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## INTRODUCTION

In the real-world situation, human thinking and reasoning involve fuzzy information. In order to cope with unreliable and incomplete information, Zadeh introduced the concept of fuzzy set theory in 1965.

Fuzzy set is a mapping on a set  $X$  to  $I = [0,1]$

In 1968, the notion of fuzzy topology was first proposed by Chang and then modified by Lowen in 1976.

In 1975, Zadeh proposed the concept of second order fuzzy set (map from a set  $X$  to  $I^I$ ). The second order fuzzy sets are characterized by a membership function, the grade (fuzzy grade) of which is a fuzzy set in the unit interval  $[0,1]$  rather than a point in  $[0,1]$ .

A wide variety of human decision making is based on double sided or bipolar judgemental thinking on a positive side and a negative side. For example, friendship and hostility, cooperation and competition, common interests and conflict interests, likelihood and unlikelihood, etc,... are often two sides in decision and coordination.

In 1994, Zhang introduced the notion of a bipolar fuzzy set. Azhagappan and Kamaraj (2016) introduced the concept of bipolar fuzzy topology by following Chang's fuzzy topology. J. Kim, S. K. Samanta, P. K. Lim. J. G. Lee and K. Hur (2019) introduced the same concept by following Lowen's fuzzy topology.

Using second order fuzzy sets Kalaichelvi (2007) extended first order fuzzy topology of Chang and Lowen to second order fuzzy topology.

Matrix algebra is a mathematical language which makes simultaneous equations easier to express and solve. It can be applied to both the creation of a mathematical model of the structure and the acquisition of a concise statement of a structural problem. Thomason introduced the concept of fuzzy matrices in 1977. In 2019, M. Pal and Sanjib Mondal proposed the concept of bipolar fuzzy matrices. Several results about the power-convergence and transitive closure of BFM's are examined.

Multi-criteria decision making (MCDM) is a process to make an optimal choice that has the highest degree of achievement from a set of alternatives which are characterized in

terms of multiple conflicting criteria. Hwang and Yoon (1981) devised the Technique for the Order of Preference by Similarity to Ideal Solution (TOPSIS) method, which is one of the most advantageous and successful approaches for resolving MCDM problems. In 2022, Natthinee Deetae proposed method a new methodology BFS-TOPSIS, which considers the positive and negative decisions of the decision-maker. The BFS-TOPSIS is a combination of bipolar value sets with the fuzzy TOPSIS method and uses a score function to select the best alternative.

In this thesis, the concepts of second order bipolar fuzzy set and second order bipolar fuzzy topology are introduced and various second order bipolar fuzzy structures are studied. Also an optimal solution is obtained for the decision making problem involving second order bipolar fuzzy matrix using TOPSIS method.