Avinashilingam Institute for Home Science and Higher Education for Women,
(Deemed to be University), Coimbatore – 641 043

Master's Degree Examination – November 2018
Semester I

Class : I PG | Time : 3 hrs
Major: Mathematics | Max.Marks: 60

17MMAC03 Graph Theory

Part A

Choose the correct Answer **10 x ½ = 5**

1. An edge having the same vertex as both its end vertices is called a _____
   a) edge set b) vertex set c) loop d) vertex connected
2. A graph in which all vertices are of equal degree is called a _____ graph
   a) complete b) connected c) null d) regular
3. The set of vertices of a subgraph is the _____ of vertices of the original graph
   a) same set b) subset c) same set or subset d) None of these
4. In a _____ , an edge appears only once
   a) path b) walk c) circuit d) path, walk and circuit
5. Fusion reduces the number of vertices by _____
   a) one b) two c) three d) None
6. Hamiltonian path is obtained by removing _____ from a Hamiltonian circuit
   a) one vertex b) one edge c) two vertices d) two edge
7. A connected graph without any circuits is known as _____
   a) disconnected graph b) closed walk c) open walk d) tree
8. The length of the shortest path between two vertices in a graph is called as _____
   a) distance b) center c) radius d) diameter
9. A cut-set always cuts a graph into _____ graph(s)
   a) one b) two c) three d) many
10. Every cut-set in a nonseparable graph with more than two vertices contains _____two edges
    a) atmost b) atleast c) exactly d) None of these

Part B

Answer ALL questions. **5 X 4 = 20 marks**

Each question should not exceed 200 words or one page

11.a) Nine members of a new club meet each day for lunch at a round table. They decide to sit such that every member has different neighbours at each lunch. How many days can this arrangement last?

(or)

b) Draw the graph of three houses – three utilities problem.

12. a) State the properties of two isomorphic graphs.

(or)

b) Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two nonempty, disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in subset V_1 and the other in subset V_2.

13. a) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits.

(or)

b) Define an Euler graph and give an example.

14. a) Prove that a graph G with n vertices, $n-1$ edges, and no circuits is connected

(or)

b) Prove that every connected graph has atleast one spanning tree.

15. a) Prove that in a connected graph G, any minimal set of edges containing at least one branch of every spanning tree of G is a cut-set.

(or)

b) Explain briefly about edge connectivity in a graph.

Part C

Answer ALL questions. **5 X 7 = 35 marks**

Each question should not exceed 600 words or three pages.

16.a) Analyse how graph theory is applied electrical network problems.

(or)

b) Prove that the number of vertices of odd degree in a graph is always even.

17. a) Demonstrate the application of graph colouring with four colours in Instant Insanity problem.

(or)

b) Prove that a simple graph with n vertices and k components can have at most

$(n-k)(n-k+1)/2$ edges.

18. a) Prove that connected graph is an Euler graph if and only if all its vertices are of even degree.

(or)

b) Prove that, in a complete graph with n vertices there are $(n-1)$ edge-disjoint Hamiltonian circuits, if n is an odd number ≥ 3.

19. a) Prove that a tree with n vertices has $(n-1)$ edges.

(or)

b) Illustrate and prove that every tree has either one or two centers.

20. a) Every circuit has an even number of edges in common with any cut-set. Illustrate.

(or)

b) Apply the concepts of edge and vertex connectivity in graphs to solve a telephone line problem with 8 vertices and 16 edges.

****