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Generalized Beta Homeomorphisms in Intuitionistic Fuzzy Topological Spaces

D. Jayanthi
Department of Mathematics, Avinashilingam University, Coimbatore, Tamilnadu, India
jayanthimaths@rediffmail.com

Abstract

In this paper we introduce the new class of homeomorphisms called generalized beta homeomorphisms in intuitionistic fuzzy topological spaces. We also introduce M-generalized beta homeomorphisms in intuitionistic fuzzy topological spaces and investigate some of the properties. We provide the relation between intuitionistic fuzzy generalized beta homeomorphisms and intuitionistic fuzzy M-generalized beta homeomorphisms. Also we prove that the set of all M-generalized beta homeomorphisms forms a group under the operation of composition of maps.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy generalized beta $T_{1/2}$ space, intuitionistic fuzzy generalized beta homeomorphisms and intuitionistic fuzzy M-generalized beta homeomorphisms.

I. Introduction

Zadeh [6] introduced fuzzy sets. After that Atanassov [1] introduced intuitionistic fuzzy sets. Using the notion of intuitionistic fuzzy sets, Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. The notion of homeomorphisms plays a vital role in intuitionistic fuzzy topology as well as in topology. Here we introduce the new class of homeomorphisms called generalized beta homeomorphisms in intuitionistic fuzzy topological spaces. We also introduce the M-generalized beta homeomorphisms in intuitionistic fuzzy topological spaces and investigate some of the properties. We provide the relation between intuitionistic fuzzy generalized beta homeomorphisms and intuitionistic fuzzy M-generalized beta homeomorphisms. Also we prove that the set of all M-generalized beta homeomorphisms forms a group under the operation of composition of maps.

II. Preliminaries

Definition 2.1: [1] An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form
$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$

where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$. Then

a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$

d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$

e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$. The intuitionistic fuzzy sets $0_\sim = \{\langle x, 0, 1\rangle / x \in X\}$ and $1_\sim = \{\langle x, 1, 0\rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [2] An *intuitionistic fuzzy topology* (IFT for short) on X is a family τ of IFSs in X satisfying the following axioms.
(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.
In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4:[2] Let (X, τ) be an IFTS and A = ▪ x, μ_A, ν_A ▪ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by
int (A) = $\cup$ {G / G is an IFOS in X and G $\subseteq$ A} cl (A) = $\cap$ {K / K is an IFCS in X and A $\subseteq$ K}

Definition 2.5:[2] An IFS A = ▪ x, μ_A, ν_A ▪ in an IFTS (X, τ) is said to be an
(i) *intuitionistic fuzzy beta closed set* (IFβCS for short) if int(cl(int(A))) $\subseteq$ A.
intuitionistic fuzzy beta open set (IFβOS for short) if A $\subseteq$ cl(int(cl(A))).

Definition 2.6:[3] Let A be an IFS in an IFTS (X, τ). Then the beta interior and the beta closure of A are defined by
βint (A) = $\cup$ {G / G is an IFβOS in X and G $\subseteq$ A}. βcl (A) = $\cap$ {K / K is an IFβCS in X and A $\subseteq$ K}.
We have for any IFS A in (X, τ), $\beta cl(A^c) = (\beta int(A))^c$ and $\beta int(A^c) = (\beta cl(A))^c$ [3].

Definition 2.7:[3] An IFS A in an IFTS (X,τ) is said to be an *intuitionistic fuzzy generalized beta closed set* (IFGβCS for short) if βcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS in (X, τ).

Every IFβCS is an IFGβCS but the converse may not be true in general [3].

Definition 2.8:[3] The complement A^c of *intuitionistic fuzzy generalized beta open set* an IFGβCS A in an IFTS (X, τ) is called an (IFGβOS for short) in X
Definition 2.9:[3] If every IFGβCS in (X, τ) is an IFβCS in (X, τ), then the space can be called as an *intuitionistic fuzzy βT1/2 space* (IFβ$T_{1/2}$ space for short).

Definition 2.10:[4] A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy generalized beta continuous mapping* (IFGβ continuous mapping for short) if $f^{-1}(V)$ is an IFGβCS in (X, τ) for every IFCS V of (Y, σ).

Definition 2.11:[4] A mapping f: (X, τ) → (Y,σ) is called *intuitionistic fuzzy generalized beta irresolute* (IFGβ irresolute) mapping if $f^{-1}(V)$ is an IFGβCS in (X, τ) for every IFGβCS V of (Y, σ).

Definition 2.12:[5] A map f: X → Y is called an *intuitionistic fuzzy generalized beta closed mapping* (IFGβCM for short) if f(A) is an IFGβCS in Y for each IFCS A in X.

Definition 2.13:[5] A mapping f: X → Y is said to be an *intuitionistic fuzzy generalized beta open mapping* (IFGβOM for short) if f(A) is an IFGβOS in Y for each IFOS in X.

Definition 2.14:[5] A mapping f : X → Y is said to be an *intuitionistic fuzzy M-generalized beta closed mapping* (IFMGβCM, for short) if f(A) is an IFGβCS in Y for every IFGβCS A in X.

III. Generalized Beta Homeomorphisms in Intuitionistic Fuzzy Topological Spaces

In this section we introduce intuitionistic fuzzy generalized beta homeomorphisms and investigate some properties.
Definition 3.1: Let f: X → Y be a bijective mapping. Then f is said to be an intuitionistic fuzzy generalized beta homeomorphism (IFGβHM for short) if f is both an IFGβ continuous mapping and an IFGβOM.

Example 3.2: Let X = {a, b}, Y = {u, v} and G_1 = ▪ x, $(0.5_a, 0.6_b)$, $(0.5_a, 0.4_b)$▪ , G_2 = ▪ y, $(0.2_u, 0.3_v)$, $(0.8_u, 0.7_v)$▪ . Then τ = $\{0_\sim, G_1, 1_\sim\}$and σ = $\{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGβHM.

Theorem 3.3: Let f: X → Y be a bijective mapping. If f is an IFGβ continuous mapping, then the following are equivalent:
(i) f is an IFGβOM (ii) f is an IFGβHM (iii) f is an IFGβCM.
Proof: Straightforward.

Remark 3.4: The composition of two IFGβHMs need not be an IFGβHM in general.

Example 3.5: Let X = {a, b}, Y = {c, d}and Z = {e, f}. Let G_1 = ▪ x, $(0.5_a, 0.6_b)$, $(0.5_a, 0.4_b)$▪ , G_2 = ▪ x, $(0.8_a, 0.7_b)$, $(0.2_a, 0.3_b)$▪ , G_3 = ▪ y, $(0.8_c, 0.9_d)$, $(0.2_c, 0.1_d)$▪ , G_4 = ▪ z, $(04_e, 0.3_f)$, $(0.6_e, 0.7_f)$▪ and G_5 = ▪ z, $(0.2_e, 0.2_f)$, $(0.8_e, 0.8_f)$▪ and Then τ = $\{0_\sim, G_1, G_2, 1_\sim\}$, σ = $\{0_\sim, G_3, 1_\sim\}$and η = $\{0_\sim, G_4, G_5, 1\}$ are IFTs on X, Y and Z respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = c and f(b) = d and g: (Y, σ) → (Z, η) by g(c)= d and g(d) = e . Then f and g are IFGβHMs but g o f : X → Z is not an IFGβHM, since gof is not an IFGβ continuous mapping, since G_4^c = ▪ z, $(0.6_e, 0.7_f)$, $(0.4_e, 0.3_f)$▪ is an IFCS in Z but $(g \text{ o } f)^{-1}(G_4^c)$ = ▪ x, $(0.6_a, 0.7_b)$, $(0.4_a, 0.3_b)$▪ is not an IFGβCS in X, since $(g \text{ o } f)^{-1}(G_4^c)$ = ▪ x, $(0.6_a, 0.7_b)$, $(0.4_a, 0.3_b)$▪ $\subseteq G_2$ but $\beta cl((g \text{ o } f)^{-1}(G_4^c)) = 1_\sim \not\subset G_2$.

Definition 3.6: Let f: X → Y be a bijective mapping. Then f is said to be an intuitionistic fuzzy M generalized beta homeomorphism (IFMGβHM for short) if f is both an IFGβ irresolute mapping and an IFMGβOM.

The family of all IFMGβHMs in X is denoted by IFMGβHM(X).

Theorem 3.7: Every IFMGβHM is an IFGβHM but not conversely.

Proof: Let f: X → Y be an IFMGβHM. Let A ⊆ Y be an IFCS. Then A is an IFGβCS in Y. By hypothesis, $f^{-1}(A)$ is an IFGβCS in X. Hence f is an IFGβ continuous mapping. Let B ⊆ X be an IFOS. Then B is an IFGβOS in X. By hypothesis, f (B) is an IFGβOS in Y. Hence f is an IFGβOM. Thus f is an IFGβHM.

Example 3.8: Let X = {a, b}, Y = {u, v} and G_1 = ▪ x, $(0.4_a, 0.6_b)$, $(0.6_a, 0.4_b)$▪ ,G_2 = ▪ x, $(0.5_a, 0.7_b)$, $(0.5_a, 0.3_b)$▪ , G_3 = ▪ y, $(0.2_u, 0.3_v)$, $(0.8_u, 0.7_v)$▪ , then τ = $\{0_\sim, G_1, G_2, 1_\sim\}$and σ = $\{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFGβHM but not an IFMGβHM, since A = ▪ y, $(0.4_u, 0.7_v)$, $(0.6_u, 0.3_v)$▪ is an IFGβCS in Y but $f^{-1}(A)$ is not an IFGβCS in X, since $f^{-1}(A)$ = ▪ x, $(0.4_a, 0.7_b)$, $(0.6_a, 0.3_b)$▪ $\subseteq G_2$ but $\beta cl(f^{-1}(A)) = 1_\sim \not\subset G_2$.

Theorem 3.9: The composition of two IFMGβHMs is an IFMGβHM.

Proof: Let f: X → Y and g: Y → Z be any two IFMGSPHMs. Let A ⊆ Z be an IFGβCS. Then by hypothesis, $g^{-1}(A)$ is an IFGβCS in Y. Again by hypothesis, $f^{-1}(g^{-1}(A))$ is an IFGβCS in X. Therefore gof is an IFGβ irresolute mapping. Now let B ⊆ X be an IFGβOS. Then by hypothesis, f(B) is an IFGβOS in Y and also g(f(B)) is an IFGβOS in Z. This implies gof is an IFMGβOM. Hence gof is an IFMGβHM.

Theorem 3.10: Let f: X → Y be a bijective mapping. If f is an IFGβ irresolute mapping, then the following are equivalent:
(i) f is an IFMGβOM (ii) f is an IFMGβHM (iii) f is an IFMGβCM.
Proof: Straightforward.

Theorem 3.11: The set of all IFMGβHMs in an IFTS (X, τ) is a group under the composition of maps.

Proof: Define a binary operation ∗ : IFMGβHM(X) x IFMGβHM(X) → IFMGβHM(X) by f ∗ g = g o f for every f, g ∈

IFMGβHM(X) and o is the usual operation of composition of maps. Since g ∈ IFMGβHM(X) and f ∈ IFMGβHM(X), by Theorem 3.9, g o f ∈ IFMGβHM(X). We know that the composition of maps is associative. The identity map I: (X, τ) → (X, τ) belonging to IFMGβHM(X) is the identity element. If f ∈ IFMGβHM(X), then f^{-1} ∈ IFMGβHM(X). Therefore $f \text{ o } f^{-1} = f^{-1} \text{ o } f = I$ and so the inverse exists for each element of IFMGβHM(X). Hence (IFMGβHM(X), o) is a group under the composition of maps.

Theorem 3.12: If f : X → Y is an IFMGβHM, then $g\beta cl(f^{-1}(B)) \subseteq f^{-1}(\beta cl(B))$ for every IFS B in Y.

Proof: Let B ⊆ Y. Then βcl(B) is an IFGβCS in Y.. Since f is an IFGβ irresolute mapping, $f^{-1}(\beta cl(B))$ is an IFGβCS in X. This implies $g\beta cl(f^{-1}(\beta cl(B))) = f^{-1}(\beta cl(B))$ Now $g\beta cl(f^{-1}(B)) \subseteq g\beta cl(f^{-1}(\beta cl(B))) = f^{-1}(\beta cl(B))$.

Theorem 3.13: If f : X → Y is an IFMGβHM, where X and Y are IFβ$T_{1/2}$ spaces, then $\beta cl(f^{-1}(B)) = f^{-1}(\beta cl(B))$ for every IFS B in Y.

Proof: Since f is an IFMGβHM, f is an IFGβ irresolute mapping. Since βcl(f(B)) is an IFGβCS in Y, $f^{-1}(\beta cl(f(B)))$ is an IFGβCS in X. Since X is an IFβ$T_{1/2}$ space, $f^{-1}(\beta cl(f(B)))$ is an IFβCS in X. Now, $f^{-1}(B) \subseteq f^{-1}(\beta cl(B)) \subseteq \beta cl(f^{-1}(\beta cl(B)))$. We have $\beta cl(f^{-1}(B)) \subseteq \beta cl(f^{-1}(\beta cl(B))) = f^{-1}(\beta cl(B))$. This implies $\beta cl(f^{-1}(B)) \subseteq f^{-1}(\beta cl(B))$ ----- (*). Again since f is an IFMGβHM, f^{-1} is IFGβ irresolute mapping. Since $\beta cl(f^{-1}(B))$ is an IFGβCS in X, $(f^{-1})^{-1}(\beta cl(f^{-1}(B))) = f(\beta cl(f^{-1}(B)))$, is an IFGβCS in Y. Now $B \subseteq (f^{-1})^{-1}(f^{-1}(B)) \subseteq (f^{-1})^{-1}(\beta cl(f^{-1}(B))) = f(\beta cl(f^{-1}(B)))$. Therefore $\beta cl(B) \subseteq \beta cl(f(\beta cl(f^{-1}(B)))) = f(\beta cl(f^{-1}(B))$, since Y is an IFβ$T_{1/2}$ space. Hence $f^{-1}(\beta cl(B)) \subseteq f^{-1}(f(\beta cl(f^{-1}(B)) \subseteq \beta cl(f^{-1}(B))$. That is $f^{-1}(\beta cl(B)) \subseteq \beta cl(f^{-1}(B))$ ------ (**). Thus from (*) and (**) we get $\beta cl(f^{-1}(B)) = f^{-1}(\beta cl(B))$ and hence the proof.

Corollary 3.14: If f : X → Y is an IFMGβHM, where X and Y are IFβ$T_{1/2}$ spaces, then βcl(f (B)) = f(βcl(B)) for every IFS B in X.

Proof: Since f is an IFMGβHM, f^{-1} is also an IFMGβHM. Therefore by Theorem 3.13 $\beta cl((f^{-1})^{-1}(B)) = (f^{-1})^{-1}(\beta cl(B))$ for every B ⊆ X. That is βcl(f (B)) = f(βcl(B)) for every IFS B in X.

Corollary 3.15: If f : X → Y is an IFMGβHM, where X and Y are IFβ$T_{1/2}$ spaces, then βint(f (B)) = f(βint(B)) for every IFS B in X.

Proof: For any IFS B ⊆ X, $\beta int(B) = (\beta cl(B^c))^c$. By Corollary 3.14, $f(\beta int(B)) = f(\beta cl(B^c))^c = (f(\beta cl(B^c)))^c = (\beta cl(f(B^c)))^c = \beta int(f(B^c))^c = \beta int(f(B^c)^c) = \beta int(f(B))$.

Corollary 3.16: If f : X → Y is an IFMGβHM, where X and Y are IFβ$T_{1/2}$ spaces, then $\beta int(f^{-1}(B)) = f^{-1}(\beta int(B))$ for every IFS B in Y

Proof: Since f is an IFMGβHM, f^{-1} is also an IFMGβHM, the proof directly follows from Corollary 3.15.

IV. References

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