venturing 8, 183c-195.

9) Bygrave, W.D., 1993. Theory building in the entrepreneurship paradigm. J. Bus. Venturing 8, 255 - 280.

10) Campbell, C.A., 1992. A decision theory model for entrepreneurial acts. Entrepreneurship. Theory Practice 17 (1), 21 - 27.

11) Carrillo, J.E., Gaimon, C., 2000. Improving manufacturing performance through process change and knowledge creation. Manage. Sci. 46 (2), 265 - 288.

12) Cenamor, J. and Frishammar, J. (2021), "Openness in platform ecosystems: innovation strategies for complementary products", Research Policy, Vol. 50 No. 1, 104148.

13) Cirule, I. and Uvarova, I. (2022), "Open innovation and determinants of technology-driven sustainable value creation in incubated startups", Journal of Open Innovation: Technology, Market, and Complexity, Vol. 8 No. 3, p. 162.

14) Cooper, A.C., 1993. Challenges in predicting new firm performance. J. Bus. Venturing 8, 241–253.

15) Corvello, V., Felicetti, A.M., Steiber, A. and Al ange, S. (2023), "Startup collaboration units as knowledge brokers in Corporate Innovation Ecosystems: a study in the automotive industry", Journal of Innovation and Knowledge, Vol. 8 No. 1, 100303.

16) Cressy, R., 1992. The theory of the opportunistic entrepreneur. Small Bus. Econ. 4 (4), 267-271.

17) De Groote, J.K. and Backmann, J. (2019), "Initiating open innovation collaborations between incumbents and startups: how can David and Goliath get along?", International Journal of Innovation Management, Vol. 24 No. 2, 2050011.

18) Dubin, R., 1978. Theory Development. Free Press, New York, NY.

19) Gifford, S., 1992a. Innovation, firm size and growth in a centralized organization. Rand J. Econ. 23, 284 - 298.

20) Gifford, S., 1997. On the relationship between the venture capitalists and the entrepreneur. J. Bus. Venturing 12, 459 - 482.

21) Giglio, C., Corvello, V., Coniglio, I.M., Kraus, S. and Gast, J. (2023), "Cooperation between large companies and startups: an overview of the current state of research", European Management Journal. doi: 10.1016/J.EMJ.2023.08.002.

22) Guo, S., & Zhao, H. (2017). Fuzzy best-worst multi-criteria decision-making method and its applications. Knowledge-Based Systems, 121, 23-31.

23) Malik, A.K., Yadav, S.K. and Yadav, S.R. (2012) Optimization Techniques, I. K International Pub. Pvt. Ltd., New Delhi.

24) Mi, X., Tang, M., Liao, H., Shen, W., & Lev, B. (2019). The state-of-the-art survey on integrations and applications of the Best Worst Method (BWM). European Journal of Operational Research, 278(1), 1–20.

25) Nemhauser, G.L., Rinnooy Kan, A.H.G., Todd, M.J. (Eds.), 1994. Optimization. Handbooks in Operations Research and Management Science, vol. 1. North-Holland, Amsterdam.

26) Quaiser, R.M and Srivastava, P.R (2024) "Outbound Open innovation effectiveness measurement between big organization and Startups using Fuzzy MCDM", Management Decision.

27) Razaei, J (2015) Best Worst multi criteria decision making method. Omega, 53 (1), 49 -57.

28) Rezaei, J. (2016). Best-worst multi-criteria decision-making method: Some properties and a linear model. Omega, 64, 126 - 130.

29) Simon, H.A., 1947. Administrative Behavior: A Study of Decision-Making Processes in Administrative Organization. Macmillan, New York, NY.

30) W.K.M. Brauers, Multi-objective contractor's ranking by applying the MOORA method, J. Bus. Econ. Manag. (4) (2008) 245 - 255.

31) West III, G.P., 1997. Frameworks for research and theory development in entrepreneurship. In: Dosier, L.N., Keys, J.B. (Eds.), Academy of Management Best Papers Proceedings. Georgia Southern University, States boro, GA, pp. 113 - 117.

32) Yadav, S.R. and Malik, A.K. (2014). Operations Research, Oxford University Press, New Delhi.