Chapter 3

λ_g^α-Continuous Maps and λ_g^α-Irresolute Maps in Topological Spaces

3.1 Introduction

The major perception in the study of general topology is done with the help of continuity and irresoluteness i.e., using the inverse images of mappings in topological spaces. The examination of inverse image serves as a handy tool to analyze the respective closed sets and open sets productively. This work was formally initiated by Levine [1960] by naming it as strongly continuous maps and in a similar direction many researchers like Arya and Gupta [1974], Jain [1980], Noiri [1984] respectively introduced weaker forms of Levine's strongly continuous maps namely completely continuous maps, totally continuous maps, perfectly continuous maps in topological spaces. Later, Mashhour et al. [1983] and Balachandran et al. [1991] characterized α-continuous maps and g-continuous maps in topological spaces. Devi et al. [1997] formulated αg-continuous and $g\alpha$-continuous maps in topological spaces and examined several fundamental properties and theorems. Further, on the introduction of λ-closed sets, λ-continuity was introduced and explored by Francisco G. Arenas et al. [1997]. Crossley and Hildebrand [1972] paved the path to work on the interesting concept of irresoluteness. Sundaram [1991], Devi et al. [1997] and many other researchers have also given many generalizations of irresolute maps. Further, Caldas et al. [2007 b] defined λ-irresolute maps and analyzed its properties.

This chapter is dedicated to define λ_g^α-continuity and λ_g^α-irresoluteness in topological spaces with the familiarity of the already existing mappings. The definitions of λ_g^α-continuous maps, λ_g^α-irresolute maps, quasi λ_g^α-continuous maps, perfectly λ_g^α-continuous maps, totally λ_g^α-continuous maps and strongly λ_g^α-continuous maps are proposed with accurate examples. Dependency and independency relationships between the newly defined maps and the previously existing maps are derived subsequently. Counter examples are provided to ascertain the independencies. Vital properties and theorems associated with the defined maps are also derived. Finally, compositions of the defined mappings are analyzed.

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3.2 λ_g^α-Continuous Maps

In this section, λ_g^α-continuous maps have been defined, their interrelationships are derived and their fundamental properties are analyzed. With the results obtained, we procure that λ_g^α-continuity is weaker than λ-continuity and stronger than $g\Lambda$-continuity.

Definition 3.2.1 A map $u: (M, \mu) \rightarrow (N, \nu)$ is called a **λ_g^α-continuous map** if the inverse image of every closed set in (N, ν) is λ_g^α-closed in (M, μ), i.e., if $u^{-1}(T)$ is a λ_g^α-closed set in (M, μ) for every closed set T in (N, ν).

Example 3.2.2 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, \{j\}, \{i, j\}, N\}$. Let $u: (M, \mu) \rightarrow (N, \nu)$ be a map defined by $u(i) = i, u(j) = k$ and $u(k) = j$. Then u is a λ_g^α-continuous map, since the inverse image of every closed set in (N, ν) is λ_g^α-closed in (M, μ).

Theorem 3.2.3 A map $u: (M, \mu) \rightarrow (N, \nu)$ is λ_g^α-continuous if and only if $u^{-1}(T)$ is a λ_g^α-open set in (M, μ) for each open set T in (N, ν).

Proof: (**Necessity)** Let T be an open set in (N, ν). Then $N \setminus T$ is closed in (N, ν). Since u is λ_g^α-continuous, $u^{-1}(N \setminus T) = M \setminus u^{-1}(T)$ is λ_g^α-closed in (M, μ) implies $u^{-1}(T)$ is λ_g^α-open in (M, μ).

Sufficiency: Suppose that $u^{-1}(T)$ is λ_g^α-open in (M, μ) for each open set T in (N, ν). Let S be a closed set in (N, ν). Then $N \setminus S$ is open in (N, ν). Since $u^{-1}(N \setminus S) = M \setminus u^{-1}(S)$ is

λ_g^α-open in (M,μ) which implies that $u^{-1}(S)$ is λ_g^α-closed in (M,μ). Hence u is a λ_g^α-continuous map.

Proposition 3.2.4

(i) Every λ-continuous map $u:(M,\mu)\to(N,\nu)$ is a λ_g^α-continuous map, but not conversely.

(ii) Every continuous map $u:(M,\mu)\to(N,\nu)$ is a λ_g^α-continuous map, but not conversely.

(iii) Every contra continuous map $u:(M,\mu)\to(N,\nu)$ is a λ_g^α-continuous map, but not conversely.

Proof: By Proposition 2.2.3, 2.2.5 and 2.2.7, the statements of this proposition hold good.

Example 3.2.5 Let $M=N=\{i,j,k\}$, $\mu=\{\phi,\{i,j\},M\}$ and $\nu=\{\phi,\{i\},\{i,j\},\{i,k\},N\}$. Let $u:(M,\mu)\to(N,\nu)$ be a map defined by $u(i)=j, u(j)=i$ and $u(k)=k$. Then u is a λ_g^α-continuous map but not a λ-continuous map, since for the closed set $\{j\}$ in (N,ν), $u^{-1}(\{j\})=\{i\}$ is not a λ-closed set in (M,μ).

Example 3.2.6 Let $M=N=\{i,j,k\}$, $\mu=\{\phi,\{i\},\{j\},\{i,j\},\{i,k\},M\}$ and $\nu=\{\phi,\{i\},\{j\},\{i,j\},N\}$. Let $u:(M,\mu)\to(N,\nu)$ be a map defined by $u(i)=k, u(j)=i$ and $u(k)=j$. Then u is a λ_g^α-continuous map but not a continuous map, since for the closed set $\{k\}$ in (N,ν), $u^{-1}(\{k\})=\{i\}$ is not a closed set in (M,μ).

Example 3.2.7 Consider M,N,μ and ν as in Example 3.2.6. Let $u:(M,\mu)\to(N,\nu)$ be a map defined by $u(i)=j, u(j)=i$ and $u(k)=k$. Then u is a λ_g^α-continuous map but not a contra continuous map, since for the closed set $\{k\}$ in (N,ν), $u^{-1}(\{k\})=\{k\}$ is not an open set in (M,μ).

Definition 3.2.8 A map $u:(M,\mu)\to(N,\nu)$ is called a ***g*Λ-continuous map** if the inverse image of every closed set in (N,ν) is $g\Lambda$-closed in (M,ν), i.e., if $u^{-1}(T)$ is a $g\Lambda$-closed set in (M,μ) for every closed set T in (N,ν).

Proposition 3.2.9 Every λ_g^α-continuous map $u:(M,\mu)\to(N,\nu)$ is a $g\Lambda$-continuous map, but not conversely.

Proof: Follows from the Proposition 2.2.11.

Example 3.2.10 Let $M=N=\{i,j,k,l\}$, $\mu=\{\phi,\{i\},M\}$ and $\nu=\{\phi,\{i,j\},N\}$. Let $u:(M,\mu)\to(N,\nu)$ be a map defined by $u(i)=j, u(j)=k, u(k)=l$ and $u(l)=i$. Then u is a $g\Lambda$-continuous map but not a λ_g^α-continuous map, since for the closed set $\{k,l\}$ in (N,ν), $u^{-1}(\{k,l\})=\{j,k\}$ is not a λ_g^α-closed set in (M,μ).

Proposition 3.2.11 Every λ-irresolute map $u:(M,\mu)\to(N,\nu)$ is a λ_g^α-continuous map, but not conversely.

Proof: Let T be a closed set in (N,ν). Since every closed set is λ-closed, T is λ-closed in (N,ν). Since u is λ-irresolute, $u^{-1}(T)$ is λ-closed in (M,μ). By Proposition 2.2.3 we have $u^{-1}(T)$ is λ_g^α-closed in (M,μ). Thus u is a λ_g^α-continuous map.

Example 3.2.12 Let $M=N=\{i,j,k,l\}$, $\mu=\{\phi,\{i,j\},M\}$ and $\nu=\{\phi,\{i,j,k\},N\}$. Let $u:(M,\mu)\to(N,\nu)$ be a map defined by $u(i)=k, u(j)=l, u(k)=i$ and $u(l)=j$. Then u is a λ_g^α-continuous map but not a λ-irresolute map, since for the λ-closed set $\{l\}$ in (N,ν), $u^{-1}(\{l\})=\{j\}$ is not a λ-closed set in (M,μ).

Remark 3.2.13 g-continuous maps and λ_g^α-continuous maps are independent of each other as observed from the following examples.

Example 3.2.14 Let $M=N=\{i,j,k\}$, $\mu=\{\phi,\{i\},\{i,j\},M\}$ and $\nu=\{\phi,\{i\},\{j\},\{i,j\},N\}$. Let $u:(M,\mu)\to(N,\nu)$ be a map defined by $u(i)=j, u(j)=i$ and $u(k)=k$. Then u is a g-continuous map but not a λ_g^α-continuous map, since for the closed set $\{j,k\}$ in (N,ν), $u^{-1}(\{j,k\})=\{i,k\}$ is not a λ_g^α-closed set in (M,ν).

Example 3.2.15 Let $M=N=\{i,j,k\}$, $\mu=\{\phi,\{i\},\{i,j\},\{i,k\},M\}$ and $\nu=\{\phi,\{i\},\{j\},\{i,j\},\{i,k\},N\}$. Then the identity map $u:(M,\mu)\to(N,\nu)$ is a λ_g^α-continuous map but not a g-continuous map, since for the closed set $\{i, k\}$ in (N, ν), $u^{-1}(\{i, k\}) = \{i, k\}$ is not a g-closed set in (M, μ).

Remark 3.2.16 α-continuous maps and λ_g^α- continuous maps are independent of each other as observed from the following examples.

Example 3.2.17 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, \{j, k\}, N\}$. Then the identity map $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-continuous map but not an α-continuous map, since for the closed set $\{i\}$ in (N, ν), $u^{-1}(\{i\}) = \{i\}$ is not an α-closed set in (M, μ).

Example 3.2.18 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, M\}$ and $\nu = \{\phi, \{i, j, k\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = k, u(j) = l, u(k) = i$ and $u(l) = j$. Then u is an α-continuous map but not a λ_g^α- continuous map, since for the closed set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{j\}$ is not a λ_g^α-closed set in (M, μ).

Remark 3.2.19 αg-continuous maps and λ_g^α-continuous maps are independent of each other as observed from the following examples.

Example 3.2.20 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be the map defined by $u(i) = j, u(j) = i, u(k) = k$ and $u(l) = l$. Then u is an αg-continuous map but not a λ_g^α-continuous map, since for the closed set $\{j, k, l\}$ in (N, ν), $u^{-1}(\{j, k, l\}) = \{i, k, l\}$ is not a λ_g^α-closed set in (M, ν).

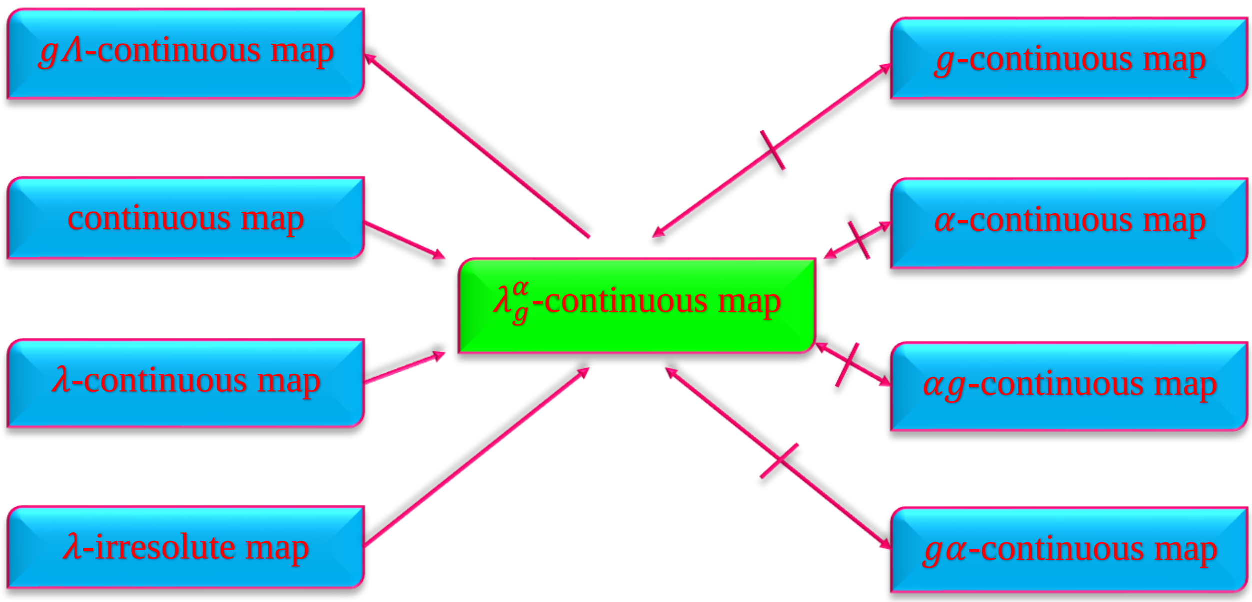
Example 3.2.21 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{j\}, \{i, j\}, \{i, j, k\}, \{i, j, l\}, M\}$ and $\nu = \{\phi, \{i\}, \{k\}, \{i, j\}, \{i, k\}, \{i, j, k\}, \{i, k, l\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = i, u(j) = l, u(k) = k$ and $u(l) = j$. Then u is a λ_g^α-continuous map but not an αg-continuous map, since for the closed set $\{j, l\}$ in (N, ν), $u^{-1}(\{j, l\}) = \{j, l\}$ is not an αg-closed set in (M, μ).

Remark 3.2.22 $g\alpha$-continuous maps and λ_g^α-continuous maps are independent of each other as observed from the following examples.

Example 3.2.23 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i, j, k\}, N\}$. Then the identity map $u: (M, \mu) \to (N, \nu)$ is a $g\alpha$-continuous map but not a λ_g^α-continuous map, since for the closed set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{l\}$ is not a λ_g^α-closed set in (M, μ).

Example 3.2.24 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{k\}, \{i, j\}, \{i, k\}, \{i, j, k\}, \{i, k, l\}, M\}$ and $\nu = \{\phi, \{i, j, k\}, M\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = j, u(j) = k, u(k) = l$ and $u(l) = i$. Then u is a λ_g^α-continuous map but not a $g\alpha$-continuous map, since for the closed set $\{l\}$ in (N, ν), $u^{-1}(\{l\}) = \{k\}$ is not a $g\alpha$-closed set in (M, μ).

Remark 3.2.25 The above relationships are presented in the following diagram.



Proposition 3.2.26 Every α-continuous (resp. g-continuous, αg-continuous) map $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-continuous map, whenever the domain (M, μ) is an α-space (resp. $T_{1/2}$-space, ${}_\alpha T_b$-space).

Proof: Follows from Proposition 2.2.29, Proposition 2.2.30 and Proposition 2.2.33.

Proposition 3.2.27 Every λ_g^α-continuous map $u: (M, \mu) \to (N, \nu)$ is a g-continuous map, whenever the domain (M, μ) is a partition space.

Proof: Let T be a closed set in (N, ν). Since u is λ_g^α-continuous, $u^{-1}(T)$ is λ_g^α-closed in (M, μ). Since (M, μ) is a partition space, by Proposition 2.2.38, we have $u^{-1}(T)$ is g-closed in (M, μ). Hence u is a g-continuous map.

Proposition 3.2.28 Every map $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-continuous map, whenever the domain (M, μ) is a door space or a $T_{1/2}$-space.

Proof: Follows from Proposition 2.2.34 and Proposition 2.2.31.

Theorem 3.2.29 If $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-continuous map, then $u(\lambda_g^\alpha cl(P)) \subseteq cl(u(P))$ for every subset P of (M, μ).

Proof: Let P be any subset of (M, μ). Then $cl(u(P))$ is closed in (N, ν). Since u is λ_g^α-continuous, $u^{-1}(cl(u(P))$ is λ_g^α-closed in (M, μ). Using the fact that $u(P) \subseteq cl(u(P))$, $P = u^{-1}\big(u(P)\big) \subseteq u^{-1}(cl(u(P))$ implies $u^{-1}(cl(u(P))$ is a λ_g^α-closed set containing P. By definition of λ_g^α-closure, $\lambda_g^\alpha cl(P) \subseteq u^{-1}(cl\big(u(P)\big))$ implies $u(\lambda_g^\alpha cl(P)) \subseteq cl(u(P))$.

Proposition 3.2.30 If $u: (M, \mu) \to (N, \nu)$ is a continuous map, then $u(\lambda_g^\alpha cl(P)) \subseteq cl(u(P))$ for every subset P of (M, μ).

Proof: Follows from Proposition 3.2.4 and Theorem 3.2.29.

Theorem 3.2.31 Let $P \subseteq M$ and $m \in M$. Then $m \in \lambda_g^\alpha cl(P)$ if and only if $U \cap P \neq \phi$, for every λ_g^α-open set U containing m.

Proof: Assume that $m \in \lambda_g^\alpha cl(P)$ and suppose $\exists$ a λ_g^α-open set U in (M, μ) containing m such that $U \cap P = \phi$. Now $M \setminus U$ is λ_g^α-closed in (M, μ) containing P. Then $P \subseteq M \setminus U \Rightarrow \lambda_g^\alpha cl(P) \subseteq M \setminus U$. Hence $m \notin \lambda_g^\alpha cl(P)$, which contradicts the assumption. Hence $U \cap P \neq \phi$.

Conversely, assume that $U \cap P \neq \phi$ for every λ_g^α-open set U in (M, μ) containing m. Suppose that $m \notin \lambda_g^\alpha cl(P)$ then $\exists$ a λ_g^α-closed set T containing P such that $m \notin T$. Then $m \in M \setminus T$ and $M \setminus T$ is a λ_g^α-open set. By assumption, $(M \setminus T) \cap P \neq \phi$. Since $P \subseteq T$, $(M \setminus T) \cap P = \phi$, which is a contradiction. Therefore $m \in \lambda_g^\alpha cl(P)$.

Theorem 3.2.32 Let $u: (M, \mu) \to (N, \nu)$ be a map from a topological space (M, μ) into a topological space (N, ν). Then the following statements are equivalent.

(i) For each point m in (M, μ) and each open set Q in (N, ν) containing $u(m)$, there exists a λ_g^α-open set P in (M, μ) containing m such that $u(P) \subseteq Q$.

(ii) For every subset S of (M, μ), $u(\lambda_g^\alpha cl(S)) \subseteq cl(u(S))$.

(iii) For every subset T of (N, ν), $\lambda_g^\alpha cl(u^{-1}(T)) \subseteq u^{-1} cl(T)$.

Proof: (i) $\Rightarrow$ (ii) Assume that (i) holds and let $n \in u(\lambda_g^\alpha cl(S))$. Then $n = u(m)$ for some $m \in \lambda_g^\alpha cl(S) \subseteq M$. Let Q be any open set in (N, ν) containing $u(m)$ i.e., $u(m) \subseteq Q$. Then by assumption there exists a λ_g^α-open set P in (M, μ) containing m such that $u(P) \subseteq Q$. By Theorem 3.2.31 $P \cap S \neq \phi$, then $u(P \cap S) \neq \phi \Rightarrow Q \cap u(S) \neq \phi$. Hence $n = u(m) \in cl(u(S))$.

(ii) $\Rightarrow$ (i) Assume that (ii) holds. Let $m \in M$ and Q be any open set in (N, ν) containing $u(m)$. Let $S = u^{-1}(Q^c)$. Then $m \notin S$. By assumption $u(\lambda_g^\alpha cl(S)) \subseteq cl(u(S)) = cl(u(u^{-1}(Q^c))) = cl(Q^c) = Q^c$. Therefore, $u^{-1}(u(\lambda_g^\alpha cl(S))) \subseteq u^{-1}(Q^c) = S$ which implies $\lambda_g^\alpha cl(S) \subseteq S$. Hence $S = \lambda_g^\alpha cl(S)$. Since $m \notin S$, $m \notin \lambda_g^\alpha cl(S)$, then there exists a λ_g^α-open set P containing m such that $P \cap S = \phi$ and hence $u(P) \subseteq u(S^c) \subseteq Q$.

(ii) $\Rightarrow$ (iii) Assume that (ii) holds. Let T be any subset of (N, ν). Replace S by $u^{-1}(T)$ in (ii), we obtain $u(\lambda_g^\alpha cl(u^{-1}(T))) \subseteq cl(u(u^{-1}(T))) = cl(T)$. Hence $\lambda_g^\alpha cl(u^{-1}(T)) \subseteq u^{-1}(cl(T))$.

(iii) ⇒ (ii) Assume that (iii) holds. Let $T = u(S)$ where S is a subset of (M, μ). Then $\lambda_g^\alpha cl(S) = \lambda_g^\alpha cl(u^{-1}(T)) \subseteq u^{-1}\big(cl(T)\big) = u^{-1}(cl\big(u(S)\big))$. Hence $u(\lambda_g^\alpha cl(S)) \subseteq cl(u(S))$.

Remark 3.2.33 The composition of two λ_g^α-continuous maps need not be a λ_g^α-continuous map as observed from the following example.

Example 3.2.34 Let $M = N = K = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, M\}, \nu = \{\phi, \{i, j, k\}, N\}$ and $\kappa = \{\phi, \{i\}, \{i, j\}, K\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = l, u(j) = k, u(k) = j$ and $u(l) = i$ and $w: (N, \nu) \to (K, \kappa)$ be a map defined by $w(i) = k, w(j) = j, w(k) = l$ and $w(l) = i$. Then the maps u and w are both λ_g^α-continuous maps, but their composition $(w \circ u): (M, \mu) \to (K, \kappa)$ is not a λ_g^α-continuous map, since for the closed set $\{k, l\}$ in (K, κ) $(w \circ u)^{-1}(\{k, l\}) = u^{-1}(w^{-1}(\{k, l\})) = u^{-1}(\{i, k\}) = \{j, l\}$ is not a λ_g^α-closed set in (M, μ).

Proposition 3.2.35 The composition of two continuous maps is a λ_g^α-continuous map.

Proof: Follows from Proposition 3.2.4.

Proposition 3.2.36 The composition of two λ-irresolute maps is a λ_g^α-continuous map.

Proof: Let $u: (M, \mu) \to (N, \nu)$ and $w: (N, \nu) \to (K, \kappa)$ be the λ-irresolute maps and T be a closed set in (K, κ). By Lemma 1.1.8 (ii), T is also a λ-closed set in (K, κ). Then $w^{-1}(T)$ is a λ-closed set in (N, ν). Then $u^{-1}(w^{-1}(T)) = (w \circ u)^{-1}(T)$ is a λ-closed set in (M, μ). By Proposition 2.2.3, $(w \circ u)^{-1}(T)$ is λ_g^α-closed. Hence $(w \circ u): (M, \mu) \to (K, \kappa)$ is a λ_g^α-continuous map.

Proposition 3.2.37 If $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-continuous map and $w: (N, \nu) \to (K, \kappa)$ is a continuous map then their composition $(w \circ u): (M, \mu) \to (K, \kappa)$ is a λ_g^α-continuous map.

Proof: Let T be a closed set in (K,κ). Then $w^{-1}(T)$ is a closed set in (N,ν) as w is continuous. Since u is λ_g^α-continuous, $(w\circ u)^{-1}(T)=u^{-1}(w^{-1}(T))$ is a λ_g^α-closed set in (M,μ). Hence $(w\circ u)$ is a λ_g^α-continuous map.

Remark 3.2.38 Proposition 3.2.37 is true even if $u:(M,\mu)\to(N,\nu)$ is a λ-continuous map.

Remark 3.2.39 If $u:(M,\mu)\to(N,\nu)$ is a continuous map and $w:(N,\nu)\to(K,\kappa)$ is a λ_g^α-continuous map then their composition $(w\circ u):(M,\mu)\to(K,\kappa)$ need not be a λ_g^α-continuous map.

Example 3.2.40 Let $M=N=K=\{i,j,k\}$, $\mu=\{\phi,\{i\},\{i,j\},M\}$, $\nu=\{\phi,\{i,j\},N\}$ and $\kappa=\{\phi,\{i\},\{j\},\{i,j\},K\}$. Let $u:(M,\mu)\to(N,\nu)$ and $w:(N,\nu)\to(K,\kappa)$ be the identity maps. Then u is a continuous map and w is a λ_g^α-continuous map but their composition $(w\circ u):(M,\mu)\to(N,\nu)$ is not a λ_g^α-continuous map, since for the closed set $\{i,k\}$ in (K,κ), $(w\circ u)^{-1}(\{i,k\})=u^{-1}(w^{-1}(\{i,k\}))=u^{-1}(\{i,k\})=\{i,k\}$ is not a λ_g^α-closed set in (M,μ).

Proposition 3.2.41 If $u:(M,\mu)\to(N,\nu)$ is a λ-irresolute map and $w:(N,\nu)\to(K,\kappa)$ is a λ-continuous map then their composition $(w\circ u):(M,\mu)\to(K,\kappa)$ is a λ_g^α-continuous map.

Proof: Let T be a closed set in (K,κ). Then $w^{-1}(T)$ is a λ-closed set in (N,ν) as w is λ-continuous. Since u is λ-irresolute, $(w\circ u)^{-1}(T)=u^{-1}(w^{-1}(T))$ is a λ-closed set in (M,μ). By Proposition 2.2.3, $(w\circ u)^{-1}(T)$ is a λ_g^α-closed set in (M,μ). Hence $(w\circ u)$ is a λ_g^α-continuous map.

Proposition 3.2.42 If a map $u:(M,\mu)\to(N,\nu)$ is a contra continuous map and $w:(N,\nu)\to(K,\kappa)$ be a continuous map then their composition $(w\circ u):(M,\mu)\to(K,\kappa)$ is a λ_g^α-continuous map.

Proof: Obvious by Proposition 2.2.7.

Proposition 3.2.43 If $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-continuous map and $w: (N, \nu) \to (K, \kappa)$ is a continuous map then their composition $(w \circ u): (M, \mu) \to (K, \kappa)$ is a g-continuous map whenever the domain (M, μ) is a partition space.

Proof: Follows from the Proposition 2.2.38.

Proposition 3.2.44 If $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-continuous map and $w: (N, \nu) \to (K, \kappa)$ is an α-continuous (resp. g-continuous, αg-continuous) map then their composition $(w \circ u): (M, \mu) \to (K, \kappa)$ is a λ_g^α-continuous map whenever (N, ν) is an α-space (resp. $T_{1/2}$-space, ${}_\alpha T_b$-space).

Proof: Let T be a closed set in (K, κ). Since w is α-continuous (resp. g-continuous, αg-continuous), $w^{-1}(T)$ is α-closed (resp. g-closed, αg-closed) in (N, ν). As (N, ν) is an α-space (resp. $T_{1/2}$-space, ${}_\alpha T_b$-space), $w^{-1}(T)$ is a closed set in (N, ν). Since u is λ_g^α-continuous, $(w \circ u)^{-1}(T) = u^{-1}(w^{-1}(T))$ is λ_g^α-closed in (M, μ). Hence $(w \circ u)$ is a λ_g^α-continuous map.

Proposition 3.2.45 If $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-continuous map and $w: (N, \nu) \to (K, \kappa)$ is a continuous map then their composition $(w \circ u): (M, \mu) \to (K, \kappa)$ is a λ-continuous map whenever the domain (M, μ) is a T_1-space.

Proof: Follows from the Proposition 2.3.13.

Proposition 3.2.46 Let $u: (M, \mu) \to (N, \nu)$ and $w: (N, \nu) \to (K, \kappa)$ be bijective maps.

(i) If $(w \circ u): (M, \mu) \to (K, \kappa)$ is a λ-continuous map and u is a λ-closed map then w is a λ_g^α-continuous map.

(ii) If $(w \circ u): (M, \mu) \to (K, \kappa)$ is a λ-irresolute map and u is a λ-closed map then w is a λ_g^α-continuous map.

(iii) If $(w \circ u): (M, \mu) \to (K, \kappa)$ is a continuous map and u is a λ-closed map then w is a λ_g^α-continuous map.

Proof:

(i) Let T be a closed set in (K,κ). Since $(w \circ u)$ is a λ-continuous map, $(w \circ u)^{-1}(T) = u^{-1}(w^{-1}(T))$ is λ-closed in (M,μ). Since u is a λ-closed map $u(u^{-1}(w^{-1}(T))) = w^{-1}(T)$ is a λ-closed set in (N,ν). By Proposition 2.2.3, $u(u^{-1}(w^{-1}(T))) = w^{-1}(T)$ is also a λ_g^α-closed set in (N,ν). Thus w is a λ_g^α-continuous map.

(ii) Let T be a closed set in (K,κ). By Lemma 1.1.8, T is also a λ-closed set in (K,κ). Since $(w \circ u)$ is a λ-irresolute map, $(w \circ u)^{-1}(T) = u^{-1}(w^{-1}(T))$ is λ-closed in (M,μ). Since u is a λ-closed map $u(u^{-1}(w^{-1}(T))) = w^{-1}(T)$ is a λ-closed set in (N,ν). By Proposition 2.2.3, $u(u^{-1}(w^{-1}(T))) = w^{-1}(T)$ is also a λ_g^α-closed set in (N,ν). Thus w is a λ_g^α-continuous map.

(iii) Let T be a closed set in (K,κ). Since $(w \circ u)$ is a continuous map, $(w \circ u)^{-1}(T) = u^{-1}(w^{-1}(T))$ is closed in (M,μ). By Lemma 1.1.8, $(w \circ u)^{-1}(T) = u^{-1}(w^{-1}(T))$ is also a λ-closed set in (M,μ). Since u is a λ-closed map $u(u^{-1}(w^{-1}(T))) = w^{-1}(T)$ is a λ-closed set in (N,ν). By Proposition 2.2.3, $u(u^{-1}(w^{-1}(T))) = w^{-1}(T)$ is also a λ_g^α-closed set in (N,ν). Thus w is a λ_g^α-continuous map.

3.3 λ_g^α-Irresolute Maps

In this section, λ_g^α-irresolute maps have been defined and its vital properties have been derived and examined.

Definition 3.3.1 A map $u:(M,\mu) \to (N,\nu)$ is called λ_g^α-irresolute if the inverse image of every λ_g^α-closed set in (N,ν) is λ_g^α-closed in (M,μ), i.e., $u^{-1}(T)$ is λ_g^α-closed in (M,μ) for every λ_g^α-closed set T in (N,ν).

Example 3.3.2 Let $M = N = \{i,j,k\}$, $\mu = \{\phi,\{i,j\},M\}$ and $\mu = \{\phi,\{i\},N\}$. Let $u:(M,\mu) \to (N,\nu)$ be a map defined by $u(i) = j, u(j) = k$ and $u(k) = i$. Then u is a λ_g^α-irresolute map.

Proposition 3.3.3 A map $u:(M,\mu)\rightarrow(N,\nu)$ is λ_g^α-irresolute if and only if $u^{-1}(S)$ is λ_g^α-open in (M,μ) for every λ_g^α-open set S in (N,ν).

Proof: (Necessity) Let S be a λ_g^α-open set in (N,ν). Then $N\setminus S$ is a λ_g^α-closed set in (N,ν). By definition $u^{-1}(N\setminus S)=M\setminus u^{-1}(S)$ is a λ_g^α-closed set in (M,μ). Thus $u^{-1}(S)$ is a λ_g^α-open set in (M,μ).

Sufficiency: Follows from the Definition 3.3.1.

Proposition 3.3.4 Every λ_g^α-irresolute map $u:(M,\mu)\rightarrow(N,\nu)$ is a λ_g^α-continuous map, but not conversely.

Proof: Let T be any closed set in (N,ν). By Proposition 2.2.5, T is also a λ_g^α-closed set in (N,ν). As u is λ_g^α-irresolute, $u^{-1}(T)$ is λ_g^α-closed in (M,μ). Hence u is a λ_g^α-continuous map.

Example 3.3.5 Let $M=N=\{i,j,k,l,m\}$, $\mu=\{\phi,\{i\},\{j\},\{i,j\},\{j,k\},\{i,j,k\},\{j,k,l\},$ $\{i,j,k,l\},\{j,k,l,m\},M\}$ and $\nu=\{\phi,\{i\},\{j,k\},\{i,j,k\},N\}$. Let $u:(M,\mu)\rightarrow(N,\nu)$ be a map defined by $u(i)=i,u(j)=j,u(k)=k,u(l)=m$ and $u(m)=l$. Then u is a λ_g^α-continuous map but not a λ_g^α-irresolute map, since for the λ_g^α-closed set $\{j,l,m\}$ in (N,ν), $u^{-1}(\{j,l,m\})=\{j,l,m\}$ is not a λ_g^α-closed set in (M,μ).

Remark 3.3.6 λ_g^α-irresolute maps and λ-irresolute maps are independent of each other as observed from the following examples.

Example 3.3.7 Let $M=N=\{i,j,k\},\mu=\{\phi,\{i\},\{i,j\},M\}$ and $\nu=\{\phi,\{i,j\},N\}$. Let $u:(M,\mu)\rightarrow(N,\nu)$ be a map defined by $u(i)=k,u(j)=j$ and $u(k)=i$. Then u is a λ-irresolute map but not a λ_g^α-irresolute map, since for the λ_g^α-closed set $\{i,k\}$ in (N,ν), $u^{-1}(\{i,k\})=\{i,k\}$ is not a λ_g^α-closed set in (M,μ).

Example 3.3.8 Let $M=N=\{i,j,k\}$, $\mu=\{\phi,\{i,j\},M\}$ and $\nu=\{\phi,\{i\},\{i,j\},N\}$. Let $u:(M,\mu)\rightarrow(N,\nu)$ be a map defined by $u(i)=k,u(j)=j$ and $u(k)=i$. Then u is a λ_g^α-

irresolute map but not a λ-irresolute map, since for the λ-closed set $\{k\}$ in (N, ν), $u^{-1}(\{k\}) = \{i\}$ is not a λ-closed set in (M, μ).

Theorem 3.3.9 If $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-irresolute map then u is g-continuous whenever the domain (M, μ) is a partition space.

Proof: Follows from Proposition 2.2.38.

Remark 3.3.10 αg-irresloute maps and λ_g^α-irresolute maps are independent of each other as observed from the following examples.

Example 3.3.11 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, M\}$ and $\nu = \{\phi, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = i, u(j) = k$ and $u(k) = j$. Then u is an αg-irresolute map but not a λ_g^α-irresolute map, since for the λ_g^α-closed set $\{j\}$ in (N, ν), $u^{-1}(\{j\}) = \{k\}$ is not a λ_g^α-closed set in (M, μ).

Example 3.3.12 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = i, u(j) = k$ and $u(k) = j$. Then u is a λ_g^α-irresolute map but not an αg-irresloute map, since for the αg-closed set $\{k\}$ in (N, ν), $u^{-1}(\{k\}) = \{j\}$ is not an αg-closed set in (M, μ).

Remark 3.3.13 $g\alpha$-irresolute maps and λ_g^α-irresolute maps are independent of each other as observed from the following examples.

Example 3.3.14 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = i, u(j) = k$ and $u(k) = j$. Then u is a $g\alpha$-irresolute map but not a λ_g^α-irresolute map, since for the λ_g^α-closed set $\{k\}$ in (N, ν), $u^{-1}(\{k\}) = \{j\}$ is not a λ_g^α-closed set in (M, μ).

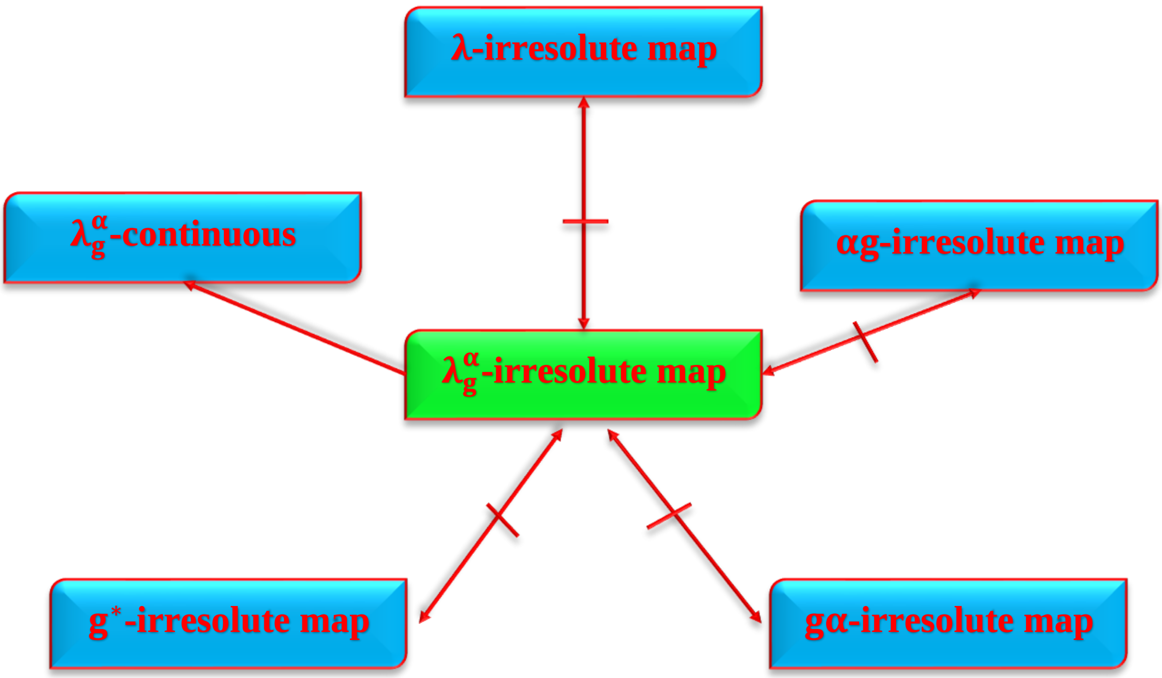
Example 3.3.15 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, \{j\}, \{i, j\}, N\}$. Then the identity map $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-irresolute map but not a $g\alpha$-irresolute map, since for the $g\alpha$-closed set $\{i,k\}$ in (N,ν), $u^{-1}(\{i,k\}) = \{i,k\}$ is not a $g\alpha$-closed set in (M,μ).

Remark 3.3.16 g^*-irresolute maps and λ_g^α-irresolute maps are independent of each other as observed from the following examples.

Example 3.3.17 Let $M = N = \{i,j,k\}$, $\mu = \{\phi,\{i\},\{i,j\},M\}$ and $\nu = \{\phi,\{i,j\},N\}$. Then the identity map $u:(M,\mu) \to (N,\nu)$ is a g^*-irresolute map but not a λ_g^α-irresolute map, since for the λ_g^α-closed set $\{i,k\}$ in (N,ν), $u^{-1}(\{i,k\}) = \{i,k\}$ is not a λ_g^α-closed set in (M,μ).

Example 3.3.18 Let $M = N = \{i,j,k\}$, $\mu = \{\phi,\{i\},\{j\},\{i,j\},M\}$ and $\nu = \{\phi,\{i\},\{j\},\{i,j\},\{i,k\},N\}$. Then the identity map $u:(M,\mu) \to (N,\nu)$ is a λ_g^α-irresolute map but not a g^*-irresolute map, since for the g^*-closed set $\{j\}$ in (N,ν), $u^{-1}(\{j\}) = \{j\}$ is not a g^*-closed set in (M,μ).

Remark 3.3.19 The above derived interrelationships are depicted in the following diagram.



Proposition 3.3.20 If a map $u:(M,\mu)\to(N,\nu)$ is λ_g^α-irresolute then for every subset G of (M,μ) such that $u(G)$ is λ_g^α-closed in (N,ν), $u(\lambda_g^\alpha cl(G))\subseteq\lambda_g^\alpha cl(u(G))$.

Proof: Let G be a subset of (M,μ) such that $u(G)$ is a λ_g^α-closed set in (N,ν). By the property of λ_g^α-closure, we have $\lambda_g^\alpha cl(u(G))$ is also a λ_g^α-closed set in (N,ν). Since u is a λ_g^α-irresolute map, $u^{-1}(\lambda_g^\alpha cl(u(G)))$ is λ_g^α-closed in (M,μ). Using the fact that $G\subseteq G$, we have $G=u^{-1}(u(G))\subseteq u^{-1}(\lambda_g^\alpha cl(u(G)))$. Therefore, $\lambda_g^\alpha cl(G)\subseteq u^{-1}\big(\lambda_g^\alpha cl(u(G))\big)\Rightarrow u(\lambda_g^\alpha cl(G))\subseteq u(u^{-1}(\lambda_g^\alpha cl(u(G))))=\lambda_g^\alpha cl(u(G))$.

Proposition 3.3.21 If a map $u:(M,\mu)\to(N,\nu)$ is λ_g^α-irresolute then for every λ_g^α-closed subset H of (N,ν), $\lambda_g^\alpha cl(u^{-1}(H))\subseteq u^{-1}(\lambda_g^\alpha cl(H))$.

Proof: Let H be a λ_g^α-closed set in (N,ν). Then $\lambda_g^\alpha cl(H)$ is λ_g^α-closed in (N,ν). Since u is a λ_g^α-irresolute map, $u^{-1}(\lambda_g^\alpha cl(H))$ is λ_g^α-closed in (M,μ). Since $H\subseteq\lambda_g^\alpha cl(H)$, $u^{-1}(H)\subseteq u^{-1}(\lambda_g^\alpha cl(H))$. By Definition of λ_g^α-closure, $\lambda_g^\alpha cl(u^{-1}(H))=u^{-1}(H)\subseteq u^{-1}(\lambda_g^\alpha cl(H))$.

Proposition 3.3.22 Let $u:(M,\mu)\to(N,\nu)$ be a λ_g^α-irresolute map and $w:(N,\nu)\to(K,\kappa)$ be a λ_g^α-continuous map. Then $(w\circ u):(M,\mu)\to(K,\kappa)$ is a λ_g^α-continuous map.

Proof: Let T be any closed set in (K,κ). Since w is λ_g^α-continuous, $w^{-1}(T)$ is λ_g^α-closed in (N,ν). As u is λ_g^α-irresolute, inverse image of every λ_g^α-closed set in (N,ν) is λ_g^α-closed in (M,μ). Therefore $(w\circ u)^{-1}(T)=u^{-1}(w^{-1}(T))$ is a λ_g^α-closed set in (M,μ). Hence $(w\circ u)$ is a λ_g^α-continuous map.

Proposition 3.3.23 Let $u:(M,\mu)\to(N,\nu)$ be a λ_g^α-irresolute map and $w:(N,\nu)\to(K,\kappa)$ be a continuous (resp. λ-continuous) map. Then $(w\circ u):(M,\mu)\to(K,\kappa)$ is a λ_g^α-continuous map.

Proof: Let T be any closed set in (K,κ). Since w is continuous (resp. λ-continuous), $w^{-1}(T)$ is closed (resp. λ-closed) in (N,ν). As every closed (resp. λ-closed) set is λ_g^α-

closed, we have $w^{-1}(T)$ is λ_g^α-closed in (N,ν). As u is λ_g^α-irresolute, $(w \circ u)^{-1}(T) = u^{-1}(w^{-1}(T))$ is a λ_g^α-closed set in (M,μ). Hence $(w \circ u)$ is a λ_g^α-continuous map.

Proposition 3.3.24 The composition of two λ_g^α-irresolute maps is a λ_g^α-irresolute map.

Proof: Let $u: (M,\mu) \to (N,\nu)$ and $w: (N,\nu) \to (K,\kappa)$ be two λ_g^α-irresolute maps. Let T be a λ_g^α-closed set in (K,κ). Since w is a λ_g^α-irresolute map, $w^{-1}(T)$ is λ_g^α-closed in (N,ν). As u is λ_g^α-irresolute, $(w \circ u)^{-1}(T) = u^{-1}(w^{-1}(T))$ is λ_g^α-closed in (M,μ). Thus $(w \circ u): (M,\mu) \to (K,\kappa)$ is a λ_g^α-irresolute map.

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3.4 Special Types of λ_g^α-Continuous Maps

This section deals with the special types of continuities namely, quasi λ_g^α-continuous maps, perfectly λ_g^α-continuous maps, totally λ_g^α-continuous maps and strongly λ_g^α-continuous maps. Their interrelationships and the remarkable properties have been accomplished.

Definition 3.4.1 A map $u: (M,\mu) \to (N,\nu)$ is called **quasi λ_g^α-continuous** if the inverse image of every λ_g^α-closed set in (N,ν) is closed in (M,μ) i.e., if $u^{-1}(S)$ is closed in (M,μ) for every λ_g^α-closed set S in (N,ν).

Example 3.4.2 Let $M = N = \{i,j,k\}$, $\mu = \{\phi, \{i\}, \{j,k\}, M\}$ and $\nu = \{\phi, \{i\}, N\}$. Let $u: (M,\mu) \to (N,\nu)$ be a map defined by $u(i) = i, u(j) = k$ and $u(k) = j$. Then u is a quasi λ_g^α-continuous map.

Theorem 3.4.3 A map $u: (M,\mu) \to (N,\nu)$ is quasi λ_g^α-continuous if the inverse image of every λ_g^α-open set in (N,ν) is open in (M,μ) and vice versa.

Proof: Let T be any λ_g^α-open set in (N, ν). Then $N \setminus T$ is λ_g^α-closed in (N, ν). By assumption, $u^{-1}(N \setminus T) = M \setminus (u^{-1}(T))$ is closed in (M, μ). Hence $u^{-1}(T)$ is open in (M, μ).

On the other side, let S be any λ_g^α-closed set in (N, ν). Then $N \setminus S$ is λ_g^α-open in (N, ν), $u^{-1}(N \setminus S) = M \setminus (u^{-1}(S))$ is open in $(M, \mu) \Rightarrow u^{-1}(S)$ is closed in (M, μ). Hence u is a quasi λ_g^α-continuous map.

Definition 3.4.4 A map $u: (M, \mu) \to (N, \nu)$ is called **perfectly λ_g^α-continuous** if the inverse image of every λ_g^α-closed set in (N, ν) is clopen in (M, μ) i.e., if $u^{-1}(S)$ is clopen in (M, μ) for every λ_g^α-closed set S in (N, ν).

Example 3.4.5 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{j, k\}, M\}$ and $\nu = \{\phi, \{i\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = i, u(j) = k$ and $u(k) = j$. Then u is a perfectly λ_g^α-continuous map.

Theorem 3.4.6 A map $u: (M, \mu) \to (N, \nu)$ is perfectly λ_g^α-continuous if the inverse image of every λ_g^α-open set in (N, ν) is clopen in (M, μ) and vice versa.

Proof: Let T be any λ_g^α-open set in (N, ν). Then $N \setminus T$ is λ_g^α-closed in (N, ν). Since u is perfectly λ_g^α-continuous $u^{-1}(N \setminus T) = M \setminus (u^{-1}(T))$ is clopen in (M, μ). Hence $u^{-1}(T)$ is clopen in (M, μ).

On the other side, let S be λ_g^α-closed set in (N, ν), then $N \setminus S$ is λ_g^α-open in (N, ν). From the assumption, $u^{-1}(N \setminus S) = M \setminus (u^{-1}(S))$ is clopen in $(M, \mu) \Rightarrow u^{-1}(S)$ is clopen in (M, μ). Hence u is a perfectly λ_g^α-continuous map.

Proposition 3.4.7 If $u: (M, \mu) \to (N, \nu)$ is a perfectly λ_g^α-continuous map, then it is a quasi λ_g^α-continuous map.

Proof: Suppose that S is a λ_g^α-closed set in (N, ν). Since u is perfectly λ_g^α-continuous, $u^{-1}(S)$ is clopen in (M, μ). Hence u is a quasi λ_g^α-continuous map.

Proposition 3.4.8 Every strongly continuous map $u: (M, \mu) \to (N, \nu)$ is a quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) map.

Proof: Suppose that S is a λ_g^α-closed set in (N, ν). As u is strongly continuous, for any subset S, $u^{-1}(S)$ is both open and closed in (M, μ). Hence u is quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) map.

Theorem 3.4.9 Consider the discrete topological space (M, μ) and any topological space (N, ν), then for any map $u: (M, \mu) \to (N, \nu)$, the statements

(i) u is perfectly λ_g^α-continuous

(ii) u is quasi λ_g^α-continuous

are equivalent.

Proof: (i)⇒(ii) Follows from Proposition 3.4.7.

(ii)⇒(i) Let S be any λ_g^α-closed set in (N, ν). By assumption, $u^{-1}(S)$ is closed in (M, μ). Since (M, μ) is a discrete space, $u^{-1}(S)$ is open in (M, μ). Hence $u^{-1}(S)$ is clopen in (M, μ). Hence u is a perfectly λ_g^α-continuous map.

Proposition 3.4.10 Every quasi λ_g^α-continuous map $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-continuous map.

Proof: Let S be a closed set in (N, ν). Using Proposition 2.2.5, S is λ_g^α-closed in (N, ν). Since u is quasi λ_g^α-continuous, $u^{-1}(S)$ is closed in (M, μ). Again, using Proposition 2.2.5, $u^{-1}(S)$ is λ_g^α-closed in (M, μ). Hence u is a λ_g^α-continuous map.

The subsequent example shows that the converse of Proposition 3.4.10 may not hold good.

Example 3.4.11 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{j\}, \{i, j\}, \{i, k\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, \{i, k\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = j, u(j) = i$ and $u(k) = k$. Then u is a λ_g^α-continuous map but not a quasi λ_g^α-continuous map, since for the λ_g^α-closed set $\{i, j\}$ in (N, ν), $u^{-1}(\{i, j\}) = \{i, j\}$ is not closed in (M, μ).

Proposition 3.4.12 Let $u: (M, \mu) \to (N, \nu)$ and $w: (N, \nu) \to (K, \kappa)$ be any two maps. Then $(w \circ u): (M, \mu) \longrightarrow (K, \kappa)$ is quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) map, whenever

(i) w is a strongly continuous map and u is a quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) map

(ii) w is a quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) map and u is a continuous map

Proof: (i) Let S be any λ_g^α-closed set in (K, κ). As w is strongly continuous, $w^{-1}(S)$ is both open and closed in $(N, \nu) \Rightarrow w^{-1}(S)$ is λ_g^α-closed in (N, ν). Since u is quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous), $(w \circ u)^{-1}(S) = u^{-1}(w^{-1}(S))$ is closed (resp. clopen) in (M, μ). Hence $(w \circ u)$ is a quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) map.

(ii) Let S be any λ_g^α-closed set in (K, κ). As w is quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous), $w^{-1}(S)$ is closed (resp. clopen) in (N, ν). Since u is continuous, $(w \circ u)^{-1}(S) = u^{-1}(w^{-1}(S))$ is closed (resp. clopen) in (M, μ). Hence $(w \circ u)$ is a quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) map.

Proposition 3.4.13 If $u: (M, \mu) \to (N, \nu)$ is a quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) map and $w: (N, \nu) \to (K, \kappa)$ is a perfectly λ_g^α-continuous (resp. quasi λ_g^α-continuous) map, then $(w \circ u): (M, \mu) \to (K, \kappa)$ is a quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) map.

Proof: Follows from Proposition 2.2.5.

Proposition 3.4.14 If $u: (M, \mu) \to (N, \nu)$ and $w: (N, \nu) \to (K, \kappa)$ are quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) maps, then $(w \circ u): (M, \mu) \to (K, \kappa)$ is also a quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) map.

Proof: Let S be any λ_g^α-closed set in (K, κ). As w is quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous), $w^{-1}(S)$ is closed (resp. clopen) in $(N, \nu) \Rightarrow w^{-1}(S)$ is λ_g^α-closed in (N, ν). Since u is quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous), $(w \circ u)^{-1}(S) = u^{-1}(w^{-1}(S))$ is closed (resp. clopen) in (M, μ) and hence $(w \circ u)$ is also a quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) map.

Definition 3.4.15 A subset $\mathcal{A}$ of (M, μ) is called $\boldsymbol{\lambda_g^\alpha}$**-clopen** if it is both λ_g^α-open and λ_g^α-closed in (M, μ).

Example 3.4.16 Let $M = \{i, j, k\}$ and $\mu = \{\phi, \{i\}, \{i, j\}, \{i, k\}, M\}$. Then the subset $\{j, k\}$ of (M, μ) is a λ_g^α-clopen set.

Definition 3.4.17 A map $u: (M, \mu) \to (N, \nu)$ is called **totally** $\boldsymbol{\lambda_g^\alpha}$**-continuous** if the inverse image of every open set in (N, ν) is λ_g^α-clopen in (M, μ) i.e., if $u^{-1}(T)$ is a λ_g^α-clopen set in (M, μ) for every open set T in (N, ν).

Example 3.4.18 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{i, j\}, \{i, k\}, M\}$ and $\nu = \{\phi, \{i\}, \{j\}, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = j, u(j) = k$ and $u(k) = i$. Then u is a totally λ_g^α-continuous map.

Proposition 3.4.19 A map $u: (M, \mu) \to (N, \nu)$ is totally λ_g^α-continuous if the inverse image of every closed subset of (N, ν) is a λ_g^α-clopen subset of (M, μ) and vice versa.

Proof: Let S be any closed set in $(N, \nu) \Rightarrow N \setminus S$ is open in (N, ν). As u is totally λ_g^α-continuous, $u^{-1}(N \setminus S) = M \setminus (u^{-1}(S))$ is λ_g^α-clopen in $(M, \mu) \Rightarrow u^{-1}(S)$ is λ_g^α-clopen in (M, μ).

On the other side, let T be any open set in $(N, \nu) \Rightarrow N \setminus T$ is closed in (N, ν). By assumption, $u^{-1}(N \setminus T) = M \setminus u^{-1}(T)$ is λ_g^α-clopen in $(M, \mu) \Rightarrow u^{-1}(T)$ is λ_g^α-clopen in (M, μ). Hence u is totally λ_g^α-continuous.

Proposition 3.4.20 Every totally λ_g^α-continuous map $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-continuous map.

Proof: Follows from the Definitions 3.4.17 and 3.2.1.

The subsequent example shows that the converse part of Proposition 3.4.20 may not hold good.

Example 3.4.21 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, \{j\}, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = i, u(j) = k$ and $u(k) = j$. Then u is a λ_g^α-continuous map but not a totally λ_g^α-continuous map, since for the open set $\{i, j\}$ in (N, ν), $u^{-1}(\{i, j\}) = \{i, k\}$ is not λ_g^α-clopen in (M, μ).

Definition 3.4.22 A map $u: (M, \mu) \to (N, \nu)$ is called **strongly λ_g^α-continuous** if the inverse image of every subset in (N, ν) is λ_g^α-clopen in (M, μ) or if $u^{-1}(S)$ is a λ_g^α-clopen set in (M, μ) for every subset S in (N, ν).

Example 3.4.23 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{i, j\}, \{i, k\}, M\}$ and $\nu = \{\phi, \{i\}, \{j, k\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = k, u(j) = j$ and $u(k) = i$. Then u is a strongly λ_g^α-continuous map.

Proposition 3.4.24 Every strongly λ_g^α-continuous map $u: (M, \mu) \to (N, \nu)$ is a totally λ_g^α-continuous map.

Proof: Follows from the Definitions 3.4.17 and 3.4.22.

The subsequent example shows that the converse of Proposition 3.4.24 may not hold good.

Example 3.4.25 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, \{j\}, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be the identity map. Then u is a totally λ_g^α-continuous map but not a strongly λ_g^α-continuous map, since for the set $\{i, k\}$ in (N, ν), $u^{-1}(\{i, k\}) = \{i, k\}$ is not λ_g^α-clopen in (M, μ).

Theorem 3.4.26 Let $u: (M, \mu) \to (N, \nu)$ be a totally λ_g^α-continuous map where (N, ν) is a discrete topological space. Then u is a strongly λ_g^α-continuous map.

Proof: Let T be any subset of (N, ν). As (N, ν) is a discrete topological space, T is open in $(N, \nu) \Rightarrow u^{-1}(S)$ is λ_g^α-clopen in (M, μ), as u is totally λ_g^α-continuous. Hence u is a strongly λ_g^α-continuous map.

Proposition 3.4.27 Every perfectly λ_g^α-continuous map $u: (M, \mu) \to (N, \nu)$ is a totally λ_g^α-continuous map.

Proof: Let T be any open set in (N, ν). Then T is also a λ_g^α-open set in (N, ν). Since u is perfectly λ_g^α-continuous, $u^{-1}(T)$ is clopen in $(M, \mu) \Rightarrow u^{-1}(T)$ is λ_g^α-clopen in (M, μ). Hence u is a totally λ_g^α-continuous map.

The subsequent example shows that the converse of Proposition 3.4.27 may not hold good.

Example 3.4.28 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, M\}$ and $\nu = \{\phi, \{i\}, \{j, k\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = i, u(j) = k$ and $u(k) = j$. Then u is a totally λ_g^α-continuous map but not a perfectly λ_g^α-continuous map, since for the λ_g^α-closed set $\{k\}$ in (N, ν), $u^{-1}(\{k\}) = \{j\}$ is not clopen in (M, μ).

Proposition 3.4.29 Every quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous) map $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-irresolute map.

Proof: Let S be any λ_g^α-closed set in (N,ν). As u is quasi λ_g^α-continuous (resp. perfectly λ_g^α-continuous), $u^{-1}(T)$ is closed (resp. clopen) in $(M,\mu) \Rightarrow u^{-1}(T)$ is λ_g^α-closed in (M,μ). Hence u is a λ_g^α-irresolute map.

The subsequent examples show that the converse of Proposition 3.4.29 may not hold good.

Example 3.4.30 Let $M = N = \{i,j,k\}$, $\mu = \{\phi,\{i\},\{j\},\{i,j\},\{i,k\},M\}$ and $\nu = \{\phi,\{i\},\{i,j\},\{i,k\},N\}$. Let $u:(M,\mu) \to (N,\nu)$ be a map defined by $u(i) = j, u(j) = i$ and $u(k) = k$. Then u is a λ_g^α-irresolute map but not a quasi λ_g^α-continuous map, since for the λ_g^α-closed set $\{i,j\}$ in (N,ν), $u^{-1}(\{i,j\}) = \{i,j\}$ is not closed in (M,μ).

Example 3.4.31 Consider M,N,μ,ν and u as in Example 3.4.30. Then u is a λ_g^α-irresolute map but not a perfectly λ_g^α-continuous map, since for the λ_g^α-closed set $\{k\}$ in (N,ν), $u^{-1}(\{k\}) = \{k\}$ is not clopen in (M,μ).

Remark 3.4.32 Perfectly λ_g^α-continuous maps and strongly λ_g^α-continuous maps are independent as observed from their respective definitions. In the same way, quasi λ_g^α-continuous maps and totally λ_g^α-continuous maps are also independent.

Remark 3.4.33 The above implications are depicted in the diagram below.