Introduction

INTRODUCTION

"If I feel Unhappy, I do Mathematics to become happy. If I am happy, I do Mathematics to keep myself happy"

- *Alfred Renyi*

In 1999, Molodtsov [23] initiated a novel concept of Soft Set Theory, which is completely a new approach for modeling vagueness and uncertainties. Soft Set Theory has a rich potential for application in solving practical problems in Economics, Social Sciences, Medical Sciences etc. Applications of Soft Set Theory in other disciplines and in real life problems are now catching momentum. Molodtsov [23] successfully applied Soft Theory into several directions, such as Smoothness of Functions, Game Theory, Operations Research, Riemann Integration, Perron Integration, Theory of Probability, Theory of Measurement and so on. Maji et al. [19] gave first practical application of Soft Sets in decision making problems.

Maji et al. [17] initiated the concept of Fuzzy Soft sets with some properties regarding union, intersection and complement of a Fuzzy Soft Set, De Morgan's Law etc. These results were further revised and improved by Ahmad and Kharal [2]. They defined arbitrary fuzzy soft union and intersection and proved De Morgan's Inclusions and De Morgan's Laws in Fuzzy Soft Set Theory. Cagman, Citak and Eugi Noglu [24] defined the concept of Fuzzy Parameterized Fuzzy Soft Sets and illustrated their applications.

In 1975, Zadeh [40] introduced the concept of Interval-Valued Fuzzy Sets. In 2009, by combining the Interval-Valued Fuzzy Sets and Soft Sets, Yang et al. [38] introduced the concept of Interval-Valued Fuzzy Soft Sets. In 2011, Alkhazaled et al. [29] introduced the concept of Fuzzy Parameterized Interval-Valued Fuzzy Soft Sets (*fpivfss*). They defined *fpivfss*-aggregation operator to

form *fpivfss*-decision making method that allows constructing more efficient process.

In 2010, Pinaki Majumdar and S.K.Samanta generalised [26] the concept of Fuzzy Soft Sets and introduced Generalised Fuzzy Soft Sets. Shawkat Alkhazaleh and Abdul Razak Salleh [31] obtained a new Soft Set model named as Generalised Interval-Valued Fuzzy Soft Set by combining the Generalised Fuzzy Soft Set and Interval-Valued Fuzzy set. It can be viewed as an Interval-Valued extension of the Generalised Fuzzy Soft Set Theory or a generalisation of the Interval-Valued Fuzzy extension of the Generalised Fuzzy Soft Set Theory or a generalisation of the Interval-Valued Fuzzy Soft Set Theory.

All these different forms of Fuzzy Soft Sets have applications in real life problems. The applications of Fuzzy Soft Set Theory in many disciplines and in real life situations have been studied by many researchers – Arindam Chandhuri, Dr.Kajal De, Dr.Dipak Chatterjee, A.R.Roy, P.K.Maji, Tridiv Jyoti Neog, Dusmanta Kumar Sut, B.Chetia, P.K.Das, Naim Cagman, Filiz Citak, Serdar Enginoglu and many others.

The main aim of this thesis is to study Fuzzy Soft Sets with application to real life problems.

The plan of study is as follows:

1) Fuzzy Soft Sets
2) Fuzzy Parameterized Fuzzy Soft Sets
3) Interval-Valued Fuzzy Soft Sets
4) Fuzzy Parameterized Interval-Valued Fuzzy Soft Sets
5) Generalised Fuzzy Soft Sets
6) Generalised Interval-Valued Fuzzy Soft Sets
7) Application of Fuzzy Soft Sets to Investment Decision Making Problem

8) Small-Scale Business Opportunities for Women Entrepreneurs – A Decision Making Model Using Fuzzy Soft Sets
9) Application of Interval-Valued Fuzzy Soft Sets in the Analysis of the Factors Influencing High Scores in Higher Secondary Examinations

The First chapter is devoted to the study of Fuzzy Soft Sets.

"Let U be the Universe and E be the set of parameters. A pair (F, E) is called **Soft Set** if F is a mapping of E into the set of all subsets of U.

A **Fuzzy Set** in U is a function with domain U and the values in the closed interval I = [0, 1].

The pair (F, E) is called a **Fuzzy Soft Set** if F is a mapping of E into the set of all Fuzzy subsets of U."

In this chapter basic definitions and properties regarding Fuzzy Soft Sets are given with intersecting examples. Important properties proved here are given in Theorems (1.22, 1.23, 1.24, 1.25, 1.26, 1.29, 1.30, 1.31 and 1.38)

In chapter II, Fuzzy Parameterized Fuzzy Soft Sets (*fpfs*-sets) are defined (Definition 2.1) and properties regarding *fpfs*-sets are studied with intersecting examples.

Chapter III deals with Interval-Valued Fuzzy Soft Sets. In this chapter fundamental definitions regarding Interval-Valued Fuzzy Soft Sets are given with interesting examples. The complement, "AND", and "OR" operations on the Interval-Valued Fuzzy Soft Sets are illustrated with examples (Examples 3.20 and 3.21). The De Morgan's, Associative and Distributive laws of the Interval-Valued Fuzzy Soft Sets are also proved.

In chapter IV, the concept of Fuzzy Parameterized Interval-Valued Fuzzy Soft Sets (*fpivfss*) are introduced (Definition 4.1) and their operations are studied.

Here, *fpivfss*-aggregation operator to form *fpivfss*-decision making method that allows constructing more efficient decision process is also defined.

Chapter V deals with Generalised Fuzzy Soft Sets (GFSS). In this chapter, the definition of Generalised Fuzzy Soft Set introduced by Pinaki Majumdar and S.K.Samanta is given (Definition 5.1) and some of its properties are studied. Relations on Generalised Fuzzy Soft Sets are also defined and an application of the Generalised Fuzzy Soft Relation in a decision making problem is given (Example 5.21).

In chapter VI, the concept of Generalised Interval-Valued Fuzzy Soft Sets introduced by Shawkat Alkhazaleh and Abdul Razak Salleh (Definition 6.1) is studied. The complement, union, intersection, "AND", and "OR" operations are also defined on these Fuzzy Soft Sets and analyzed.

Decision making is the process of choosing the best among the alternatives available. It involves identification of various alternatives and systematic analysis of each alternative to identify one that serves best the accomplishment of the desired objective. The decision making process involves a series of activities viz. definition of goals to be achieved, identification of alternatives, analysis of each alternative, selection of the best alternative and evaluation of the outcome. Whether it is an ordinary day-to-day decision or critical and sensitive decision, application of suitable decision making models ensures that the decision taken are rational and logical and serves best the objectives intended for.

The following are the applications of Fuzzy Soft Sets in real life problems selected for the study:

1) Application of Fuzzy Soft Sets to Investment Decision Making Problem.
2) Small-Scale Business Opportunities for Women Entrepreneurs – A Decision Making Model Using Fuzzy Soft Sets.

3) Application of Interval-Valued Fuzzy Soft Sets in the analysis of the Factors Influencing High Scores in Higher Secondary Examinations.

It is worth mentioning that the author of this thesis published three articles related to this topic as detailed below:

1) "Application of Fuzzy Soft Sets to Investment Decision Making Problem", International Journal of Mathematical Sciences and Applications, Vol.1, No.3, 2011, pp.1583-1586 [14].
2) "Small-Scale Business Opportunities for Women Entrepreneurs – A Decision Making Model Using Fuzzy Soft Sets", Proceedings of International Conference on Mathematics and its Application – A New Wave (ICMANW), December 21-22, 2011, pp.155-159 [15].
3) "Application of Interval Fuzzy Number Matrices and Interval-Valued Fuzzy Soft Sets in the analysis of the Factors Influencing High Scores in Higher Secondary Examinations", International Journal of Mathematical Sciences and Applications, Vol.2, No.2, 2012, pp.777-780 [16].

Chapter VII deals with Application of Fuzzy Soft Sets to Investment Decision Making Problem.

Investment refers to activity involving commitment of funds, usually surplus funds, with an intention to earn return at an expected rate. An investor has various alternative avenues of investment for his savings to flow in accordance with his preference. A wise investor knows that money is losing its value by passage of time due to rise in prices or inflation. So he prefers to invest his funds in select avenues that fetch return, atleast sufficient enough to compensate such inflationary effects. The author of this thesis attempted to apply fuzzy soft sets to investment decision making problem based on the data collected from female

employees working in both government and private sector undertakings located in Coimbatore city, Tamil Nadu, India. The details are given in this chapter.

In chapter VIII, A Decision Making Model Using Fuzzy Soft Sets is constructed regarding Small-Scale Business Opportunities for Women Entrepreneurs.

In recent years, empowerment of women has been recognized as vital in improving the status of Indian women. They constitute one of the most important target groups for the provision of social welfare service. It is a proved maxim that women who are empowered economically and socially become a strong and vigorous force for the removal of poverty and for the overall development of the society. The consistent efforts of both central and state governments and the liberal financial assistance through banks and other financial institutions encouraged women to involve themselves in gainful self-employment activities that fetch earnings and uplift the socio economic condition of such women entrepreneurs.

Though a vast number of self-employment opportunities are available, it is the individual to select a particular avenue that suits best her capabilities and expectations. A venture best selected is half the success attained. There are avenues which are capital intensive, risky, skill oriented, seasonal, less remunerative and so on. Many of the women entrepreneurs find it difficult to decide upon the venture they have to select among the vast alternatives available. To facilitate scientific decision making, the author attempted to develop decision making model using Fuzzy Soft Sets based on the data collected from women entrepreneurs of Coimbatore city. The study has been confined to small-scale ventures mostly preferred by the population in the study area. The various features (expectations) that influenced the womenfolk to carry on the ventures of their

choice have been taken into account to develop the model. The details are given in this chapter.

Chapter IX is devoted to the study of Application of Interval-Valued Fuzzy Soft Sets in the analysis of the Factors Influencing High Scores in Higher Secondary Examinations.

High scores in the Higher Secondary Examinations have become the high order priority in the academic life of a student in the present scenario. The parents have more concern and they leave no stone unturned to get their wards a pass with comparatively higher score which enables them to join a course, particularly a professional course, in a reputed institution. Many students perform upto the expectations, though not all. The author of this thesis intended to analyze the factors contributing high scores in the higher secondary examination and also to identify the prime factor by collecting the data from students, academicians and parents using Interval-Valued Fuzzy Soft Sets. The details are given in this chapter.