

## **$\alpha$ -CUTS OF TRIANGULAR FUZZY NUMBERS AND $\alpha$ -CUTS OF TRIANGULAR FUZZY NUMBER MATRICES**

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### **ABSTRACT**

*In this paper, some new elementary operators on  $\alpha$ -cuts of Triangular Fuzzy Numbers (TFNs) and some new operators on  $\alpha$ -cuts of Triangular Fuzzy Number Matrices (TFNMs) are defined. Using these operators, some important properties are proved.*

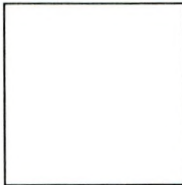
### **INTRODUCTION**

Real world decision making problems are very often uncertain (or) vague in a number of ways. In 1965, Zadeh [5] introduced the concept of fuzzy set theory to meet those problems. The fuzzyness can be represented by different ways. One of the most useful representation is the membership function. Depending on the nature of the membership function the fuzzy numbers can be classified in different forms, such as Triangular Fuzzy Numbers (TFNs), Trapezoidal Fuzzy Numbers, Interval Fuzzy Numbers etc. Fuzzy matrices play an important role in scientific development. Fuzzy matrices were introduced by M.G.Thomason [4]. A.K.Shymal and M.Pal introduced Triangular Fuzzy Number Matrices [2]. Two new operators and some properties of fuzzy matrices over these new operators are given in [1]. Some new operators on triangular fuzzy numbers and triangular fuzzy number matrices are given in [3].

In this paper, some new elementary operators on  $\alpha$ -cuts of Triangular Fuzzy Numbers (TFNs) and some new operators on  $\alpha$ -cuts of Triangular Fuzzy Number Matrices (TFNMs) are defined. Using these operators, some important properties are proved.

#### **1. Basic Definitions and Properties**

**Definition : 1.1 [2]**

A **Triangular Fuzzy Number** denoted by  =  $\langle m, \alpha, \beta \rangle$  has the membership function.

$$\mu_M(x) = \begin{cases} 0 & \text{for } x \leq m - \alpha \\ 1 - \frac{m-x}{\alpha} & \text{for } m - \alpha < x < m \\ 1 & \text{for } x = m \\ 1 - \frac{x-m}{\beta} & \text{for } m < x < m + \beta \\ 0 & \text{for } x \geq m + \beta \end{cases}$$

The point  $m$ , with membership grade of 1, is called the mean value and  $\alpha, \beta$  are the left hand and right hand spreads of  $M$  respectively.

**Example : 1.2**

$\tilde{M} = \langle 3, 6, 7 \rangle$  is a triangular fuzzy number.

**Definition : 1.3 [2]**

A **Triangular Fuzzy Number Matrix** of order  $m \times n$  is defined as  $M = (\tilde{M}_{ij})_{m \times n}$  where  $\tilde{M}_{ij} = \langle m_{ij}, \alpha_{ij}, \beta_{ij} \rangle$

**Example : 1.4**

$$M = \begin{bmatrix} \langle 2, 3, 4 \rangle & \langle 4, 6, 7 \rangle & \langle 3, 6, 7 \rangle \\ \langle 0, 1, 0 \rangle & \langle 1, 2, 3 \rangle & \langle 6, 8, 10 \rangle \\ \langle 7, 9, 11 \rangle & \langle 0, 0, 0 \rangle & \langle 5, 3, 4 \rangle \end{bmatrix} \text{ and}$$

$$N = \begin{bmatrix} \langle 0.3, 0.3, 0.6 \rangle & \langle 0.8, 0.1, 0.3 \rangle \\ \langle 0.4, 0.9, 0.5 \rangle & \langle 0.2, 0.1, 0.3 \rangle \end{bmatrix} \text{ are triangular fuzzy number matrices.}$$

**Definition : 1.5 [3]**

Let  $\tilde{M} = \langle m, \rho, \beta \rangle$  and  $\tilde{N} = \langle n, \gamma, \delta \rangle$  be two Triangular Fuzzy Numbers where  $m, \rho, \beta, n, \gamma, \delta \in [0,1]$ . Then the following operators are defined.

- (1)  $\tilde{M} \oplus \tilde{N} = \langle m + n - mn, \rho + \gamma - \rho\gamma, \beta + \delta - \beta\delta \rangle$
- (2)  $\tilde{M} \vee \tilde{N} = \langle m \vee n, \rho \vee \gamma, \beta \vee \delta \rangle$  where  $x \vee y$  means  $\max \{x, y\}$ . That is,  $x \vee y = \max \{x, y\}$ .
- (3)  $\tilde{M} \ominus \tilde{N} = \langle m \ominus n, \rho \ominus \gamma, \beta \ominus \delta \rangle$  where  $x \ominus y = \begin{cases} x, & \text{if } x > y \\ 0, & \text{if } x \leq y \end{cases}$
- (4)  $\tilde{M} \geq \tilde{N}$  if  $m \geq n, \rho \geq \gamma$  and  $\beta \geq \delta$
- (5) For  $\alpha \in [0, 1]$ ,  $\tilde{M} \geq \alpha$  if  $m \geq \alpha, \rho \geq \alpha, \beta \geq \alpha$  and  $\alpha \geq \tilde{M}$  if  $\alpha \geq m, \alpha \geq \rho, \alpha \geq \beta$ .

**Definition : 1.6 [3]**

## α-CUTS OF TRIANGULAR FUZZY NUMBERS AND α-CUTS...

Let  $M = (\tilde{M}_{ij})_{m \times n}$  and  $N = (\tilde{N}_{ij})_{m \times n}$  be two Triangular Fuzzy Number Matrices of the same order.

Then the following operators are defined.

- (1)  $M \oplus N = (\tilde{M}_{ij} \oplus \tilde{N}_{ij})$
- (2)  $M \vee N = (\tilde{M}_{ij} \vee \tilde{N}_{ij})$
- (3)  $M \ominus N = (\tilde{M}_{ij} \ominus \tilde{N}_{ij})$
- (4)  $M \geq N$  iff  $\tilde{M}_{ij} \geq \tilde{N}_{ij} \quad \forall i = 1 \text{ to } m, j = 1 \text{ to } n.$

### 2. α-Cuts of Triangular Fuzzy Numbers and α-Cuts of Triangular Fuzzy Number Matrices

**Definition : 2.1 [1]**

For all  $x, \alpha \in [0, 1]$  the **upper α-cut of x** is denoted as  $x^{(\alpha)}$  and is defined as

$$x^{(\alpha)} = \begin{cases} 1, & \text{if } x \geq \alpha \\ 0, & \text{if } x < \alpha \end{cases}$$

and the **lower α-cut of x** is denoted as  $x_{(\alpha)}$  and is defined as  $x_{(\alpha)} = \begin{cases} x, & \text{if } x \geq \alpha \\ 0, & \text{if } x < \alpha \end{cases}$

Now we define α-cuts of the Triangular Fuzzy Numbers and Triangular Fuzzy Number Matrices.

**Definition : 2.2**

For  $\alpha \in [0, 1]$ , the **upper α-cut of the Triangular Fuzzy Number**  $\tilde{M} = \langle m, \rho, \beta \rangle$  is defined as

$$\tilde{M}^{(\alpha)} = \langle m^{(\alpha)}, \rho^{(\alpha)}, \beta^{(\alpha)} \rangle$$

and the **lower α-cut of**  $\tilde{M}$  is defined as

$$\tilde{M}_{(\alpha)} = \langle m_{(\alpha)}, \rho_{(\alpha)}, \beta_{(\alpha)} \rangle$$

**Example : 2.3**

Consider the triangular fuzzy number  $\tilde{M}$  as follows :

$$\tilde{M} = \langle 0.5, 0.1, 0.4 \rangle. \text{ By taking } \alpha = 0.5, \text{ we get}$$

$$\tilde{M}_{(\alpha)} = \langle 0.5, 0, 0 \rangle \text{ and}$$

$$\tilde{M}^{(\alpha)} = \langle 1, 0, 0 \rangle$$

**Definition : 2.4**

The **upper α-cut of a Triangular Fuzzy Number Matrix**  $M = (\tilde{M}_{ij})_{m \times n}$  is defined as

$$M^{(\alpha)} = (\tilde{M}_{ij}^{(\alpha)})_{m \times n}$$

and the **lower α-cut of M** is defined as

$$M_{(\alpha)} = (\tilde{M}_{ij(\alpha)})_{m \times n}$$

**Example : 2.5**

Consider the triangular fuzzy number matrix M as follows :

$$M = \begin{bmatrix} \langle 0.5, 0.1, 0.4 \rangle & \langle 0.1, 0.7, 0.6 \rangle \\ \langle 0.3, 0.5, 0.2 \rangle & \langle 0.6, 0.3, 0.65 \rangle \end{bmatrix}$$

Then by taking  $\alpha = 0.5$ , we get

$$M^{(\alpha)} = \begin{bmatrix} \langle 0.5, 0, 0 \rangle & \langle 0, 0.7, 0.6 \rangle \\ \langle 0, 0.5, 0 \rangle & \langle 0.6, 0, 0.65 \rangle \end{bmatrix} \text{ and}$$

$$M^{(\alpha)} = \begin{bmatrix} \langle 1, 0, 0 \rangle & \langle 0, 1, 1 \rangle \\ \langle 0, 1, 0 \rangle & \langle 1, 0, 1 \rangle \end{bmatrix}$$

**Theorem : 2.6**

For any two triangular fuzzy number matrices M and N,

- (i)  $(M \ominus N)^{(\alpha)} \geq M^{(\alpha)} \ominus N^{(\alpha)}$
- (ii)  $(M \vee N)^{(\alpha)} = M^{(\alpha)} \vee N^{(\alpha)}$
- (iii)  $(M \oplus N)^{(\alpha)} \geq M^{(\alpha)} \oplus N^{(\alpha)}$

**Proof**

Let  $M = (\tilde{M}_{ij})_{m \times n}$  where  $\tilde{M}_{ij} = \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle$  and

$N = (\tilde{N}_{ij})_{m \times n}$  where  $\tilde{N}_{ij} = \langle n_{ij}, \gamma_{ij}, \delta_{ij} \rangle$

- (i) Let  $\tilde{E}_{ij}$  and  $\tilde{D}_{ij}$  be the  $ij^{\text{th}}$  elements of  $M^{(\alpha)} \ominus N^{(\alpha)}$  and  $(M \ominus N)^{(\alpha)}$ .

$$\therefore \tilde{E}_{ij} = \tilde{M}_{ij}^{(\alpha)} \ominus \tilde{N}_{ij}^{(\alpha)} \text{ and } \tilde{D}_{ij} = (\tilde{M}_{ij} \ominus \tilde{N}_{ij})^{(\alpha)}$$

**Case 1 :  $\tilde{M}_{ij} \geq \tilde{N}_{ij} \geq \alpha$**

$$\langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle \geq \langle n_{ij}, \gamma_{ij}, \delta_{ij} \rangle \geq \alpha$$

$$\therefore m_{ij} \geq n_{ij} \geq \alpha, \rho_{ij} \geq \gamma_{ij} \geq \alpha, \beta_{ij} \geq \delta_{ij} \geq \alpha.$$

$$\begin{aligned} \text{Therefore, } \tilde{D}_{ij} &= (\tilde{M}_{ij} \ominus \tilde{N}_{ij})^{(\alpha)} \\ &= \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle^{(\alpha)} \\ &= \langle m_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \rangle \\ &= \langle 1, 1, 1 \rangle \end{aligned}$$

$$\begin{aligned} \text{and } \tilde{E}_{ij} &= \tilde{M}_{ij}^{(\alpha)} \ominus \tilde{N}_{ij}^{(\alpha)} \\ &= \langle m_{ij}^{(\alpha)} \ominus n_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)} \ominus \gamma_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \ominus \delta_{ij}^{(\alpha)} \rangle \\ &= \langle 1 \ominus 1, 1 \ominus 1, 1 \ominus 1 \rangle \\ &= \langle 0, 0, 0 \rangle \end{aligned}$$

$$\text{i.e., } \tilde{D}_{ij} > \tilde{E}_{ij}$$

**Case 2 :  $\tilde{M}_{ij} \geq \alpha \geq \tilde{N}_{ij}$**

$$\begin{aligned} \therefore \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle &\geq \alpha \geq \langle n_{ij}, \gamma_{ij}, \delta_{ij} \rangle \\ \therefore m_{ij} \geq \alpha \geq n_{ij}, \rho_{ij} \geq \alpha \geq \gamma_{ij}, \beta_{ij} \geq \alpha \geq \delta_{ij}. \end{aligned}$$

Then

$$\begin{aligned} \tilde{D}_{ij} &= (\tilde{M}_{ij} \ominus \tilde{N}_{ij})^{(\alpha)} \\ &= \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle^{(\alpha)} \\ &= \langle m_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \rangle \\ &= \langle 1, 1, 1 \rangle \end{aligned}$$

$$\begin{aligned} \text{and } \tilde{E}_{ij} &= \tilde{M}_{ij}^{(\alpha)} \ominus \tilde{N}_{ij}^{(\alpha)} \\ &= \langle m_{ij}^{(\alpha)} \ominus n_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)} \ominus \gamma_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \ominus \delta_{ij}^{(\alpha)} \rangle \\ &= \langle 1 \ominus 0, 1 \ominus 0, 1 \ominus 0 \rangle \\ &= \langle 1, 1, 1 \rangle \end{aligned}$$

$$\therefore \tilde{D}_{ij} = \tilde{E}_{ij}$$

**Case 3** :  $\alpha \geq \tilde{M}_{ij} \geq \tilde{N}_{ij}$

$$\begin{aligned} \alpha &\geq \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle \geq \langle n_{ij}, \gamma_{ij}, \delta_{ij} \rangle \\ \therefore \alpha \geq m_{ij} \geq n_{ij}, \alpha \geq \rho_{ij} \geq \gamma_{ij}, \alpha \geq \beta_{ij} \geq \delta_{ij}. \end{aligned}$$

Here

$$\begin{aligned} \tilde{D}_{ij} &= (\tilde{M}_{ij} \ominus \tilde{N}_{ij})^{(\alpha)} \\ &= \langle m_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \rangle \\ &= \langle 0, 0, 0 \rangle \end{aligned}$$

$$\begin{aligned} \text{and } \tilde{E}_{ij} &= \tilde{M}_{ij}^{(\alpha)} \ominus \tilde{N}_{ij}^{(\alpha)} \\ &= \langle m_{ij}^{(\alpha)} \ominus n_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)} \ominus \gamma_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \ominus \delta_{ij}^{(\alpha)} \rangle \\ &= \langle 0 \ominus 0, 0 \ominus 0, 0 \ominus 0 \rangle \\ &= \langle 0, 0, 0 \rangle \end{aligned}$$

$$\text{i.e., } \tilde{D}_{ij} = \tilde{E}_{ij}$$

$$\therefore \text{In all the cases, } \tilde{D}_{ij} \geq \tilde{E}_{ij}$$

$$\therefore (\tilde{M}_{ij} \ominus \tilde{N}_{ij})^{(\alpha)} \geq \tilde{M}_{ij}^{(\alpha)} \ominus \tilde{N}_{ij}^{(\alpha)}$$

$$\therefore (M \ominus N)^{(\alpha)} \geq M^{(\alpha)} \ominus N^{(\alpha)}.$$

(ii) Let  $\tilde{C}_{ij}$  and  $\tilde{D}_{ij}$  be the  $ij^{\text{th}}$  elements of  $M^{(\alpha)} \vee N^{(\alpha)}$  and  $(M \vee N)^{(\alpha)}$ .

$$\therefore \tilde{C}_{ij} = \tilde{M}_{ij}^{(\alpha)} \vee \tilde{N}_{ij}^{(\alpha)} \text{ and } \tilde{D}_{ij} = (\tilde{M}_{ij} \vee \tilde{N}_{ij})^{(\alpha)}$$

**Case 1** :  $\tilde{M}_{ij} \geq \tilde{N}_{ij} \geq \alpha$

$$\text{i.e., } \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle \geq \langle n_{ij}, \gamma_{ij}, \delta_{ij} \rangle \geq \alpha$$

$$\therefore m_{ij} \geq n_{ij} \geq \alpha, \rho_{ij} \geq \gamma_{ij} \geq \alpha \text{ and } \beta_{ij} \geq \delta_{ij} \geq \alpha.$$

$$\begin{aligned} \therefore \tilde{D}_{ij} &= (\tilde{M}_{ij} \vee \tilde{N}_{ij})^{(\alpha)} \\ &= \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle^{(\alpha)} \end{aligned}$$

$$\begin{aligned}
 &= \langle m_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \rangle \\
 &= \langle 1, 1, 1 \rangle \\
 \text{and } \tilde{C}_{ij} &= \tilde{M}_{ij}^{(\alpha)} \vee \tilde{N}_{ij}^{(\alpha)} \\
 &= \langle m_{ij}^{(\alpha)} \vee n_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)} \vee \gamma_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \vee \delta_{ij}^{(\alpha)} \rangle \\
 &= \langle 1 \vee 1, 1 \vee 1, 1 \vee 1 \rangle \\
 &= \langle 1, 1, 1 \rangle \\
 \text{i.e., } \tilde{D}_{ij} &= \tilde{C}_{ij}.
 \end{aligned}$$

**Case 2 :**  $\tilde{M}_{ij} \geq \alpha \geq \tilde{N}_{ij}$

$$\begin{aligned}
 \therefore \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle &\geq \alpha \geq \langle n_{ij}, \gamma_{ij}, \delta_{ij} \rangle \\
 \therefore m_{ij} \geq \alpha \geq n_{ij}, \rho_{ij} &\geq \alpha \geq \gamma_{ij}, \beta_{ij} \geq \alpha \geq \delta_{ij}.
 \end{aligned}$$

Then

$$\begin{aligned}
 \tilde{D}_{ij} &= (\tilde{M}_{ij} \vee \tilde{N}_{ij})^{(\alpha)} \\
 &= \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle^{(\alpha)} \\
 &= \langle m_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \rangle \\
 &= \langle 1, 1, 1 \rangle
 \end{aligned}$$

$$\begin{aligned}
 \text{and } \tilde{C}_{ij} &= \tilde{M}_{ij}^{(\alpha)} \vee \tilde{N}_{ij}^{(\alpha)} \\
 &= \langle m_{ij}^{(\alpha)} \vee n_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)} \vee \gamma_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \vee \delta_{ij}^{(\alpha)} \rangle \\
 &= \langle 1 \vee 0, 1 \vee 0, 1 \vee 0 \rangle \\
 &= \langle 1, 1, 1 \rangle
 \end{aligned}$$

$$\therefore \tilde{D}_{ij} = \tilde{C}_{ij}$$

**Case 3 :**  $\alpha \geq \tilde{M}_{ij} \geq \tilde{N}_{ij}$

$$\begin{aligned}
 \text{i.e., } \alpha &\geq \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle \geq \langle n_{ij}, \gamma_{ij}, \delta_{ij} \rangle \\
 \therefore \alpha &\geq m_{ij} \geq n_{ij}, \alpha \geq \rho_{ij} \geq \gamma_{ij}, \alpha \geq \beta_{ij} \geq \delta_{ij}.
 \end{aligned}$$

Then

$$\begin{aligned}
 \tilde{D}_{ij} &= (\tilde{M}_{ij} \vee \tilde{N}_{ij})^{(\alpha)} \\
 &= \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle^{(\alpha)} \\
 &= \langle m_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \rangle \\
 &= \langle 0, 0, 0 \rangle
 \end{aligned}$$

$$\begin{aligned}
 \text{and } \tilde{C}_{ij} &= (\tilde{M}_{ij}^{(\alpha)} \vee \tilde{N}_{ij}^{(\alpha)}) \\
 &= \langle m_{ij}^{(\alpha)} \vee n_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)} \vee \gamma_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \vee \delta_{ij}^{(\alpha)} \rangle \\
 &= \langle 0 \vee 0, 0 \vee 0, 0 \vee 0 \rangle \\
 &= \langle 0, 0, 0 \rangle
 \end{aligned}$$

$$\text{i.e., } \tilde{D}_{ij} = \tilde{C}_{ij}.$$

$$\therefore \text{In all the cases, } \tilde{D}_{ij} = \tilde{C}_{ij}$$

$$\therefore (\tilde{M}_{ij} \vee \tilde{N}_{ij})^{(\alpha)} = \tilde{M}_{ij}^{(\alpha)} \vee \tilde{N}_{ij}^{(\alpha)}$$

$$\therefore (M \vee N)^{(\alpha)} = M^{(\alpha)} \vee N^{(\alpha)}.$$

(iii) Let  $\tilde{P}_{ij}$  and  $\tilde{Q}_{ij}$  be the  $ij^{\text{th}}$  elements of  $M^{(\alpha)} \oplus N^{(\alpha)}$  and  $(M \oplus N)^{(\alpha)}$ .

$$\therefore \tilde{P}_{ij} = \tilde{M}_{ij}^{(\alpha)} \oplus \tilde{N}_{ij}^{(\alpha)} \text{ and } \tilde{Q}_{ij} = (\tilde{M}_{ij} \oplus \tilde{N}_{ij})^{(\alpha)}.$$

**Case 1 :**  $\tilde{M}_{ij} \geq \tilde{N}_{ij} \geq \alpha$

$$\therefore \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle \geq \langle n_{ij}, \gamma_{ij}, \delta_{ij} \rangle \geq \alpha$$

$$\therefore m_{ij} \geq n_{ij} \geq \alpha, \rho_{ij} \geq \gamma_{ij} \geq \alpha \text{ and } \beta_{ij} \geq \delta_{ij} \geq \alpha.$$

$$\begin{aligned} \text{Then } \tilde{Q}_{ij} &= (\tilde{M}_{ij} \oplus \tilde{N}_{ij})^{(\alpha)} \\ &= \langle m_{ij} + n_{ij} - m_{ij} \cdot n_{ij}, \rho_{ij} + \gamma_{ij} - \rho_{ij} \cdot \gamma_{ij}, \beta_{ij} + \delta_{ij} - \beta_{ij} \cdot \delta_{ij} \rangle^{(\alpha)} \\ &= \langle m_{ij} + n_{ij} (1 - m_{ij}), \rho_{ij} + \gamma_{ij} (1 - \rho_{ij}), \beta_{ij} + \delta_{ij} (1 - \beta_{ij}) \rangle^{(\alpha)} \\ &\geq \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle^{(\alpha)} \\ &> \langle m_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \rangle \\ &= \langle 1, 1, 1 \rangle \end{aligned}$$

$$\begin{aligned} \text{and } \tilde{P}_{ij} &= \tilde{M}_{ij}^{(\alpha)} \oplus \tilde{N}_{ij}^{(\alpha)} \\ &= \langle m_{ij}^{(\alpha)} \oplus n_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)} \oplus \gamma_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \oplus \delta_{ij}^{(\alpha)} \rangle \\ &= \langle m_{ij}^{(\alpha)} + n_{ij}^{(\alpha)} - m_{ij}^{(\alpha)} \cdot n_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)} + \gamma_{ij}^{(\alpha)} - \rho_{ij}^{(\alpha)} \cdot \gamma_{ij}^{(\alpha)}, \\ &\quad \beta_{ij}^{(\alpha)} + \delta_{ij}^{(\alpha)} - \beta_{ij}^{(\alpha)} \cdot \delta_{ij}^{(\alpha)} \rangle \\ &= \langle 1 + 1 - 1, 1 + 1 - 1, 1 + 1 - 1 \rangle \\ &= \langle 1, 1, 1 \rangle \\ &\therefore \tilde{Q}_{ij} > \tilde{P}_{ij}. \end{aligned}$$

**Case 2 :**  $\tilde{M}_{ij} \geq \alpha \geq \tilde{N}_{ij}$

$$\therefore \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle \geq \alpha \geq \langle n_{ij}, \gamma_{ij}, \delta_{ij} \rangle$$

$$\therefore m_{ij} \geq \alpha \geq n_{ij}, \rho_{ij} \geq \alpha \geq \gamma_{ij}, \beta_{ij} \geq \alpha \geq \delta_{ij}.$$

Then

$$\begin{aligned} \tilde{Q}_{ij} &= (\tilde{M}_{ij} \oplus \tilde{N}_{ij})^{(\alpha)} \\ &= \langle m_{ij} + n_{ij} - m_{ij} \cdot n_{ij}, \rho_{ij} + \gamma_{ij} - \rho_{ij} \cdot \gamma_{ij}, \beta_{ij} + \delta_{ij} - \beta_{ij} \cdot \delta_{ij} \rangle^{(\alpha)} \\ &= \langle m_{ij} + n_{ij} (1 - m_{ij}), \rho_{ij} + \gamma_{ij} (1 - \rho_{ij}), \beta_{ij} + \delta_{ij} (1 - \beta_{ij}) \rangle^{(\alpha)} \\ &\geq \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle^{(\alpha)} \\ &> \langle m_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \rangle \\ &= \langle 1, 1, 1 \rangle \end{aligned}$$

$$\begin{aligned} \text{and } \tilde{P}_{ij} &= \tilde{M}_{ij}^{(\alpha)} \oplus \tilde{N}_{ij}^{(\alpha)} \\ &= \langle m_{ij}^{(\alpha)} \oplus n_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)} \oplus \gamma_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \oplus \delta_{ij}^{(\alpha)} \rangle \\ &= \langle m_{ij}^{(\alpha)} + n_{ij}^{(\alpha)} - m_{ij}^{(\alpha)} \cdot n_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)} + \gamma_{ij}^{(\alpha)} - \rho_{ij}^{(\alpha)} \cdot \gamma_{ij}^{(\alpha)}, \\ &\quad \beta_{ij}^{(\alpha)} + \delta_{ij}^{(\alpha)} - \beta_{ij}^{(\alpha)} \cdot \delta_{ij}^{(\alpha)} \rangle \end{aligned}$$

$$\begin{aligned} & \beta_{ij}^{(\alpha)} + \delta_{ij}^{(\alpha)} - \beta_{ij}^{(\alpha)} \cdot \delta_{ij}^{(\alpha)} > \\ & = \langle 1+0-0, 1+0-0, 1+0-0 \rangle \\ & = \langle 1, 1, 1 \rangle \\ & \therefore \tilde{Q}_{ij} > \tilde{P}_{ij}. \end{aligned}$$

**Case 3 :**  $\alpha \geq \tilde{M}_{ij} \geq \tilde{N}_{ij}$

$$\begin{aligned} \text{i.e., } \alpha & \geq \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle \geq \langle n_{ij}, \gamma_{ij}, \delta_{ij} \rangle \\ \therefore \alpha & \geq m_{ij} \geq n_{ij}, \alpha \geq \rho_{ij} \geq \gamma_{ij}, \alpha \geq \beta_{ij} \geq \delta_{ij}. \end{aligned}$$

Then

$$\begin{aligned} \tilde{Q}_{ij} & = (\tilde{M}_{ij} \oplus \tilde{N}_{ij})^{(\alpha)} \\ & = \langle m_{ij} + n_{ij} - m_{ij} \cdot n_{ij}, \rho_{ij} + \gamma_{ij} - \rho_{ij} \cdot \gamma_{ij}, \beta_{ij} + \delta_{ij} - \beta_{ij} \cdot \delta_{ij} \rangle^{(\alpha)} \\ & = \langle m_{ij} + n_{ij} (1 - m_{ij}), \rho_{ij} + \gamma_{ij} (1 - \rho_{ij}), \beta_{ij} + \delta_{ij} (1 - \beta_{ij}) \rangle^{(\alpha)} \\ & \geq \langle m_{ij}, \rho_{ij}, \beta_{ij} \rangle^{(\alpha)} \\ & > \langle m_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \rangle \\ & = \langle 0, 0, 0 \rangle \end{aligned}$$

$$\begin{aligned} \text{and } \tilde{P}_{ij} & = \tilde{M}_{ij}^{(\alpha)} \oplus \tilde{N}_{ij}^{(\alpha)} \\ & = \langle m_{ij}^{(\alpha)} \oplus n_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)} \oplus \gamma_{ij}^{(\alpha)}, \beta_{ij}^{(\alpha)} \oplus \delta_{ij}^{(\alpha)} \rangle \\ & = \langle m_{ij}^{(\alpha)} + n_{ij}^{(\alpha)} - m_{ij}^{(\alpha)} \cdot n_{ij}^{(\alpha)}, \rho_{ij}^{(\alpha)} + \gamma_{ij}^{(\alpha)} - \rho_{ij}^{(\alpha)} \cdot \gamma_{ij}^{(\alpha)}, \\ & \quad \beta_{ij}^{(\alpha)} + \delta_{ij}^{(\alpha)} - \beta_{ij}^{(\alpha)} \cdot \delta_{ij}^{(\alpha)} \rangle \\ & = \langle 0+0-0, 0+0-0, 0+0-0 \rangle \\ & = \langle 0, 0, 0 \rangle \\ & \therefore \tilde{Q}_{ij} = \tilde{P}_{ij}. \end{aligned}$$

$\therefore$  In all the cases,  $\tilde{Q}_{ij} \geq \tilde{P}_{ij}$

$$\therefore (\tilde{M}_{ij} \oplus \tilde{N}_{ij})^{(\alpha)} \geq \tilde{M}_{ij}^{(\alpha)} \oplus \tilde{N}_{ij}^{(\alpha)}$$

Thus,  $(M \oplus N)^{(\alpha)} \geq M^{(\alpha)} \oplus N^{(\alpha)}$ .

In a similar manner, for the lower cut of triangular fuzzy number matrices, one can prove the following theorem :

**Theorem : 2.7**

For any two triangular fuzzy number matrices M and N

- (i)  $(M \vee N)_{(\alpha)} = M_{(\alpha)} \vee N_{(\alpha)}$
- (ii)  $(M \ominus N)_{(\alpha)} = M_{(\alpha)} \ominus N_{(\alpha)}$
- (iii)  $(M \oplus N)_{(\alpha)} \geq M_{(\alpha)} \oplus N_{(\alpha)}$

**References**

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