5

γ Generalized Homeomorphisms in Intuitionistic Fuzzy Topological Space

5.1 Introduction

The notion of homeomorphisms plays an important role in topological spaces. Maki et al. (1991) introduced generalized homeomorphisms and studied their properties. Benchalli, Jenifer Karnel and Siddapur (2012) introduced fuzzy b-generalized homeomorphism in fuzzy topological spaces. Santhi and Sakthivel (2011b) have introduced intuitionistic fuzzy alpha generalized homeomorphism and intuitionistic fuzzy M-alpha generalized homeomorphism in intuitionistic fuzzy topological spaces. In this chapter we have introduced intuitionistic fuzzy γ generalized homeomorphism and intuitionistic fuzzy M - γ generalized homeomorphism in intuitionistic fuzzy topological spaces. Also we have proved that the set of all intuitionistic fuzzy M - γ generalized homeomorphism forms a group under the operation of composition of maps.

5.2 Intuitionistic fuzzy γ generalized homeomorphisms

In this section we have introduced intuitionistic fuzzy γ generalized homeomorphism and investigated some of their properties.

Definition 5.2.1: A bijection mapping f: (X, τ) → (Y,σ) is called an ***intuitionistic fuzzy γ generalized*** (IFγG) ***homeomorphism*** if f is both an IFγG continuous mapping and an IFγG open mapping.

Example 5.2.2: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$ where $\mu_{G_1}(a) = 0.3_a$, $\mu_{G_1}(b) = 0.2_b$, $\nu_{G_1}(a) = 0.7_a$, $\nu_{G_1}(b) = 0.8_b$, $G_2 = \langle y, (0.4_u, 0.5_v), (0.6_u, 0.5_v)\rangle$ where $\mu_{G_2}(u) = 0.4_a$, $\mu_{G_2}(v) = 0.5_b$, $\nu_{G_2}(u) = 0.6_a$, $\nu_{G_2}(v) = 0.5_b$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG homeomorphism.

Proposition 5.2.3: Every IF homeomorphism is an IFγG homeomorphism but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF homeomorphism. Then f is both an IF continuous mapping and an IF open mapping. This implies f is both an IFγG continuous mapping and an IFγG open mapping. Hence f is an IFγG homeomorphism.

Example 5.2.4: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.5_v), (0.6_u, 0.5_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG homeomorphism but not an IF homeomorphism. For consider the IFCS $G_2^c = \langle y, (0.6_u, 0.5_v), (0.4_u, 0.5_v) \rangle$ in Y. Then $f^{-1}(G_2^c) = \langle x, (0.6_a, 0.5_b), (0.4_a, 0.5_b)$ is not an IFCS in X, as $cl(f^{-1}(G_2^c)) = G_1^c \neq f^{-1}(G_2^c)$. This implies f is not an IF continuous mapping. Hence f is not an IF homeomorphism.

Proposition 5.2.5: Every IFS homeomorphism is an IFγG homeomorphism but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFS homeomorphism. Then f is both an IFS continuous mapping and an IFS open mapping. This implies f is both an IFγG continuous mapping and an IFγG open mapping. Hence f is an IFγG homeomorphism.

Example 5.2.6: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)\rangle$, $G_2 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG homeomorphism but not an IFS homeomorphism. For, consider the IFCS $G_2^c = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v) \rangle$ in Y. Then $f^{-1}(G_2^c) = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_2^c))) = 1_\sim \not\subseteq f^{-1}(G_2^c)$. This implies f is not an IFS continuous mapping. Hence f is not an IFS homeomorphism.

Proposition 5.2.7: Every IFP homeomorphism is an IFγG homeomorphism but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFP homeomorphism. Then f is both an IFP continuous mapping and an IFP open mapping. This implies f is both an IFγG continuous mapping and an IFγG open mapping. Hence f is an IFγG homeomorphism.

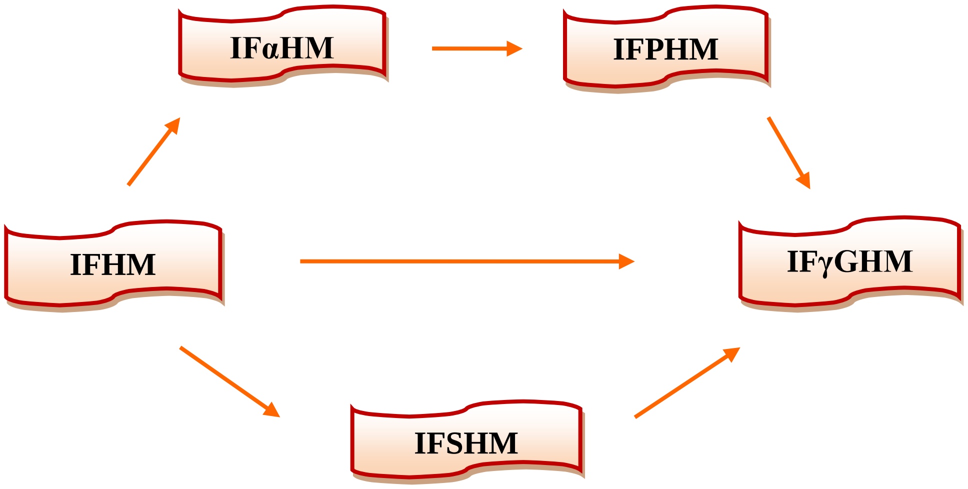
Example 5.2.8: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.5_v), (0.6_u, 0.5_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG homeomorphism but not an IFP homeomorphism. For, consider the IFCS $G_2^c = \langle y, (0.6_u, 0.5_v), (0.4_u, 0.5_v) \rangle$ in Y. Then $f^{-1}(G_2^c) = \langle x, (0.6_a, 0.5_b), (0.4_a, 0.5_b)$ is not an IFPCS in X, as $cl(int(f^{-1}(G_2^c))) = cl(G_1) = G_1^c \nsubseteq f^{-1}(G_2^c)$. This implies f is not an IFP continuous mapping. Hence f is not an IFP homeomorphism.

Proposition 5.2.9: Every IFα homeomorphism is an IFγG homeomorphism but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFα homeomorphism. Then f is both an IFα continuous mapping and an IFα open mapping. This implies f is both an IFγG continuous mapping and an IFγG open mapping. Therefore f is an IFγG homeomorphism.

Example 5.2.10: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)\rangle$, $G_2 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFγG homeomorphism but not an IFα homeomorphism. For, consider the IFCS $G_2^c = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v) \rangle$ in Y. Then $f^{-1}(G_2^c) = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)$ is not an IFαCS in X, as $cl(int(cl(f^{-1}(G_2^c)))) = cl(int(1_\sim)) = 1_\sim \nsubseteq f^{-1}(G_2^c)$. This implies f is not an IFα continuous mapping. Hence f is not an IFα homeomorphism.

The relation between various types of intuitionistic fuzzy homeomorphisms is given in the following diagram. In this diagram "HM" means homeomorphism.



The reverse implications are not true in general in the above diagram.

Proposition 5.2.11: Let f : (X, τ) → (Y, σ) be a bijective mapping from an IFTS (X, τ) into an IFTS (Y, σ), then the following statements are equivalent:

(i) f is an IFγG open mapping,

(ii) f is an IFγG closed mapping,

(iii) f^{-1}: (Y, σ) → (X, τ) is an IFγG continuous mapping.

Proof: (i) ⇒ (ii): Let A be an IFCS in X, then A^c is an IFOS in X. By hypothesis, $f(A^c) = (f(A))^c$ is an IFγGOS in Y. Therefore f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

(ii) ⇒ (iii): Let B be an IFCS in X. Since f is an IFγG closed mapping, $f(A) = (f^{-1})^{-1}(A)$ is an IFγGCS in Y. Hence f^{-1} is an IFγG continuous mapping.

(iii) ⇒ (i): Let A be an IFOS in X. By hypothesis, $(f^{-1})^{-1}(A) = f(A)$ is an IFγGOS in Y. Hence f is an IFγG open mapping.

Proposition 5.2.12: Let f: (X, τ) → (Y, σ) be an IFγG homeomorphism, then f is an IF homeomorphism if X and Y are $IF\gamma_c\ T_{1/2}$ space.

Proof: Let B be an IFCS in Y. Then $f^{-1}(B)$ is an IFγGCS in X, by hypothesis. Since X is an $IF\gamma_cT_{1/2}$ space, $f^{-1}(B)$ is an IFCS in X. Hence f is an IF continuous mapping. By hypothesis f^{-1}: (Y, σ) → (X, τ) is a IFγG continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}(A) = f(A)$ is an IFγGCS in Y, by hypothesis. Since Y is an $IF\gamma_cT_{1/2}$ space, f(A) is an IFCS in Y. Hence f^{-1} is an IF continuous mapping. Therefore the mapping f is an IF homeomorphism.

Proposition 5.2.13: Let f: (X, τ) → (Y, σ) be an IFγG homeomorphism, then f is an IFP homeomorphism if X and Y are $IF\gamma_pT_{1/2}$ space.

Proof: Let B be an IFCS in Y. Then $f^{-1}(B)$ is an IFγGCS in X, by hypothesis. Since X is an $IF\gamma_pT_{1/2}$ space, $f^{-1}(B)$ is an IFPCS in X. Hence f is an IFP continuous mapping. Similarly f^{-1} is also an IFP continuous mapping. Therefore the mapping f is an IFP homeomorphism.

Proposition 5.2.14: Let f: (X, τ) → (Y, σ) be a bijective mapping. If f is an IFγG continuous mapping, then the following are equivalent:
(i) f is an IFγG closed mapping,
(ii) f is an IFγG open mapping,
(iii) f is an IFγG homeomorphism.

Proof: (i) ⟹ (ii): Let f: (X, τ) → (Y, σ) be a bijective mapping and let f be an IFγG closed mapping. Then f^{-1}: (Y, σ) → (X, τ) is an IFγG continuous mapping. That is the inverse image of every IFOS in X is an IFγGOS in Y. Hence f is an IFγG open mapping.

(ii) ⟹ (iii): Let f: (X, τ) → (Y, σ) be a bijective mapping and an IFγG open mapping. Then f^{-1}: (Y, σ) → (X, τ) is an IFγG continuous mapping and therefore f is an IFγG open mapping. Hence f is both an IFγG continuous mapping and an IFγG open mapping. Therefore f is an IFγG homeomorphism.

(iii) ⟹ (i): Let f be an IFγG homeomorphism. That is f is both an IFγG continuous mapping and an IFγG open mapping. As $f(A^c) = (f(A))^c$ for a bijective mapping, f is an IFγG closed mapping.

Remark 5.2.15: The composition of two IFγG homeomorphisms need not be an IFγG homeomorphism in general.

Example 5.2.16: Let X = {a, b}, Y = {u, v}, Z = {p, q}. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ $\delta = \{0_\sim, G_4, 1_\sim\}$ are IFTs on X, Y and Z respectively, where $G_1 = \langle x, (0.5_a, 0.7_b), (0.5_a, 0.3_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_3 = \langle y, (0.4_u, 0.5_v), (0.6_u, 0.5_v)\rangle$, $G_4 = \langle z, (0.4_p, 0.3_q), (0.6_p, 0.7_q)\rangle$. Then (X, τ), (Y, σ) and (Z, δ) are IFTSs. Now define a bijective mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v and g: (Y, σ) → (Z, δ) by g(u) = p and g(v) = q.

IFγO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] /$ either $\nu_a < 0.5$ or $\nu_b < 0.6$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1\}$.

IFγC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] /$ either $\mu_a < 0.5$ or $\mu_b < 0.6$, $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1\}$.

IFγO(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1] / 0 \le \mu_u + \nu_u \le 1$ and $0 \le \mu_v + \nu_v \le 1\}$.

IFγC(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1] / 0 \le \mu_u + \nu_u \le 1$ and $0 \le \mu_v + \nu_v \le 1\}$.

Then f is both an IFγG continuous mapping and an IFγG open mapping. Also g is both an IFγG continuous mapping and an IFγG open mapping. Hence f and g are IFγG homeomorphisms. But the composition $g \circ f$: X → Z is not an IFγG homeomorphism, since $g \circ f$ is not an IFγG continuous mapping.

Proposition 5.2.17: Let f: (X, τ) → (Y, σ) and g : (Y, σ) → (Z, δ) be two IFγG homeomorphisms and (Y, σ) is an IFγ_c $T_{1/2}$ space. Then $g \circ f$ is an IFγG homeomorphism.

Proof: Let A be an IFCS in Z. Since g : (Y, σ) → (Z, δ) is an IFγG continuous mapping, $g^{-1}(A)$ is an IFγGCS in Y. Then $g^{-1}(A)$ is an IFCS in Y as (Y, σ) is an IFγ_c $T_{1/2}$ space.

Also since f: (X, τ) → (Y, σ) is an IFγG continuous mapping, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is an IFγGCS in X. Hence g $\circ$ f is an IFγG continuous mapping.

Let A be an IFCS in X. Since f^{-1}: (Y, σ) → (X, τ) is an IFγG continuous mapping, $(f^{-1})^{-1}(A) = f(A)$ is an IFγGCS in Y. Then f(A) is an IFCS in Y as (Y, σ) is an IFγ_c $T_{1/2}$ space. Also since g^{-1}: (Z, δ)→ (Y, σ) is an IFγG continuous mapping, $(g^{-1})^{-1}(f(A)) = g(f(A)) = (g \circ f)(A)$ is an IFγGCS in Z. Therefore $((g \circ f)^{-1})^{-1}(A) = (g\circ f)(A)$ is an IFγGCS in Z. Hence $(g \circ f)^{-1}$ is an IFγG continuous mapping. Thus g $\circ$ f is an IFγG homeomorphism.

5.3. Intuitionistic fuzzy M - γ generalized homeomorphisms

Definition 5.3.1: Let f: (X, τ) → (Y,σ) be a bijective mapping. Then f is said to be an ***intuitionistic fuzzy M-γ generalized*** (IFM-γG) ***homeomorphism*** if f is both an IFγG irresolute mapping and an IFM-γG open mapping.

Example 5.3.2: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is both an IFγG irresolute mapping and an IFM-γG open mapping. Hence f is an IFM-γG homeomorphism.

Proposition 5.3.3: Every IFM-γG homeomorphism is an IFγG homeomorphism but not conversely in general.

Proof: Assume that f: (X, τ) → (Y, σ) be an IFM-γG homeomorphism. Let A ⊆ Y be an IFCS. Then A is an IFγGCS in Y. By hypothesis $f^{-1}(A)$ is an IFγGCS in X. Hence f is an IFγG continuous mapping. Let B ⊆ X be an IFOS. Then B is an IFγGOS in X. By hypothesis f(B) is an IFγGOS in Y. Hence f is an IFγG open mapping. Thus f is an IFγG homeomorphism.

Example 5.3.4: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.5_v), (0.6_u, 0.5_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a bijective mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

IFγO(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]/$ either $\mu_a < 0.5$ or $\mu_b < 0.6$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$.

IFγC(X) = $\{0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1]/$ either $\mu_a < 0.5$ or $\mu_b < 0.6$, $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$.

IFγO(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]/\ 0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$.

IFγC(Y) = $\{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1]/\ 0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$.

Then f is an IFγG homeomorphism but not an IFM-γG homeomorphism, since the IFS A= ⟨y, (0.7, 0.8), (0.3, 0.2)⟩ in Y is an IFγGCS in Y, but $f^{-1}(A)$ is not an IFγGCS in X. This implies f is not an IFγG irresolute mapping.

Proposition 5.3.5: If f: (X, τ) → (Y, σ) is an IFM-γG homeomorphism, then $\gamma gcl(f^{-1}(B)) \subseteq f^{-1}(\gamma cl(B))$ for every IFS B in Y.

Proof: Let B ⊆ Y. Then γcl(B) is an IFγCS in Y. This implies γcl(B) is an IFγGCS in Y. Since f is an IFγG irresolute mapping, $f^{-1}(\gamma cl(B))$ is an IFγGCS in X. This implies $\gamma gcl(f^{-1}(\gamma cl(B))) = f^{-1}(\gamma cl(B))$. Now $\gamma gcl(f^{-1}(B)) \subseteq \gamma gcl(f^{-1}(\gamma cl(B))) = f^{-1}(\gamma cl(B))$. Hence $\gamma gcl(f^{-1}(B)) \subseteq f^{-1}(\gamma cl(B))$ for every IFS B in Y.

Proposition 5.3.6: If f: (X, τ) → (Y, σ) is an IFM-γG homeomorphism, where X and Y are $IF\gamma_\gamma\ T_{1/2}$ spaces, then $\gamma cl(f^{-1}(B)) = f^{-1}(\gamma cl(B))$ for every IFS B in Y.

Proof: Since f is an IFM-γG homeomorphism, f is an IFγG irresolute mapping. Let B ⊆ Y. Then since γcl(B) is an IFγGCS in Y, $f^{-1}(\gamma cl(B))$ is an IFγGCS in X. Since X is an $IF\gamma_\gamma\ T_{1/2}$ spaces, $f^{-1}(\gamma cl(B))$ is an IFγCS in X. Now $f^{-1}(B) \subseteq f^{-1}(\gamma cl(B))$. We have

$\gamma cl(f^{-1}(B)) \subseteq \gamma cl(f^{-1}(\gamma cl(B))) = f^{-1}(\gamma cl(B)))$. This implies $\gamma cl(f^{-1}(B)) \subseteq f^{-1}(\gamma cl(B))$___(1). Again since f is an IFM-γG homeomorphism, f^{-1} is an IFγG irresolute mapping. Since $\gamma cl(f^{-1}(B))$ is an IFγGCS in X. This implies $(f^{-1})^{-1}(\gamma cl(f^{-1}(B))) = f(\gamma cl(f^{-1}(B)))$ is an IFγGCS in Y. Now $B \subseteq (f^{-1})^{-1}(f^{-1}(B)) \subseteq (f^{-1})^{-1}(\gamma cl(f^{-1}(B))) = f(\gamma cl(f^{-1}(B)))$. Therefore $\gamma cl(B) \subseteq \gamma cl(f(\gamma cl(f^{-1}(B)))) = f(\gamma cl(f^{-1}(B))$, since Y is an IF$\gamma_\gamma$ $T_{1/2}$ space. Hence $f^{-1}(\gamma cl(B)) \subseteq f^{-1}(f(\gamma cl(f^{-1}(B)) \subseteq \gamma cl(f^{-1}(B))$. That is $f^{-1}(\gamma cl(B)) \subseteq \gamma cl(f^{-1}(B))$___(2). From (1) and (2) we get $\gamma cl(f^{-1}(B)) = f^{-1}(\gamma cl(B))$.

Proposition 5.3.7: If f: (X, τ) → (Y, σ) is an IFM-γG homeomorphism, where X and Y are IFγ_γ $T_{1/2}$ spaces, then $\gamma cl(f(B)) = f(\gamma cl(B))$ for every IFS B in X.

Proof: Since f is an IFM-γG homeomorphism, f^{-1} is also an IFM-γG homeomorphism. Therefore by Proposition 5.3.6, $\gamma cl((f^{-1})^{-1}(B)) = (f^{-1})^{-1}(\gamma cl(B))$ for every $B \subseteq X$. That is $\gamma cl(f(B)) = f(\gamma cl(B))$ for every IFS B in X.

Proposition 5.3.8: Let f: (X, τ) → (Y, σ) be a bijective mapping. If f is an IFγG irresolute mapping, then the following are equivalent:

(i) f is an IFM-γG closed mapping,

(ii) f is an IFM-γG open mapping,

(iii) f is an IFM-γG homeomorphism.

Proof: Straightforward.

Proposition 5.3.9: The composition of two IFM-γG homeomorphisms is an IFM-γG homeomorphism in general.

Proof: Assume that f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, δ) are any two IFM-γG homeomorphisms. Let $A \subseteq Z$ be an IFγGCS. Then by hypothesis, $g^{-1}(A)$ is an IFγGCS in Y. Again by hypothesis, $f^{-1}(g^{-1}(A))$ is an IFγGCS in X. Hence $g \circ f$ is an IFγG irresolute mapping. Now let $B \subseteq X$ be an IFγGCS in X. Then by hypothesis, f(B) is an IFγGCS in Y and also g(f(B)) is an IFγGCS in Z. This implies $g \circ f$ is an IFM-γG open mapping. Hence $g \circ f$ is an IFM-γG homeomorphism.