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n-FUZZY PROXIMITY-II
n-FUZZY PROXIMITY–BASE AND PRODUCT

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In this paper, a new concept called *n*-fuzzy proximity base is introduced in I_n^X. Its extension to a fuzzy proximity base and some of its properties are studied. The behaviour of the new concept *n*-fuzzy proximity in connection with supremum of a collection of fuzzy proximities and product of a collection of fuzzy proximities is studied in detail.

KEY WORDS : *n*-fuzzy proximity, *n*-fuzzy proximity base, fuzzy proximity, fuzzy proximity base.

Mathematics Subject Classification : 54 E 05, 54 E 15.

INTRODUCTION

The concept of fuzzy topology was first introduced by Chang, C.L. [1]. The problem of generalizing the concepts of general topological spaces to fuzzy topological spaces has been carried out by many authors. In 1979 Katsaras [3] introduced the first definition of fuzzy proximity.

The concept of fuzzy proximity base was first introduced by P. Srivastava and R.L. Gupta [5] in 1980. They used this concept to study the product of fuzzy proximities and the lattice of fuzzy proximities. In this paper, a new concept called *n*-fuzzy proximity base is introduced in I_n^X. It is shown that every *n*-fuzzy proximity base B_{n*} on X can be extended to a fuzzy proximity base [Srivastava [5]] on X and it is denoted by $E_x(B_{n*})$. It is shown that if $P(B_{n*})$ is the *n*-fuzzy proximity induced by B_{n*} and $E_x(P\ (B_{n*}))$ is the extension of $P(B_{n*})$ then $P(E_x(B_{n*})) = E_x(P(B_{n*}))$.

The behaviour of the new concept *n*-fuzzy proximity in connection with supremum of a collection of fuzzy proximities and product of a collection of fuzzy proximities is studied in detail.

PRELIMINARY RESULTS

Definition : 2.1 [2]

Let $I_n = \{0, 1/n, 2/n, ..., 1\}$. A **$I_n$-valued fuzzy set** on X is an element of the set I_n^X of all functions from X to I_n.

Definition : 2.2 [6]

A binary relation ρ_{n*} on I_n^X is called an ***n*-fuzzy proximity on X** if ρ_{n*} satisfies the following axioms.

For any $f, g, h \in I_n^X$.

(FP_{n*} 1) $f\, \rho_{n*}\, g$ implies $g\, \rho_{n*}\, f$

(FP_{n*} 2) $(f \vee h)\ \rho_{n*}\, g$ iff $f\, \rho_{n*}\, g$ or $h\, \rho_{n*}\, g$

(FP_{n*} 3) $f\, \rho_{n*}\, g$ implies $f \neq \mathbf{0}$ and $g \neq \mathbf{0}$

(FP_{n*} 4) $f\, \bar{\rho}_{n*}\, g$ implies that there exists an $A \subseteq X$ such that $f\, \bar{\rho}_{n*}\, \chi_A$ and $(1 - \chi_A)$ $\bar{\rho}_{n*}\, g$

(FP_{n*} 5) $f \wedge g \neq \mathbf{0}$ implies $f\, \rho_{n*}\, g$

The pair (X, ρ_{n*}) is called an ***n*-fuzzy proximity space**.

To extend the concept of n-fuzzy proximity to a fuzzy proximity on X, the concept of nth order approximation introduced in [2] is required. The definition and properties of nth order approximations are collected below.

Definition : 2.3

With every fuzzy set f defined on a set X and with every positive integer n, a finite fuzzy set ${}^n f$ with values in I_n is associated as follows :

For $x \in X$

(i) if $f(x) = 0$, define ${}^n f(x) = 0$.

(ii) if $1/n < f(x) \leq (i+1)/n$ define ${}^n f(x) = (i+1)/n$, for $i = 0, 1, 2, \ldots, n-1$.

${}^n f$ is called the ***n*th upper approximation of *f*.**

Proposition : 2.4

$$f(x) = i/n \Rightarrow {}^n f(x) = i/n \ \text{ for } \ i = 1, 2, \ldots, n$$

$$f \leq {}^n f \text{ for all } n.$$

$$f \leq g \Rightarrow {}^n f \leq {}^n g$$

$$f \leq {}^n g \Rightarrow {}^n f \leq {}^n g$$

$${}^n({}^n f) = {}^n f$$

$${}^n(\vee f_\lambda) = \vee ({}^n f_\lambda)$$

$${}^n\left(\bigwedge_{k=1}^{m} f_k\right) = \bigwedge_{k=1}^{m} ({}^n f_k)$$

Definition : 2.5

For each fuzzy set f on a set X, the ***n*th lower approximation $_n f$** is defined as follows :

For $x \in X$,

(i) if $f(x) = 1$ define $_n f(x) = 1$

(ii) if $i/n \leq f(x) < (i+1)/n$, define $_n f(x) = i/n$ for $i = 0, 1, 2, \ldots, n-1$.

Proposition : 2.6

(i) If $f(x) = i/n$ then $_n f(x) = i/n$, for $i = 0, 1, \ldots, n-1$

(ii) $_n f \leq f$ for all n.

(iii) $f \leq g \Rightarrow {}_n f \leq {}_n g$.

(iv) $_nf \le g \Rightarrow {}_nf \le {}_ng$

(v) $_n({}_nf) = {}_nf$

(vi) $_n(\wedge f_\lambda) = \wedge ({}_nf_\lambda)$

(vii) $_n(\overset{m}{\underset{k=1}{\vee}} f_k) = \overset{m}{\underset{k=1}{\vee}} ({}_nf_k)$

Proposition : 2.7

(i) $_n(1-f) = 1 - {}^nf$ and $^n(1-f) = 1 - {}_nf$

(ii) $^nf \le g \Rightarrow {}^nf \le {}_ng$

(iii) $f \le {}_ng \Rightarrow {}^nf \le {}_ng, f \le {}^ng \Rightarrow {}_nf \le g \le {}^ng$

(iv) $_n({}^nf) = {}^nf$

(v) $^n({}_nf) = {}_nf$

(vi) $^nf \neq \mathbf{0} \Rightarrow f \neq \mathbf{0}$

Proposition : 2.8

(i) If $f \in I_n^X$ then $_nf = f = {}^nf$.

(ii) For $A \subseteq X, \chi_A = {}^n\chi_A = {}_n\chi_A$

Proposition : 2.9

Let $\theta : X \to Y$ be a function then

(i) For all $f \in I^X$, $^n(\theta(f)) = \theta({}^nf)$

(ii) For given $f \in I^X$, $^n(\theta^{-1}(f)) = \theta^{-1}({}^nf)$

Proposition : 2.10 [6]

Given an n-fuzzy proximity ρ_{n^*}, it is extended to a fuzzy proximity $\boldsymbol{E_x(\rho_{n^*})}$ as follows :

$f(E_x(\rho_{n^*}))\, g \Leftrightarrow {}^nf\ \rho_{n^*}\ {}^ng$. Here $E_x(\rho_{n^*})$ is called **the extension of** ρ_{n^*}.

Definition [Srivastava and Gupta, 5]: 2.11

A binary relation B on I^X is called a **fuzzy proximity base** on X if it satisfies the following axioms :

For $f, g, h \in I^X$.

($FB1$) $f B g$ implies $g B f$

($FB2$) If $f B g$ and $f \le f', g \le g'$ then $f' B g'$

($FB3$) $f B g$ implies $f \neq \mathbf{0}$ and $g \neq \mathbf{0}$

($FB4$) $f \bar{B} g$ implies there exists $A \subseteq X$ such that $f \bar{B} \chi_A$ and $(1-\chi_A) \bar{B} g$

($FB5$) $f \wedge g \neq \mathbf{0}$ implies $f B g$.

n-FUZZY PROXIMITY BASE AND PRODUCT

Definition : 3.1

A binary relation B_{n^*} on I_n^X is called **an *n*-fuzzy proximity base** on X if B_{n^*} satisfies the following axioms.

For $f, g, h \in I_n^X$,

$(FB_{n*}1)$ $f\,B_{n*}\,g \Rightarrow g\,B_{n*}\,f$

$(FB_{n*}2)$ If $f\,B_{n*}\,g$ and $f \le f'$, $g \le g'$ then $f'\,B_{n*}\,g'$

$(FB_{n*}3)$ $f\,B_{n*}\,g$ implies $f \neq \mathbf{0}$ and $g \neq \mathbf{0}$

$(FB_{n*}4)$ $f\,\overline{B_{n*}}\,g$ implies there exists $A \subseteq X$ such that $f\,\overline{B_{n*}}\,\chi_A$ and $(1-\chi_A)$ $\overline{B_{n*}}\,g$

$(FB_{n*}5)$ $f \wedge g \neq \mathbf{0}$ implies $f\,B_{n*}\,g$.

Definition : 3.2

Given an n-fuzzy proximity base B_{n*} on X, it is extended to $E_x(B_{n*})$ on X as follows :

$$f\,(E_x\,(B_{n*}))\,g \;\Leftrightarrow\; {}^{n}f\,B_{n*}\,{}^{n}g$$

Theorem : 3.3

If B_{n*} is an n-fuzzy proximity base then its extension $E_x(B_{n*})$ is a fuzzy proximity base.

Proof :

Let $E_x(B_{n*})$ be a binary relation on I^X s.t. $f\,E_x(B_{n*})\,g \Leftrightarrow {}^{n}f\ B_{n*}\,{}^{n}g$.

For $f, g, h \in I^X$

(FB 1) $f(E_x(B_{n*}))\,g \Leftrightarrow {}^{n}f\ B_{n*}\ {}^{n}g$

$$\Leftrightarrow {}^{n}g\ B_{n*}\ {}^{n}f$$

$$\Leftrightarrow g\,(E_x\,(B_{n*}))\,f$$

(FB 2) $f(E_x(B_{n*}))\,g$ and $f \le f'$ and $g \le g'$

Then ${}^{n}f\ B_{n*}\,{}^{n}g$ and ${}^{n}f \le {}^{n}f'$ and ${}^{n}g \le {}^{n}g'$ [Proposition 2.4 (iii)]

$$\Rightarrow {}^{n}f'\ B_{n*}\ {}^{n}g' \;\Rightarrow f'(E_x(B_{n*}))\,g'$$

(FB 3) $f(E_x(B_{n*}))\,g \Rightarrow {}^{n}f\ B_{n*}\ {}^{n}g$

$$\Rightarrow {}^{n}f \neq \mathbf{0},\ {}^{n}g \neq \mathbf{0}$$

$$\Rightarrow f \neq \mathbf{0} \text{ and } g \neq \mathbf{0} \text{ [Proposition 2.7 (vi)]}$$

(FB 4) $f(\overline{E_x(B_{n*})})\ g \Rightarrow {}^{n}f\ \overline{B_{n*}}\ {}^{n}g$

$$\Rightarrow \text{there exists } \chi_A \text{ s.t. } {}^{n}f\ \overline{B_{n*}}\ \chi_A \text{ and } {}^{n}g\ \overline{B_{n*}}\ (1-\chi_A)$$

$$\Rightarrow {}^{n}f\ \overline{B_{n*}}\ {}^{n}\chi_A \text{ and } {}^{n}g\ \overline{B_{n*}}\ {}^{n}(1-\chi_A) \quad \text{(Proposition 2.8)}$$

$$\Rightarrow f\ (\overline{E_x(B_{n*})})\,\chi_A \text{ and } g\ (\overline{E_x(B_{n*})})\ (1-\chi_A)$$

(FB 5) $f \wedge g \neq \mathbf{0} \;\Rightarrow {}^{n}f \wedge {}^{n}g \neq \mathbf{0}$

$$\Rightarrow {}^{n}f\ B_{n*}\ {}^{n}g \Rightarrow f(E_x\,(B_{n*}))\,g$$

Hence $E_x(B_{n*})$ is a fuzzy proximity base.

Result [5, Theorem 3.2]: 3.4

Let B be a fuzzy proximity base on X and let a binary relation $P(B)$ on I^X be defined as follows :

$f\,P(B)\,g$ iff given any two families $\{f_i \mid i \in J_m\}$ and $\{g_j \mid j \in J_n\}$ with $f = \vee f_i$ and $g = \vee g_j$, there exists a pair $(i, j) \in J_m \times J_n$ such that $f_i\ B\ g_j$. [Here J_m denotes the set of the first m

natural numbers]. Then $P(B)$ is the coarsest fuzzy proximity finer than the fuzzy proximity base B.

Theorem : 3.5

Let B_{n*} be an n-fuzzy proximity base. Let $P(B_{n*})$ be defined on I_n^X as follows :

For $f, g \in I_n^X, f\,P(B_{n*})\,g$ iff given any two families $\{f_i \in I_n^X \mid i \in J_m\}$ and $\{g_j \in I_n^X \mid j \in J_n\}$ with $f = \vee f_i$ and $g = \vee g_j$, there exists a pair (i, j) in $J_m \times J_n$ s.t. $f_i\ B_{n*}\ g_j$. Then $P(B_{n*})$ is the coarsest n-fuzzy proximity on X finer than the n-fuzzy proximity base B_{n*}.

Proof : The proof is similar to that of Result 3.4.

Theorem : 3.6

Let B_{n*} be an n-fuzzy proximity base on X. Let $B = E_x(B_{n*})$ be its extension. Let $P(B)$ be the fuzzy proximity induced by B and $P(B_{n*})$ be the n-fuzzy proximity induced by B_{n*}. Then $P(B)$ is equal to the extension of $P(B_{n*})$ (*i.e.*) $P(E_x(B_{n*})) = E_x(P(B_{n*}))$.

Proof : Let $\rho = P(E_x(B_{n*}))$ be the fuzzy proximity induced by $(E_x(B_{n*}))$ and

$\rho' = E_x(P(B_{n*}))$ be the extension of $P(B_{n*})$.

First to prove $\rho \geq \rho'$

For $f, g \in I^X$, assume $f \rho g$

$f \rho g \Rightarrow {}^{n}f \rho {}^{n}g \quad [\because {}^{n}f \geq f$ and ${}^{n}g \geq g]$

Let ${}^{n}f = \vee \{f_i / i \in J_m\}$ and ${}^{n}g = \vee \{g_j / j \in J_n\}$ ... (1)

Then ${}^{n}f = {}_{n}({}^{n}f) = \vee ({}^{n}f_i)$

and ${}^{n}g = {}_{n}({}^{n}g) = \vee ({}^{n}g_j)$ [Proposition 2.7 (iv) and 2.4 (vi)]

Then ${}^{n}f\ \rho\ {}^{n}g \Rightarrow {}^{n}f\ P(E_x(B_{n*}))\ {}^{n}g$

$\Rightarrow$ there exists $(i, j) \in J_m \times J_n$ such that $({}_{n}f_i)\ E_x(B_{n*})\ ({}_{n}g_j)$

$\Rightarrow {}^{n}({}_{n}f_i)\ B_{n*}\ {}^{n}({}_{n}g_j)$

$\Rightarrow {}_{n}f_i\ B_{n*}\ {}_{n}g_j$ [Proposition 2.7 (v)]

$\Rightarrow f_i\ B_{n*}\ g_j$ [Proposition 2.6 (ii) and by FB2] ... (2)

(1) and (2) $\Rightarrow {}^{n}f\ P(B_{n*}))\ {}^{n}g \Rightarrow f\,E_x(P(B_{n*}))\,g \Rightarrow f \rho' g$

Next to prove $\rho' \geq \rho$

Assume $f \rho'$ g. Let $f = \vee \{f_i \mid i \in J_m\}$ and $g = \vee \{g_j \mid j \in J_n\}$

Then ${}^{n}f = \vee ({}^{n}f_i),\ {}^{n}g = \vee ({}^{n}g_j)$ ... (1)

$f \rho' g \Rightarrow f\,E_x(P(B_{n*}))\,g$

$\Rightarrow {}^{n}f\ P(B_{n*})\ {}^{n}g$

$\Rightarrow$ there exists a pair $(i, j) \in J_m \times J_n$ s.t. ${}^{n}f_i\ B_{n*}\ {}^{n}g_j$

$\Rightarrow f_i\ E_x\ (B_{n*})\ g_j$ ... (2)

(1) and (2) $\Rightarrow f\ P(E_x(B_{n*}))\ g \Rightarrow f \rho g$

Hence $\rho = \rho'$.

Result [5,Theorem5.1]: 3.7

Let $\{\rho_\alpha \mid \alpha \in \Omega\}$ be a non-void collection of fuzzy proximities on a set X. Then there exists a coarsest fuzzy proximity ρ on I^X such that $\rho \geq \rho_\alpha$ for every $\alpha \in \Omega$. Here ρ is denoted by $\sup \rho_\alpha$.

Theorem : 3.8

Let $\{\rho_{n^*\alpha} \mid \alpha \in \Omega\}$ be a non-void collection of n-fuzzy proximities on X. Then there exists a coarsest n-fuzzy proximity ρ_{n^*} on X s.t. $\rho_{n^*} \geq \rho_{n^*\alpha}$ for every $\alpha \in \Omega$. Here ρ_{n^*} is denoted by $\sup\limits_{\alpha \in \Omega} \{\rho_{n^*\alpha}\}$.

Proof : Let $B_{n^*} = \cap \{\rho_{n^*\alpha} \mid \alpha \in \Omega\}$. Then B_{n^*} can easily be proved to be an n-fuzzy proximity base. Then the n-fuzzy proximity $P(B_{n^*})$ induced by B_{n^*} is the coarsest n-fuzzy proximity finer than every $\rho_{n^*\alpha}$.

Theorem : 3.9

Let $\{\rho_{n^*\alpha} \mid \alpha \in \Omega\}$ be a non-empty collection of n-fuzzy proximities on I_n^X. Let $\rho_\alpha = E_x(\rho_{n^*\alpha})$, Then $E_x(\sup\limits_\alpha \rho_{n^*\alpha}) = \sup\limits_\alpha \rho_\alpha = \sup\limits_\alpha (E_x(\rho_{n^*\alpha}))$

Proof : Let $\rho = E_x(\sup\limits_\alpha \rho_{n^*\alpha})$

$$\rho' = \sup_\alpha (E_x(\rho_{n^*\alpha}))$$

First to prove $f\rho g \Rightarrow f\rho' g$, for $f, g \in I^X$

Let $f = \vee\{f_i \mid i \in J_m\}$ and $g = \vee\{g_j \mid j \in J_n\}$ ... (1)

$\therefore$ ${}^n f = \vee\{{}^n f_i\}$ and ${}^n g = \vee\{{}^n g_j\}$

Now $\sup \rho_{n^*\alpha}$ is induced by $B_{n^*} = \cap\{\rho_{n^*\alpha}\}$ and $\rho' = \sup\limits_\alpha (E_x(\rho_{n^*\alpha}))$ is induced by

$$B = \cap\{E_x(\rho_{n^*\alpha})\}.$$

Now $f\rho\, g \Rightarrow f(E_x(\sup\limits_\alpha \rho_{n^*\alpha}))\, g$

$\Rightarrow {}^n f\ (\sup\limits_\alpha \rho_{n^*\alpha})\ {}^n g$

$\Rightarrow$ there exists $(i, j) \in J_m \times J_n$ s.t. ${}^n f_i\ B_{n^*}\ {}^n g_j$

$\Rightarrow {}^n f_i\ \rho_{n^*\alpha}\ {}^n g_j,\ \forall\ \alpha$

$\Rightarrow f_i\ E_x(\rho_{n^*\alpha})\ g_j,\ \forall\ \alpha$

$\Rightarrow f_i\ (\cap_\alpha E_x(\rho_{n^*\alpha}))\ g_j$

$\Rightarrow f_i\, B\, g_j$ ... (2)

(1) and (2) $\Rightarrow f\rho' g$

$\therefore$ $\rho \geq \rho'$

Next to prove, $f\rho' g \Rightarrow f\,\rho\, g$

$f\rho' g \Rightarrow {}^n f \rho' {}^n g$ $[\because {}^n f \geq f$ and ${}^n g \geq g]$

Let $\quad {}^{n}f = \vee \, \{f_i \mid i \in J_m\}$ and ${}^{n}g = \vee \, \{g_j \mid j \in J_n\}$ ... (1)

Then ${}^{n}f = {}_{n}({}^{n}f) = \vee \, ({}_{n}f_i)$ and ${}^{n}g = {}_{n}({}^{n}g) = \vee \, ({}_{n}g_j)$ [2.7 (iv) and 2.6 (vii)]

$\therefore \quad {}^{n}f \; \rho' \; {}^{n}g \Rightarrow$ there exists $(i, j) \in J_m \times J_n$ s.t. $({}_{n}f_i) \, B \, ({}_{n}g_j)$

$\Rightarrow {}_{n}f_i \; (E_x(\rho_{n^*\alpha})) \; {}_{n}g_j, \; \forall \; \alpha$

$\Rightarrow {}^{n}({}_{n}f_i) \; (\rho_{n^*\alpha}) \; {}^{n}({}_{n}g_j), \; \forall \; \alpha$

$\Rightarrow {}_{n}f_i \; (\rho_{n^*\alpha}) \; {}_{n}g_j, \; \forall \; \alpha$ $\quad [\because {}^{n}({}_{n}f) = {}_{n}f]$

$\Rightarrow f_i \; (\rho_{n^*\alpha}) \; g_j, \; \forall \; \alpha$ $\quad [\because f_i \geq {}_{n}f_i$ and $g_j \geq {}_{n}g_j]$

$\Rightarrow f_i \, (\cap \, \rho_{n^*\alpha}) \, g_j, \; \forall \; \alpha \; \Rightarrow f_i \, (B_{n^*}) \, g_j$ ... (2)

(1) and (2) $\quad \Rightarrow {}^{n}f \, (\sup \; \rho_{n^*\alpha}) \, {}^{n}g$

$\Rightarrow f \, (E_x \, (\sup \; \rho_{n^*\alpha})) \; g$

$\Rightarrow f \rho g$

$\therefore \quad \rho' \geq \rho$. Hence $\rho = \rho'$.

Definition [Katsaras, 4] : 3.10

Let (X, ρ) and (Y, ρ') be two fuzzy proximity spaces. Then a map $\theta : (X, \rho) \to (Y, \rho')$ is called **a fuzzy proximity mapping** iff for all $f, g \in I^X$,

$$f \rho g \Leftrightarrow \theta(f) \, (\rho') \, \theta(g).$$

Definition : 3.11

Let (X, ρ_{n^*}) and (Y, ρ'_{n^*}) be two n-fuzzy proximity spaces. Then a map $\theta : (X, \rho_{n^*}) \to (Y, \rho'_{n^*})$ is called **an *n*-fuzzy proximity mapping** iff for all $f, g \in I_n^X$, $f \rho_{n^*} g \Leftrightarrow \theta(f) \, (\rho'_{n^*}) \, \theta(g)$.

In Proposition 2.10 it is proved that every n-fuzzy proximity ρ_{n^*} induces a fuzzy proximity $E_x(\rho_{n^*})$. The following theorem shows that the map $\rho_{n^*} \to E_x(\rho_{n^*})$ is functorial.

Theorem : 3.12

$\theta : (X, \rho_{n^*}) \to (Y, \rho'_{n^*})$ is an n-fuzzy proximity mapping

$\Rightarrow \theta : (X, E_x \, (\rho_{n^*})) \to (Y, E_x \, (\rho'_{n^*}))$ is a fuzzy proximity mapping.

Proof : Let $f, g \in I^X$.

Assume that $\theta : (X, \rho_{n^*}) \to (Y, \rho'_{n^*})$ is an n-fuzzy proximity mapping.

Now $\quad f \, (E_x \, (\rho_{n^*})) \, g \Rightarrow {}^{n}f \, (\rho_{n^*}) \, {}^{n}g$

$\Rightarrow \theta \, ({}^{n}f) \, (\rho'_{n^*}) \, \theta \, ({}^{n}g)$

$\Rightarrow {}^{n}(\theta \, (f)) \, (\rho'_{n^*}) \, {}^{n}(\theta \, (g))$ $\quad$ [Proposition 2.9 (i)]

$\Rightarrow \theta \, (f)) \, (E_x \, (\rho'_{n^*})) \, \theta(g)$

$\therefore \quad \theta : (X, E_x(\rho_{n^*})) \to (Y, E_x \, (\rho'_{n^*}))$ is a fuzzy proximity mapping.

Theorem [5, Theorem 7.1] : 3.13

Let X be a set and let $\mathbf{F}$ be a non-void family of functions, each member α of $\mathbf{F}$ being on X into a fuzzy proximity space (Y_α, ρ_α). Then there exists a coarsest fuzzy proximity on X such that each member of $\mathbf{F}$ is a fuzzy proximity mapping.

Theorem : 3.14

Let X be a set and $\mathfrak{F}$ be a non-void family of functions, each member α of $\mathfrak{F}$ being on X into an n-fuzzy proximity space $(Y_\alpha, \rho'_{n*\alpha})$. Then there exists a coarsest fuzzy proximity ρ_{n*} on X such that each member of $\mathfrak{F}$ is an n-fuzzy proximity mapping.

Proof : Define a binary relation B_{n*} on I_n^X s.t. for $f, g \in I_n^X$, $f B_{n*} g \Leftrightarrow \alpha(f)\,(\rho'_{n*\alpha})\,\alpha(g)$ for every $\alpha \in \mathfrak{F}$. Then B_{n*} is an n-fuzzy proximity base and $P(B_{n*})$ is the required fuzzy proximity in the theorem.

Theorem : 3.15

Let X be a set and $\mathfrak{F}$ be a non-void family functions, each member α of $\mathfrak{F}$ being on X into an n-fuzzy proximity space $(Y_\alpha, \rho'_{n*\alpha})$. Let ρ_{n*} be the coarsest n-fuzzy proximity on X such that each α is an n-fuzzy proximity mapping. Let $E_x(\rho_{n*\alpha})$ be the extension of $\rho_{n*\alpha}$, $\forall\ \alpha$. Let ρ be the coarsest fuzzy proximity on X such that each $\alpha : (X, \rho) \to (Y_\alpha, E_x(\rho'_{n*\alpha}))$ is a fuzzy proximity mapping. Then $\rho = E_x(\rho_{n*})$.

Proof : Let B_{n*} be defined on I_n^X as follows :

For $f, g \in I_n^X$, $f B_{n*}\ g \Leftrightarrow \alpha(f)\ \rho'_{n*\alpha}\ \alpha(g)$, $\forall\ \alpha \in F$. Then from Theorem 3.14, B_{n*} is an n-fuzzy proximity base and $\rho_{n*} = P(B_{n*})$ is the coarsest n-fuzzy proximity making each α : $(X, \rho_{n*}) \to (Y_\alpha, \rho'_{n*\alpha})$, an n-fuzzy proximity mapping.

Let B be defined on I^X as follows :

For $f, g \in I^X, f\ B g \Leftrightarrow \alpha(f)\ (E_x(\rho'_{n*\alpha}))\ \alpha(g)$, $\forall\ \alpha \in \mathfrak{F}$. Now from Theorem 3.13 B is a fuzzy proximity base and $\rho = P(B)$ is the coarsest fuzzy proximity making each $\alpha : (X, \rho) \to (Y_\alpha, E_x\,(\rho'_{n*\alpha}))$, a fuzzy proximity mapping.

$$
\begin{aligned}
\text{Now } f B g &\Leftrightarrow {}^n(\alpha(f))\ (\rho'_{n*\alpha})\ {}^n(\alpha(g)),\ \forall\ \alpha \in \mathfrak{F} \\
&\Leftrightarrow \alpha\,({}^n f)\ \rho'_{n*\alpha}\ \alpha({}^n g),\ \forall\ \alpha \in \mathfrak{F} \qquad \text{[Proposition 2.9 (i)]} \\
&\Leftrightarrow {}^n f\ B_{n*}\ {}^n g \Leftrightarrow f\,(E_x(B_{n*}))\,g
\end{aligned}
$$

$$\therefore \quad B = E_x(B_{n*})$$

$$
\begin{aligned}
\therefore \quad \rho &= P(B) = P(E_x\,(B_{n*})) \\
&= E_x\,(P(B_{n*}))\ \text{[Theorem 3.6)]} = E_x\,(\rho_{n*})
\end{aligned}
$$

$$\therefore \quad \rho = E_x(\rho_{n*})$$

Result [5, Theorem 7.3] : 3.16

Let $\{(X_\alpha, \rho_\alpha) \mid \alpha \in \Omega\}$ be a non-void collection of fuzzy proximity spaces and let $X = \prod\limits_{\alpha \in \Omega} X_\alpha$. The binary relation B on I^X defined by $f B g \Leftrightarrow \pi_\alpha\,(f)\ P(B)\ \pi_\alpha\,(g)$ for every $\alpha \in \Omega$ is a fuzzy proximity base on X for the product fuzzy proximity $\Pi\ \rho_\alpha$. (Here π_α denotes the projection map from X to X_α).

Definition : 3.17

Given a collection $\{(X_\alpha, \rho_{n*\alpha}) \mid \alpha \in \Omega\}$ of n-fuzzy proximity spaces, the coarsest n-fuzzy proximity on the product $X = \Pi\ X_\alpha$, such that each projection map π_α is an n-fuzzy proximity mapping, is called **the product *n*-fuzzy proximity** $\prod\limits_{\alpha} \rho_{n*\alpha}$.

Theorem : 3.18

Let $\{(X_\alpha, \rho_{n*\alpha}) \mid \alpha \in \Omega\}$ be a non-empty collection of n-fuzzy proximity spaces and let $X = \Pi X_\alpha$. The binary relation B_{n*} on I_n^X defined by $f B_{n*} g \Leftrightarrow \pi_\alpha (f)\ \rho_{n*\alpha}\ \pi_\alpha (g),\ \forall\ \alpha \in \Omega$ is an n-fuzzy proximity base on I_n^X for the product n-fuzzy proximity $\rho_{n*} = \prod_\alpha \rho_{n*\alpha}$.

Proof : Proof follows from Theorem 3.14.

Theorem : 3.19

$$\prod_\alpha (E_x (\rho_{n*\alpha})) = E_x (\prod_\alpha \rho_{n*\alpha}).$$

PROOF : Using the notations of the Theorem 3.15 and the Definition 3.17, the product of extensions of $\rho_{n*\alpha}$ and the extension of the product of n-fuzzy proximities coincide.

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