Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test II – April - 2025

II Semester

Class : I PG **Time: 2Hrs**
Major: Mathematics **Max.Marks:60**

23MMAC09 Partial Differential Equations

Course Outcomes:
CO1: solve linear and non-linear partial differential equations of first order and second order.
CO2: determine special types of first order equations.
CO3: find the solution of Hyperbolic equations.
CO4: apply the Dirichlet and Neumann boundary value problems in scientific fields.
CO5: solve various real life problems by formulating them into partial differential equations

PART A **6 x 1 = 6**

Choose the Correct Answer

1. Which of the following is a linear hyperbolic equation of the second order in two independent variables? CO3K1
   a. $\frac{\partial^2 z}{\partial x \partial y} = f(x, y, z_x, z_y)$ b. $\frac{\partial^2 z}{\partial x \partial y} = f(x, y, z, z_x, z_y)$
   c. $\frac{\partial^2 z}{\partial^2 x} + \frac{\partial^2 z}{\partial^2 y} = \frac{\partial z}{\partial x}$ d. $\frac{\partial^2 z}{\partial^2 x} + \frac{\partial^2 z}{\partial^2 y} = \frac{\partial z}{\partial y}$

2. The solution of one dimensional diffusion equation $\frac{\partial^2 z}{\partial^2 x} = \frac{1}{k}\frac{\partial z}{\partial t}$ is ________ . CO3K2
   a. $z(x,y) = c_n \cos(nx + \xi_n)e^{-n^2 ky}$ b. $z(x,y) = c_n \sin(nx + \xi_n)e^{-n^2 ky}$
   c. $z(x,t) = c_n \cos(nx + \xi_n)e^{-n^2 kt}$ d. $z(x,t) = c_n \sin(nx + \xi_n)e^{-n^2 kt}$

3. The surfaces $f(x, y, z) = c$ is equipotential if the potential function $\psi(x, y, z) = c$ is constant whenever $f(x, y, z)$ is _____ . CO4K2
   a. constant b. function of x alone
   c. function of y alone d. function of z alone

4. The potential function $\psi(z,0,0)$ due to a uniform circular wire of radius a charged with electricity of line density e is ------ . CO4K2
   a. $\frac{2\pi ea}{a^2+z^2}$ b. $\frac{2\pi ea}{\sqrt{a^2+z^2}}$ c. $\frac{\pi ea}{\sqrt{a^2+z^2}}$ d. $\frac{\pi ea}{a^2+z^2}$

5. In wave equation z vanishes on the circle r=a, then the number k must be chosen so that __.
   a. $J_m(ka) = k$ b. $J_m(ka) = a$ c. $J_m(ka) = 0$ d. $J_m(ka) = c$ CO5K2

6. The simple solution of the three dimensional wave equation is of the form _____. CO5K2
   a. $e^{(\pm i(lx+my+nz+kct)}$ b. $e^{(+i(lx+my+nz+kct)}$ c. $e^{(-i(lx+my+nz+kct)}$ d. $e^{(\pm i(lx+my+nz+kt)}$

Part-B **3 x 6 = 18**

Answer ALL the questions

7. a. By separating the variables, obtain the solution of one-dimensional wave equation. CO3K3
(or)

7. b. Briefly describe Riemann's method of solution of linear hyperbolic equations of second order in two independent variables. CO3K4

8. a. Show that the surfaces $x^2 + y^2 + z^2 = cx^{2/3}$ can be a family of equipotential surfaces, and find the general form of the corresponding potential function. CO4K2

(or)

8. b. State and prove the necessary condition for any one-parameter family of surfaces

$f(x, y, z) = c$ to be a family of equipotential surfaces. CO4K3

9. a. Obtain a general solution of the wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ assuming that the string is of

Infinite extent with initial condition $y = \eta(x)$ $\frac{\partial y}{\partial t} = v(x)$ at $t = 0$. CO5K3

(or)

9. b. A thin membrane of great extent is released from rest in the position $z = f(x, y)$. Determine

the displacement at any subsequent time. CO5K4

Part-C **3 x 12 = 36**

Answer ALL the questions

10. a. Find approximate values for the first three eigen values of a square membrane of side 2.

CO5K3

(or)

10. b. By separating the variables, show that the one-dimensional wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$

has solution of the form $A\exp(\pm inx \pm inct)$ where A and n are constants. Hence show

that functions of the form $z(x,t) = \sum_r \left(A_r \cos\frac{r\pi ct}{a} + B_r \sin\frac{r\pi ct}{a} \right) \sin\frac{r\pi x}{a}$ where A_r's and

B_r's are constants, satisfy the wave equation and the boundary conditions z(0,t)=0,

z(a,t)=0 for all t. CO3K4

11. a. A uniform circular wire of radius *a* charged with electricity of line density *e* surrounds grounded concentric spherical conductor of radius *c*. Determine the electrical charge density at any point on the conductor. CO4K3

(or)

11. b. Show that the family of right circular cone $x^2+y^2 = c\, z^2$, where c is a parameter, forms a set of equipotential surfaces, and show that the corresponding potential function is of the form A log tan (1/2)θ+B, where A and B are constants and θ is the usual polar angle. CO4K4

12. a. The points of trisection of a string are pulled aside through a distance on opposite sides of the position of equilibrium, and the string is release from rest. Derive an expression for that the string displacement of at any subsequent the string the midpoint of time and show remains at rest. CO5K3

(or)

12. b. A gas is contained in a cubical box of side a. Show that if c is the velocity of sound in the

gas, the periods of free oscillation are $\frac{2a}{e\sqrt{n_1^2 + n_2^2 + n_3^2}}$ where n_1, n_2, n_3 are integers. CO5K4

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