CHAPTER - IV

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HYBRID FUZZY ANALYTIC HIERARCHY PROCESS METHOD

The Hybrid Fuzzy Analytic Hierarchy Process (AHP) method is introduced to deal with decision-making problems in an uncertain and multiple-criteria environment. This Hybrid Fuzzy AHP model is very effective in solving both quantitative data and qualitative ratings simultaneously. The Hybrid Fuzzy AHP model overcomes the disadvantages of quantitative or qualitative approaches of other Fuzzy AHP methods.

The canonical representation of a triangular fuzzy number $Y = (c, a, b)$ is given by:

$$P(Y) = \frac{1}{6}(c + 4a + b) \qquad (17)$$

The proposed Hybrid Fuzzy AHP approach involves 11 steps shown as follows.

Step 1: **Construct a Hierarchical Analysis Structure**

The first step of Hybrid Fuzzy AHP model is to decompose a general decision making problem into sub problems in a hierarchical fashion, i.e., decompose the problem into criteria, sub-criteria and alternatives. These criteria in the hierarchical analysis structure can be divided into two categories: objective and subjective criteria. The objective criteria are defined in monetary or quantitative terms. The subjective criteria are defined in linguistic terms represented by fuzzy numbers.

Step 2: **Introduce Linguistic Variables for Importance Weight of Criteria.**

The terms of linguistic variables for importance weight of criteria could be called "equally important", "weakly important", "strongly important", "demonstrably important", "absolutely important," and so forth. These linguistic variables can be expressed in fuzzy numbers such as "equally important" = (1, 1, 1), "weakly important" = (2, 3, 4), "strongly important" = (4, 5, 6), "demonstrably important" = (6, 7, 8), and "absolutely important" = (9, 9, 9). Their reciprocals are considered as "weakly unimportant"= (1/4, 1/3, 1/2), "strongly unimportant"= (1/6, 1/5, 1/4), "demonstrably unimportant" = (1/8, 1/7, 1/6), and "absolutely unimportant"= (1/9, 1/9, 1/9) (Table 5).

Scale	Definition	Description
(1, 1, 1)	Equally important, EI	The importance of both comparative alternatives is equal
(1, 2, 3)	Intermediate values, IV	Need to compromise between EI and WI
(2, 3, 4)	Weakly important, WI	Experience and judgment weakly tend to prefer one alternative
(3, 4, 5)	Intermediate values, IV	Need to compromise between WI and SI
(4, 5, 6)	Strongly important, SI	Experience and judgment strongly tend to prefer one alternative
(5, 6, 7)	Intermediate values, IV	Need to compromise between SI and DI
(6, 7, 8)	Demonstrably important, DI	Experience and judgment demonstrably tend to prefer one alternative
(7, 8, 9)	Intermediate values, IV	Need to compromise between DI and AI
(9, 9, 9)	Absolutely important, AI	Experience and judgment absolutely tend to prefer one alternative

Table 5. Comparative importance scale of criteria

Step 3: **Introduce Linguistic Variables for Ratings of Alternative Locations.**

Terms of linguistic variables for ratings of alternative locations could be called "very poor," "poor," "fair," "good," "very good," and so forth. These linguistic variables can be expressed in fuzzy numbers such as "very poor" = (0.0, 1.0, 2.0), "poor" = (1.0, 2.0, 3.0), "fair = (2.0, 3.0, 4.0)", "good" = (3.0, 4.0, 5.0), "very good" = (5.0, 5.0, 5.0), and so forth.

Step 4: **Determine the Importance Weights of Criteria by the Decision Maker.**

Assume there are N candidate locations $(A_1, A_2, \ldots, A_N)$, I evaluation criteria $(C_1, C_2, \ldots, C_I)$ and J sub criteria $(C_{i1}, C_{i2}, \ldots, C_{ij}, \ldots, C_{iJ})$ under criteria i, where $1 \leq n \leq N$,

$1 \leq i \leq I,\ 1 \leq j \leq J$. $\widetilde{W_i} = (c_i, a_i, b_i)$ and $\widetilde{W_{ij}} = (c_{ij}, a_{ij}, b_{ij})$ are the fuzzy importance weights given by the decision maker to criteria C_i and sub criteria C_{ij}, respectively.

Step 5: **Defuzzify the Weights of Criteria and Sub Criteria. Then Calculate the Normalized Weights.**

According to (17), we can obtain the representation of fuzzy numbers $\widetilde{W_i}$ and $\widetilde{W_{ij}}$ as follows:

$$P(\widetilde{W_i}) = \frac{1}{6}(c_i + 4a_i + b_i)$$

$$P(\widetilde{W_{ij}}) = \frac{1}{6}(c_{ij} + 4a_{ij} + b_{ij})$$

The normalized weights of criteria C_i and sub criteria C_{ij} are given by:

$$W_i = \frac{P(\widetilde{W_i})}{\sum_{i=1}^{I} P(\widetilde{W_i})}$$

$$W_{ij} = \frac{P(\widetilde{W_{ij}})}{\sum_{j=1}^{J} P(\widetilde{W_{ij}})}$$

where $\sum_{i=1}^{I} W_i = 1$ and $\sum_{j=1}^{J} W_{ij} = 1$.

The weight vector is therefore formed as follows:

$$W = [W_1 \times W_{11}, W_1 \times W_{12}, \dots, W_1 \times W_{1J}, \dots, W_i \times W_{i1}, W_i \times W_{i2}, \dots, W_i \times W_{iJ}, \dots, W_I \times W_{I1}, W_I \times W_{I2}, \dots, W_I \times W_{IJ}]$$

$$= [W_{111}, W_{112}, \dots, W_{11J}, \dots. W_{ii1}, W_{ii2}, \dots, W_{iiJ}, \dots, W_{II1}, W_{II2}, \dots, W_{IIJ}]$$

Step 6: **Calculate the Maximum Eigenvalue ($\lambda_{\max}$), the Consistency Index ($C.I$), and the Consistency Rate ($C.R$) to Test the Consistency of the Intuitive Judgment.**

To Calculate λ_{max} :

Consider the criteria $C_1, C_2, \dots, C_I$. Then the pair wise comparison matrix of the criteria is given by

$$B = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1I} \\ \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{iI} \\ \cdots & \cdots & \cdots & \cdots \\ a_{I1} & a_{I2} & \cdots & a_{II} \end{bmatrix}$$

where $a_{ki} = 1/a_{ik}$, i.e., B is reciprocal.

Now consider the weight vector of the criteria,

$$W_| = [W_1, W_2, \dots, W_I]^T$$

Then

$$\lambda_{\max} = \frac{1}{n}\left[\frac{W_1{}^|}{W_1} + \frac{W_2{}^|}{W_2} + \cdots + \frac{W_I{}^|}{W_I}\right]$$

where

$$BW_| = W^| = \left[W_1{}^|, W_2{}^|, \dots, W_I{}^|\right]^T$$

Consistency Test of the Comparison matrix:

We can use the following equation to calculate the Consistency Index (CI):

$$CI = \left(\frac{\lambda_{\max} - I}{I - 1}\right)$$

The comparison matrix will be considered to be consistent if there exists,

$$CR = \frac{CI}{RI} < 0.1$$

The various values of RI are shown in Table 6

Size of matrix	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

Table 6. Values of RI.

Step 7: **Assess Alternatives under Subjective Criteria.**

The decision maker assesses alternatives under subjective criteria. Let $\widetilde{R_{ijn}} = (c_{ijn}, a_{ijn}, b_{ijn})$ be the fuzzy ratings given by the decision maker to alternative A_n under subjective sub criteria C_{ij}.

Step 8: **Assess Alternatives under Objective Criteria.**

Let $\widetilde{R^{\circ}_{ijn}} = (c_{ijn}{}^{\circ}, a_{ijn}{}^{\circ}, b_{ijn}{}^{\circ})$ be the fuzzy quantity given to alternative A_n under objective sub-criteria C_{ij}. The objective criteria are determined in various units and must be transformed into dimensionless indices (or ratings) to ensure compatibility with the linguistic ratings of subjective criteria. The alternative with the minimum cost (or maximum benefit) should have the highest rating. By (18) and (19), we can transform fuzzy quantities for objective sub criteria into fuzzy ratings:

$$\widetilde{R_{ijn}} = \left\{ \frac{\widetilde{R^{\circ}_{ijn}}}{\max_i \{b_{ijn}{}^{\circ}\}} \right\} \times 5 \quad (18)$$

where $\max_i\{b_{ijn}{}^{\circ}\} > 0$ and $\widetilde{R_{ijn}}$ denotes the transformed fuzzy rating of objective fuzzy benefit $\widetilde{R^{\circ}_{ijn}}$. $\widetilde{R_{ijn}}$ becomes larger when objective fuzzy benefit $\widetilde{R^{\circ}_{ijn}}$ is larger:

$$\widetilde{R_{ijn}} = \left\{ \frac{\min_i \{c_{ijn}{}^{\circ}\}}{\widetilde{R^{\circ}_{ijn}}} \right\} \times 5 \quad (19)$$

where $\min_i\{c_{ijn}{}^{\circ}\} > 0$ and $\widetilde{R_{ijn}}$ denotes the transformed fuzzy rating of objective fuzzy cost $\widetilde{R^{\circ}_{ijn}}$. $\widetilde{R_{ijn}}$ becomes smaller when objective fuzzy cost $\widetilde{R^{\circ}_{ijn}}$ is larger.

Step 9: **Construct a Fuzzy Rating Matrix Based on Fuzzy Ratings.**

The fuzzy rating matrix M can be concisely expressed in matrix format:

$$M = \begin{bmatrix} \widetilde{R_{111}} & \widetilde{R_{121}} & \cdots & \widetilde{R_{IJ1}} \\ \widetilde{R_{112}} & \widetilde{R_{122}} & \cdots & \widetilde{R_{IJ2}} \\ \cdots & \cdots & \cdots & \cdots \\ \widetilde{R_{11N}} & \widetilde{R_{12N}} & \cdots & \widetilde{R_{IJN}} \end{bmatrix}$$

Step 10: **Obtain the Total Fuzzy Rating**

Obtain the total fuzzy rating ($\tilde{R}$) based on the fuzzy rating matrix (M) and weight vector (W):

$$\tilde{R} = \begin{bmatrix} \widetilde{R_{111}} & \widetilde{R_{121}} & \cdots & \widetilde{R_{IJ1}} \\ \widetilde{R_{112}} & \widetilde{R_{122}} & \cdots & \widetilde{R_{IJ2}} \\ \cdots & \cdots & \cdots & \cdots \\ \widetilde{R_{11N}} & \widetilde{R_{12N}} & \cdots & \widetilde{R_{IJN}} \end{bmatrix} \otimes \begin{bmatrix} W_{111} \\ W_{112} \\ \cdots \\ W_{IIJ} \end{bmatrix} = \begin{bmatrix} \widetilde{R_1} \\ \widetilde{R_2} \\ \cdots \\ \widetilde{R_N} \end{bmatrix}$$

Step 11: Defuzzify the total fuzzy rating by (17) and then rank alternatives according to their total crisp ratings.

$$P(\tilde{R}_n) = \frac{1}{6}(c_n + 4a_n + b_n)$$

Finally, we can select easily the best alternative with the maximum total crisp ratings.