

# *INTRODUCTION*

## INTRODUCTION

Fuzzy logic is the cocaine of science.

-Professor William Kahan, UC BERKELEY.

The whole notion of fuzzy set theory is due to Lotfi Zadeh [45]. His classical paper in 1965 has opened up new insights and applications in a wide range of scientific areas. After the discover of fuzzy subsets, much attention has been paid to generalize the basic concept of classical theory in fuzzy setting and thus a model theory of fuzzy topology is developed. The notion of fuzzy subsets naturally plays a very significant role in the study of fuzzy topology, which was introduced by Chang [8] in 1968. Later, Lowen [34] redefined it which is now known as a stratified fuzzy topology.

There are several non-classical and higher order fuzzy sets all having very good application potential in the area of computer science. One of the interesting generalizations of the theory of fuzzy sets is the theory of intuitionistic fuzzy sets (IFS) introduced by Atanassov [1,2]. An IFS  $A$  for a given universal set  $X$  is defined by a degree of membership  $\mu_A(x)$  and a degree of non-membership  $\nu_A(x)$  for each  $x \in X$ , as follows:

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$$

where  $\mu_A(x) : X \rightarrow [0, 1]$ ;  $\nu_A(x) : X \rightarrow [0, 1]$  and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$ .

IFS are more useful in representing human way of thinking than ordinary fuzzy sets, mainly due to the condition that  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ . This approach supports a further step in modelling uncertainty because the addition of  $\mu_A(x)$  (degree of membership) and  $\nu_A(x)$  (degree of non-membership) does not necessarily have to be 1, which is an implicit necessity in ordinary fuzzy sets. IFS introduce a technique in which two separate opinions, supporting or refuting (more or less opposite of each

other), of a certain subject can be represented independently. This is particularly useful when people or systems may lack certainly and exact information about a subject. Any missing declaration represents an uncertainty in the information to be processed.

In studying fuzzy topological vector spaces, Katsaras [27] in 1984, first introduced the notion of fuzzy norm on a linear space. After the introduction of fuzzy topological vector space many authors Felbin [18], Kaleva [26], Cheng and Mordasen [9], Kramosil and Michelek [30], Bag and Samanta [4] gave the idea of fuzzy norm. Bag and Samanta [3] introduced  $\alpha$ -norms on a linear space corresponding to the fuzzy norm on a linear space.

Gunawan and Mashadi [22] introduced the notion of n-normed space. As a generalization of n-normed space Narayanan and Vijayabalaji [38] introduced the concept of fuzzy n-normed linear space and established a sufficient condition for an fuzzy n-normed linear space to be complete. They also introduced the notion of  $\alpha$ -n-norm on a linear space corresponding to the fuzzy n-norm on a linear space. Based on Elumalai et al. [16] and Elumalai and Souruparani [15], Narayanan and Vijayabalaji [38] introduced the notion of best approximation sets in  $\alpha$ -n-norms.

El-Hamouly [14] introduced the concept of fuzzy inner product space. Vijayabalaji and Thillaigovindan [43] introduced the notion of fuzzy n-inner product space as a generalization of n-inner product space due to Cho, Matric and Pecaric [10]. They generalized fuzzy n-normed linear space due to Narayanan and Vijayabalaji [38] fuzzy quasi n-normed linear space.

Vijayabalaji, Thillaigovindan and Young Bae Jun [44] introduced the notion of intuitionistic fuzzy n-normed linear space (i-f-n-NLS) as a generalization of fuzzy n-normed linear space and introduced Cauchy sequence and convergent sequence in i-f-n-NLS. They also introduced generalized cartesian product of the i-f-n-NLS.

Following the definition of i-f-n-norm [44], Samanta [41] introduced the concept of Intuitionistic fuzzy norm (in short IFN) over a linear space. He established a sufficient condition for an Intuitionistic fuzzy normed linear space to be complete and also proved that a finite dimensional Intuitionistic fuzzy normed linear space is complete.

In this thesis we have concentrated our attention to the study of Intuitionistic fuzzy n-normed linear space and Intuitionistic fuzzy n-inner product space. The articles chosen for our study are as follows:

- i) Fuzzy topological spaces by Chang [8].
- ii) Intuitionistic Fuzzy sets by Atanassov [1].
- iii) An introduction to intuitionistic Fuzzy topological spaces by Dogan Coker [13].
- iv) Fuzzy n-Normed linear space by Narayanan and Vijayabalaji [38].
- v) Complete Fuzzy n-normed linear space by Vijiyabalaji and Thillaigovindan [42].
- vi) Intuitionistic fuzzy n-normed linear space by Vijiyabalaji, Thillaigovindan and Young Bae Jen [44].
- vii) Fuzzy n-inner product space by Vijiyabalaji and Thillaigovindan [43].

Chapter 1 is devoted to discuss the articles "Fuzzy topological spaces" by Chang [8], "Intuitionistic fuzzy sets" by Atanassov [1] and "An introduction to intuitionistic fuzzy topological spaces" by Dogan Coker [13].

In section one of the chapter 1, preliminary definitions and results on fuzzy sets and fuzzy topological spaces that are needed for our discussion are collected.

Section two of the chapter 1 deals with the basic concept of intuitionistic fuzzy sets and various operations and relations defined over intuitionistic fuzzy sets.

Section three of chapter 1 deals with the basic concept of the image and pre-image of IFS'S and intuitionistic fuzzy topological spaces.

Chapter 2 is devoted to study the following three articles

- i) Fuzzy n-normed linear space by Narayanan and Vijayabalaji [38].
- ii) Complete Fuzzy n-normed linear space by Vijayabalaji and Thillaigovindan [42].
- iii) Fuzzy n-inner product space by Vijayabalaji and Thillaigovindan [43].

In section one of the chapter 2, preliminary definitions and results on fuzzy n-normed linear space are studied.

In section two of chapter 2, we analyse the good definition of complete fuzzy n-normed linear space by Vijayabalaji and Thillaigovindan [42].

Few interesting results discussed here are as follows:

Let  $(X, \|\bullet, \bullet, \dots, \bullet\|)$  be an n-normed linear space and  $(X, N)$  be the corresponding fuzzy n-normed linear space.

- i) In  $(X, N)$  every convergent sequence is a Cauchy sequence.
- ii)  $\{x_n\}$  is a Cauchy sequence in  $(X, \|\bullet, \bullet, \dots, \bullet\|)$  iff  $\{x_n\}$  is a Cauchy sequence in  $(X, N)$ .
- iii)  $\{x_n\}$  is a convergent sequence in  $(X, \|\bullet, \bullet, \dots, \bullet\|)$  iff  $\{x_n\}$  is a convergent sequence in  $(X, N)$ .
- iv) A fuzzy n-normed linear space  $(X, N)$  in which every Cauchy sequence has a convergent subsequence is complete.

Section three of chapter 2, is devoted to the study of best approximation sets in  $\alpha$ -n-normed space and its properties.

In section four of chapter 2, we study an ascending family of quasi  $\alpha$ -n-norms corresponding to the fuzzy quasi n-norm and its properties.

Chapter 3 is devoted to study the article "Intuitionistic fuzzy n-normed linear space" by Vijayabalaji and Thillaigovindan [44].

In section one of chapter 3, the definitions of convergence sequence, Cauchy sequence and completeness extended to intuitionistic fuzzy n-normed linear space are

studied. Results which are discussed in the section two of chapter 2 in fuzzy  $n$ -normed linear space extended to intuitionistic fuzzy  $n$ -normed linear space are studied.

In section two of chapter 3, the concept of Generalized Cartesian product of the intuitionistic fuzzy  $n$ -normed linear space is analysed. Some of the interesting properties of intuitionistic fuzzy  $n$ -normed linear space are as follows:

- i) The Generalized Cartesian product of two intuitionistic fuzzy  $n$ -normed linear spaces is an intuitionistic  $n$ -normed linear space.
- ii) The Generalized Cartesian product of intuitionistic fuzzy  $n$ -normed linear spaces is commutative.
- iii) The Generalized Cartesian product of the intuitionistic fuzzy  $n$ -normed linear spaces is distributive with respect to union, intersection and difference.

In chapter 4 we have generalized fuzzy  $n$ -inner product space due to Vijayabalaji and Thillaigovindan [43] to intuitionistic fuzzy sets.

Section one of chapter 4 we study  $n$ -inner product space and fuzzy  $n$ -inner product space.

In section two of chapter 4 the notion of intuitionistic fuzzy  $n$ -inner product space is discussed.