1. Introduction

Queueing theory is a branch of applied mathematics utilizing the concepts from the field of stochastic processes. It has been developed for the purpose of better understanding of queueing systems and to take appropriate measures to maintain the system efficiency.

1.1 Classical Queueing System

A classical queueing system can be described as customers arriving for service, waiting for service if it is not immediate, and if having waited for service, leaving the system after being served. The general structure of a queueing system is presented in Fig. 1.1.

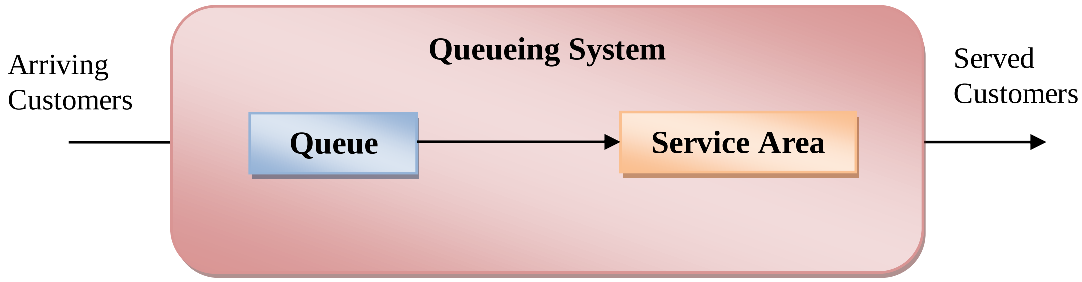

Fig. 1.1 General Structure of a Queueing System

1.2 Characteristics of a Queueing System

The basic characteristics of a queueing system which provide an adequate description are arrival pattern, service pattern, queue discipline, system capacity and service channels.

Arrival Pattern

Arrival pattern describes the way how units arrive and join the system. The arrival may be either single or batches of variable or fixed size. The source of units may be finite or infinite. The arrival patterns are often measured in terms of average number of arrivals per unit time.

The inter-arrival time is the interval between two consecutive arrivals. In case, the arrival times are known with certainity, the queueing problems are characterized as deterministic models. However, in usual queueing situations, the process of arrivals is stochastic and it is necessary to know the probability distribution associated with the successive arrivals. The most common stochastic queueing models assume that inter-arrival times follow an Exponential distribution. The arrival pattern also describes the behaviour of the customers as some customers may wait patiently in the queue and some may be impatient if it takes a long time to receive the desired service. If an arriving customer decides not to join the queue, the customer is said to have balked. If a customer leaves the queue after joining due to impatience, it is known as reneging. In case there are two or more parallel waiting lines and a customer moves from one queue to another, the customer is said to have jockeyed.

An arrival process could be stationary or non-stationary according to the probability distribution describing the arrival pattern being time independent or dependent of time.

Service Pattern

Service pattern describe the manner in which the service is rendered. The customers may be served in single or in batches of variable or fixed size and the time required for serving one customer or customers in batches is called the service time. The service pattern of customers may be stationary or non-stationary with respect to time and state dependent or independent with respect to the number of customers waiting for service. The service time may be deterministic or probabilistic.

Queue Discipline

Queue discipline refers to the manner in which customers are selected for service from the queue. The most common disciplines are first come, first served according to which the customers are served in order of their arrival. The following are the various queue disciplines:

- FIFO : first in first out (or first come first served FCFS)
- LIFO : last in, first out, usually seen in a warehouse where the items that come last are taken out first
- SIRO : service in random order

- Priority disciplines : Under this discipline, the service is offered to customers depending on the priority with the other customers. There are two situations in priority discipline called preemptive and non preemptive priority. In the preemptive case, the customer with higher priority is allowed to enter service immediately suspending the service in progress to a customer with lower priority. In non-preemptive case the higher priority goes to the head of the queue but cannot get into the service until the customer presently in service is completed.

System Capacity

In some queueing processes there is a physical limitation to the amount of waiting room so that when the line reaches a certain length, no further customers are allowed to enter until space becomes available, as a result of a service completion. These are referred to as finite queueing situations. A queue with limited waiting room can be viewed as one with forced balking.

Service Channels

The number of servers in a queueing model may be finite or infinite. The number of servers may be arranged in series, parallel or a combination of both, depending upon the nature of the services required. In parallel channels, all the channels provide identical services so that several customers may be served simultaneously. In series channels, a customer must pass through successively in several ordered channels before service is completed.

1.3 Kendall's Notation

A queueing process can be represented by the notation introduced by Kendall (1951) as A / B / C / X / Y, where A represents the inter-arrival time distribution of customers, B denotes the service time distribution, C the number of parallel servers, X represents the capacity of the system and Y denotes the queue discipline. If the queueing system has infinite capacity and the queue discipline is FIFO, then the system is denoted as A/B/C without mentioning X and Y.

The notation $M^X/G/1$ indicates a batch arrival queueing system with exponential interarrival times, general service times, single server, infinite system capacity and first come first served queue discipline.

1.4 Retrial Queueing System

In conventional queueing theory it is usually assumed that an arriving customer who cannot get service immediately either joins the waiting line or leaves the system forever. Sometimes impatient customers leave the queue but it is also assumed that they are leaving the system forever. In real life situation, such customers after some random period of time return to the system and try to get service. The standard queueing models do not consider the phenomenon of retrials and therefore cannot be applied in solving a number of practical problems. Retrial queues have been introduced to meet this inadequacy.

Retrial queueing systems are characterized by the feature that arriving customers who cannot receive service immediately may join a virtual queue called orbit to try their request after some random time. General structure of a retrial queueing system is presented in Fig. 1.2.

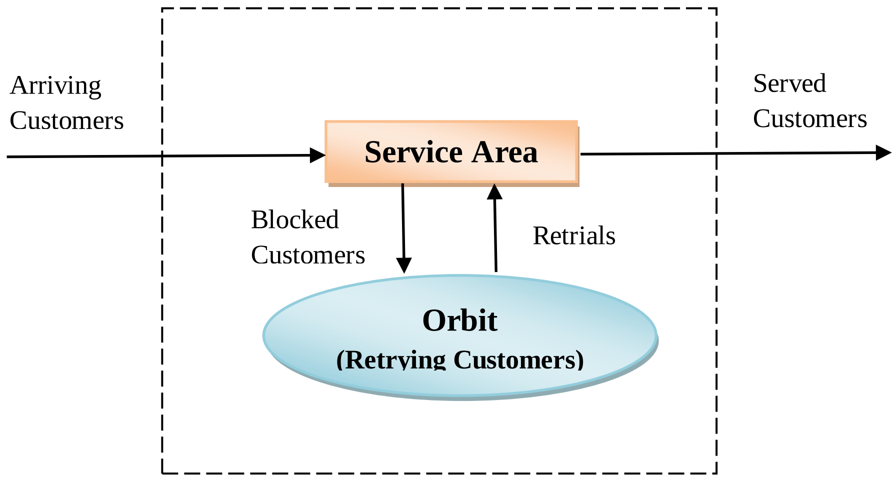

Fig. 1.2 General Structure of a Retrial Queueing System

1.5 Literature Survey

Queueing theory was developed to provide mathematical models to predict behaviour of systems that attempt to provide service for randomly arising demands. Queueing theory had its origin in 1909 when Danish telephone engineer, Erlang published his paper “The theory of probabilities and telephone conversations”. Later, Molina (1927) presented the paper “Application of the theory of probability to telephone trunking problems”. Fry (1928) extended much of Erlang’s work in a book “Probability and its Engineering Uses”. In 1930, Pollaczek studied the problem of Poisson input, arbitrary holding time and single channel and in 1932, extended the problem to multiple channel.

Kendall (1953) on imbedded chain, Tackacs (1955) on waiting time, Cox (1955) on supplementary variables, Champernowne (1956) on the use of random walks, Saaty (1960) on absorbing barriers and time dependent solutions, Little (1961) on deriving the queueing formula $L = \lambda W$, Conway and Maxwell (1962) on state dependent service and Neuts (1973) on discrete time queueing model are some remarkable work in queueing literature. The review of queueing literature can be seen in the survey papers of Prabhu (1987), Shukla and Shrivastav (2016) and Shastrakar et al. (2016).

Retrial queues

Queueing system in which arriving customers who find all servers and waiting positions occupied may retry for service after a period of time are called retrial queues. Retrial queues have been widely used to model many problems in telephone switching systems, telecommunication networks, computer networks and computer systems.

One of the earliest papers in retrial queues was On the Influence of Repeated Calls in the Theory of Probabilities of Blocking by Kosten (1947). Keilson et al. (1968) published the first result on M/G/1 retrial queue using the method of supplementary variable. The first investigation with general retrial time was done by Kapyrin (1977), in which each customer in the orbit generates a stream of repeated attempts that are independent of the customers in the orbit and the state of the server.

Review of retrial queueing literature can be found in the survey papers of Yang and Templeton (1987), Falin (1990), Kim and Kim (2016), Shekar et al. (2016) and Tuan Phung-Duc (2019), the bibliographies of Artalejo (1999a, 1999b and 2010) and the books by Falin and Templeton (1997) and Artalejo and Gomez Corral (2008).

The applications of retrial queues in science and engineering are given by Kulkarni and Liang (1997).

Queues that feature multiple entities arriving simultaneously are among the oldest models in queueing theory, and are often referred to as **batch arrival** queueing systems. Batch arrival retrial queueing model was introduced by Falin (1976). Kulkarni (1986) and Falin (1988) analysed multiple classes of customers with batch arrivals. Artalejo and Atentia (2004) analysed a single server retrial queue with batch arrivals. A detailed study on batch arrival retrial queue under both classical and constant retrial policies was done by Jain et al. (2008). Many authors including Kalyanaraman (2012), Yamamuro (2012), Singh et al. (2015) and Zirem et al. (2019) discussed retrial queueing situations with batch arrivals.

Unreliable Queueing Models

Breakdown is a remarkable and unavoidable phenomenon in the service facility of a queueing system. Queues with server subject to breakdowns and repairs are often encountered in many practical applications. The study of queueing models with repairable server can be found earlier in the articles of Avi-Itzhak and Naor (1962), Thiruvengdan (1963), Mitrany and Avi-Itzhak (1968), Neuts and Lucantoni (1979) and Cao and Cheng (1982).

Retrial queueing systems with server breakdowns and repairs were considered by Aissani (1988) and Kulkarni and Choi (1990). Aissani and Artalejo (1998) studied a single server retrial queueing system subject to active breakdown. Wang et al. (2001) obtained explicit expressions of availability, failure frequency and reliability function of the server for M/G/1 retrial queue with server breakdown. Li et al. (2006) provided BMAP/G/1 retrial queue with server breakdowns and repairs considering both from queueing view point and reliability view point. Choudhury and Deka (2008) discussed a Poisson input two phase queueing with server breakdown and repair. Jain and Charu Bhargava (2008) analysed the waiting time distribution and sensitivity analysis for a batch arrival retrial queueing model with priority subscribers and unreliable server. Choudhury and Deka (2009a) obtained the limiting distribution of the number of customers in the system at departure epoch and idle period completion epoch for an M/G/1 retrial queue with two types of heterogeneous service subject to random breakdown and repair under linear retrial policy. Falin (2010) investigated an M/G/1 retrial queue with an unreliable server and general retrial times with the help of embedded Markov chain.

In most of the papers with unreliable server it is assumed that whenever the system breaks down the repair process starts instantaneously. However, this is not the case in many real life situations. The system has to wait for repair to start. Prakash Rani et al. (2011) discussed the retrial queue with server breakdown and delayed repairs. Choudhury and Ke (2014) studied an unreliable retrial queue with delaying repair and general retrial times under Bernoulli vacation schedule and derived system size distribution at departure epoch. Gao et al. (2020) studied an M/G/1 retrial queue with two types of breakdowns and delayed repairs.

Server Vacation

In many real world systems, the server becomes unavailable for a random amount of time, referred to as vacation time. Vacation times can be used to perform tasks such as maintenance work or cleaning operations. The queueing systems with server vacations introduced by Levy and Yechialli (1975) are widely used in manufacturing systems, service systems, inventory systems and other stochastic systems. Excellent survey on vacation queueing models have been reported by Doshi (1986), Takagi (1991), Tian and Zhang (2006) and Ke et al. (2010). Bagyam and Udaya Chandrika (2010) discussed a single server retrial queueing system with two different vacation policies. Choudhury and Ke (2012) dealt with the steady state behaviour of an $M^X/G/1$ queue with general retrial time and Bernoulli vacation schedule for an unreliable server. Bagyam and Udaya Chandrika (2013a) investigated a single server state dependent batch arrival retrial queue with extended Bernoulli vacation and obtained explicit formula for the stationary distributions and performance measures of the system. Arivudainambi and Gowsalya (2018) investigated a single server retrial queueing system with two types of service and Bernoulli vacation. Gao and Wang (2019) analysed a single server retrial queue with server vacation and two waiting buffers based on ATM networks.

As the generalization of the single and multiple vacation policy, Ke and Chang (2009) introduced the concept of modified vacation in M/G/1 retrial queueing system. Chang and Ke (2009) extended the model to batch arrival retrial queues. Chen et al. (2010) studied a retrial queue with modified vacations and server breakdowns. Jain and Bhagat (2014) considered an unreliable bulk retrial queues with delayed repairs and modified vacation policy and obtained the probability generating function of the steady state queue size at random epoch using supplementary variable technique. Sumitha and Udaya Chandrika (2015a) discussed unreliable batch arrival retrial queue with delayed repair and randomized J vacations using supplementary variable techniques and verified the stochastic decomposition property. Madheswari and Suganthi (2016) examined an M/G/1 retrial queue with second optional service and starting failure under modified Bernoulli vacation. Jailaxmi et al. (2017) studied a single server retrial queue with general retrial times and modified M vacations and obtained the probability generating function of number of customers in the orbit. Yang and Wu (2019) considered the performance analysis and optimization of a retrial queue with working vacations and starting failures. Murugan and Vijaykrishnaraj (2020) discussed a bulk arrival retrial queue with starting failures and exponentially distributed multiple working vacation. Rajam and Uma (2021) investigated a batch arrival retrial queue with modified Bernoulli vacation and server utilization.

Feedback

One additional feature that has been widely discussed in retrial queueing systems is the Bernoulli feedback of customers. These queueing model arise in the stochastic modeling of many real life situations. For example, in data transmission, a packet transmitted from the source to the destination may be returned and it may continue this process until the packet is finally transmitted. This type of retransmission is called feedback. Tackacs (1963) was the first to study such a model, where the customers who completed their service feedback instantaneously to the tail of the queue with certain probability or depart from the system forever with complementary probability. The authors Choi et al. (1998), Krishna Kumar et al. (2002), Lee (2005), Li and Wang (2006) and Mokaddis et al. (2007) discussed the concept of feedback in retrial queueing systems.

Kasthuri Ramnath and Lakshmi (2011) considered a two phase multi-optional retrial queueing system in which after service completion of both the phases, the customer may join the orbit as a feedback customer or leave the system. Arivudainambi and Godhandaraman (2012) suggested a two phase batch arrival retrial queueing system with feedback and k optional vacations with practical justification. Boualem et al. (2012) proposed stochastic ordering techniques to establish various monotonicity results with respect to arrival rates, service time distributions and retrial parameters for an M/G/1 retrial queue with a Bernoulli feedback. Arivudainambi and Gowsalya (2016) analyzed a single server feedback retrial queue with two types of service and Bernoulli vacation. Rajadurai et al. (2018) studied an M/G/1 feedback retrial queue with server subject to breakdown and repair under multiple working vacation policy. Pankaj Sharma (2018) investigated a single unreliable server retrial queueing model by incorporating Bernoulli feedback and repair along with modified vacation policy.

Admission Control Policy

In many bulk arrival queueing systems, there is a restriction such that not all the customers are allowed to join the system at all times. This policy is named as admission control policy. Madan and Abu Dayyeh (2002a, 2002b) proposed restricted admissibility of arriving batches and analysed $M^X/G/1$ queueing system with Bernoulli Schedule server vacation. Madan and Choudhury (2004), Alnowibet and Tadj (2007), Choudhury (2008a) presented articles with admission control concepts. Badamchi Zadeh (2009) studied a batch arrival queueing system with two phases of heterogeneous service, restricted admissibility and server vacation. Madan (2010) investigated two stage bulk arrival queueing system with restriction on admitting an arriving batch of customers.

Admission mechanism in retrial systems was introduced by Artalejo et al. (2005). Later, Choudhury and Deka (2009b) analysed the unreliable batch arrival retrial queue with Bernoulli admission mechanism and two phase service model. Wang and Zhou (2010) derived the steady state probabilities for the $M^{[X]}/G/1$ retrial queue with starting failures, Bernoulli feedback and Bernoulli admission control for each individual customers. Purohit et al. (2012) investigated an M/M/1 retrial queue with constant retrial rate, unreliable server and threshold based recovery with state dependent arrival rates. Shweta (2013) examined the two phase bulk arrival retrial queueing model with Bernoulli admission control policy and derived the distribution of the number of customers in the retrial group using Markov chain technique. Dimitriou (2013a) considered an unreliable single server retrial queue with batch arrivals and admission control and investigated the stability condition and joint queue length distribution in steady state. Niranjan et al. (2017) analysed the bulk retrial queueing system with admission control, immediate feedback, multiple vacation and threshold.

Orbital Search

In the retrial setup, each service is preceded and followed by the server's idle time because of the ignorance of the status of the server and orbital customers by each other. Server's idle time is reduced by the introduction of search of orbital customers immediately after a service completion. Search for orbital customers was introduced by Neuts and Ramalhoto (1984) where the authors examined classical queue with search for customers immediately on termination of a service. Artalejo et al. (2002) considered a retrial queueing system with orbital search. Dudin et al. (2004) extended the model to a batch arrival retrial queue and performed the steady state analysis of the queueing system. Krishnamoorthy et al. (2005) analyzed M/G/1 retrial queue with non-persistent customers and orbital search using supplementary variable method and discussed the structure of the busy period and its analysis in terms of Laplace transform. Chakravarthy et al. (2006) studied a multi-server retrial queueing model with orbital search using direct truncation and matrix geometric approximation. Sumitha and Udaya Chandrika (2012) discussed a single server batch arrival retrial queueing system with orbital search and obtained the steady state distributions of the server state and the number of customers in the orbit. Deepak et al. (2013) considered a retrial model in which at each service completion epoch, two different search mechanisms are switched on to bring the orbital customers to service. Rajadurai et al. (2015) discussed batch arrival retrial queue with two phase service, Bernoulli vacation, delaying repair and orbital search. Dudin et al. (2017) studied a BMAP/G/1 retrial system with two types of search of customers from the orbit.

Impatient Customers

Retrial queueing model with impatient customers was proposed by Cohen (1957). Li and Zhao (2005) studied a retrial queue with impatient customers and server breakdown. Artalejo and Pla (2009) modelled a call center as the M/M/m retrial queue with exponential impatience times. Senthil Kumar and Arumuganathan (2009) analysed M/G/1 retrial queue with two essential phases of service, non-persistent customers and different vacation policies. Senthil Kumar and Arumuganathan (2010) considered a two phase retrial queue with impatient subscribers and general vacation time to analyse a communication protocol. Arrar et al. (2011) presented an article on the asymptotic behaviour of the M/G/1 batch arrival retrial queue with impatience phenomenon and obtained partial generating functions of the steady state joint distribution of the server state and the number of customers in the retrial group. Garg and Sanjeev Kumar (2012) obtained explicit time dependent probabilities of exact number of arrivals and departures from the orbit of a single server retrial queue with impatient customers. Sumitha et al. (2012) analysed a single server batch arrival retrial queue with balking, reneging, server vacation and orbital search. Bhagat and Jain (2013) investigated $M^X/G/1$ retrial queue with unreliable server by assuming the impatient customers are allowed to balk depending upon server's states. Azhagappan et al. (2018) analyzed the transient behaviour of an M/M/1 retrial queueing model where the customers in the orbit possess the reneging behaviour. Peng and Wu (2021) discussed retrial queue with impatient customers subject to disastrous failure at which times all customers are lost and provided application to telecommunication network.

Preemptive resume priority and Collision

Preemptive resume priority queues have important uses in modelling and analysing computer system and communication network. Retrial queue with priority calls can be found in the survey paper of Choi and Chang (1999). Artalejo et al. (2001) considered a retrial queueing system where customers at the retrial group have preemptive priority over customers at the waiting line. Krishna Kumar et al. (2002) analysed an M/G/1 retrial queueing system with additional phase of service and preemptive resume service discipline and obtained analytical expressions for various performance measures. Ayyappan et al. (2010) studied a retrial queueing system with single working vacation under preemptive priority service using matrix geometric technique. Senthil Kumar et al. (2013) analysed retrial queue in which two types of calls arrive according to Markovian arrival process and the service are offered with preemptive priority rule. The authors obtained performance measures using matrix analytic methods and provided numerical examples to bring out the qualitative aspects of the model. Jain and Jain (2014) used the supplementary variable technique to analyse a batch arrival priority queueing model with second optional service and server breakdown. Boutarfa and Djellab (2015) obtained pertinent performance of the $M_1/M_2/G_1,G_2/1$ retrial queue with pre-emptive priority. Using supplementary variable technique Ammar and Rajadurai (2019) examined the impact of parameters in system performance measures and cost optimization for preemptive priority retrial queueing system with disaster under working breakdown. Rajadurai et al. (2020) derived an $M^{[X]}/G/1$ priority retrial queue with feedback and working breakdowns. Gao and Wang (2020) studied the stochastic analysis of a preemptive retrial queue with orbital search and multiple vacations. Ayyappan and Udayageetha (2020) studied the transient analysis of $M^{[X1]},M^{[X2]}/G_1,G_2/1$ retrial queueing system with non-persistent customers and priority services. The server follows the preemptive priority rule subject to working breakdown, start up / close down time and Bernoulli vacation with general vacation period.

Retrial queues with collisions arise from the medium access control protocols for wireless LANs. Uncoordinated attempts by several sources to use a single server facility may result in collision. Jonin (1982) analysed a single line retrial queueing system with collision. Choi et al. (1992) discussed a retrial queueing system with constant retrial rate and collision in the specific communication protocol CSMA-CD and obtained the limiting distribution of the number of customers in the system at arbitrary time point using the theory of Markov regenerative processes. Using generating function technique, Krishna Kumar et al. (2010) studied a Markovian single server feedback retrial queue with linear retrial rate and collision of customers and obtained the joint distribution of the server state and the orbit length. Kim (2010) considered an M/M/1 retrial queue with collision and impatience and obtained the performance characteristics of the system. Wu et al. (2011) considered a discrete time Geo/G/1 retrial queue with preemptive resume and collisions and derived ergodic condition along with important performance measures. Ayyappan and Thamizhselvi (2017) studied the transient analysis of bulk arrival general service retrial queueing system with priority, collisions, Bernoulli feedback, orbital search, modified Bernoulli vacation, random breakdown and delayed repair. Lakaour et al. (2019) considered an M/M/1 retrial queue with collision and transmission errors.

Two Phase Queueing Models

Retrial queues with two phase service were discussed by Atentia and Moreno (2006), Artalejo and Gomez (2007), Srinivasan et al. (2007), Wang and Zhao (2007), Choudhury (2008b), Prakash Rani et al. (2008), Wang and Li (2008, 2009), Dimitriou and Langaris (2008, 2010) and Ramanath and Kalidass (2010). Varalakshmi et al. (2016) analysed an M/G/1 retrial queueing system with two phases of service, immediate Bernoulli feedback, single vacation and starting failures. Varalakshmi et al. (2017) analysed an unreliable two phase retrial queue immediate Bernoulli feedbacks. Murugan and Vijaykrishnaraj (2019) dealt with the study of bulk arrival retrial queue with second optional service and exponentially distributed multiple working vacation and obtained the probability generating function for the number of customers in the orbit using supplementary variable method. Nila and Sumitha (2019) studied a batch retrial queueing model with multioptional second phase, starting failures, customer impatience and orbital search.

Multistage Queueing Models

Shahkar and Badamchi Zadeh (2006) analysed vacation queueing model with multiphases of services in succession. Salehirad and Badamchi Zadeh (2009) and Abdollahi and Salehirad (2012) studied the multi phase M/G/1 queueing system with feedbacks. Bagyam and Udaya Chandrika (2013b) analysed multistage retrial queueing system with breakdown and reserved time. Bagyam and Udaya Chandrika (2013c) obtained the joint distribution of the server state and orbit length of a multi-stage retrial queueing system with feedback at each stage. Radha et al. (2014) studied a batch arrival retrial queue with K – optional stages of service, Bernoulli feedback, single vacation and random breakdown. Using generating function technique, Bagyam and Udaya Chandrika (2015) obtained important performance measures of a bulk arrival retrial queueing model with multistages of service and Bernoulli vacation. Radha et al. (2017a) considered a group arrival multistage unreliable retrial queueing system with vacation. Bagyam and Udaya Chandrika (2018) considered a bulk arrival retrial queueing system with multistages of heterogeneous services, active breakdown and delayed repair. Rajadurai et al. (2018) discussed multiphase retrial queue with optional reservice and multiple working vacations using supplementary variable technique. Radha et al. (2020) analysed a batch arrival retrial queueing model with multi optional stages of service, orbital search, extended Bernoulli vacation and stand by server. Sangeetha et al. (2020) analysed a single server bulk arrival retrial queue with multistages of heterogeneous service and feedback.

G-Queue

In recent years, interest is growing in queues with negative customers due to their applications in the telecommunication system, neutral networks, multiprocessor computer systems and manufacturing systems. The name G-queue has been adopted for the queue with negative customers in acknowledgement of Gelenbe (1989) who first introduced the concept. The negative arrivals affect the queue behaviour in a variety of ways. The arrival of negative customer – (i) eliminates all the customers in the system (disaster), (ii) removes the customer from the head of queue, including the one being in service or (iii) deletes the customer at the end of the queue. A detailed survey on queueing systems with negative arrivals can be found in Gelenbe (1991, 1994, 2000), Artalejo (2000) and Bocharov and Vishnevsky (2003).

Wang and Zhang (2009) introduced negative customers into the retrial queue and deduced the stochastic decomposition law on discrete time retrial G-queue. Aissani (2010) obtained the generating function of the number of primary customers in the stationary regime of an M/G/1 retrial queue with negative arrivals and server breakdown. Wu and Yin (2011) analysed a single server retrial queue with negative customers and non-exhaustive random vacations subject to the server breakdowns and repairs. Dimitriou (2013b) discussed retrial queue with negative arrivals, mixed priorities and multiple vacations and obtained the stability condition and the system state probabilities. Yang et al. (2013) analysed unreliable $M^X/G/1$ retrial G-queue with single vacation by using supplementary variable technique and obtained the steady state solutions for both queueing and reliability measures. Krishna Kumar et al. (2013) discussed the busy period of the M/G/1 Bernoulli feedback retrial queueing system with negative customers and derived the performance measures. Berdjoudj and Aissani (2014) analysed the M/G/1 retrial queue with negative arrivals using a Martingale technique. Peng et al. (2014) considered M/G/1 retrial G-queue with preemptive resume priority and collisions subject to server breakdowns and repairs. Sumitha and Udaya Chandrika (2015b) extended the model to batch arrival retrial queueing system. Kirupa and Udaya Chandrika (2015) studied a batch arrival retrial queue with negative customers, multi-optional service and feedback. Rajadurai et al. (2015) studied an $M^X/G/1$ unreliable retrial G-queue with orbital search, feedback and Bernoulli vacation. Li and Zhang (2017) considered an M/G/1 retrial G-queue with general retrial times, in which the server is subject to working breakdowns and repairs. Yuvarani and Saravanarajan (2017) analysed a preemptive priority retrial queue with negative customers, starting failure and J vacations. Varalakshmi et al. (2017) provided a study on M/G/1 retrial G-queue with two phases of service, immediate feedback and working vacations. Radha et al. (2017b) investigated a group arrival retrial G-queue with multi optional stages of service, orbital search and server breakdown. Sangeetha and Udaya Chandrika (2019) derived a batch arrival multistage retrial G-queue with fluctuating modes of services and feedback.

1.6 Thesis Organization

The main objective of this thesis is to analyse the steady state behaviour of single server batch arrival retrial G-queues with multistage and multi-optional services. Schematic representation of thesis is presented in Fig. 1.3.

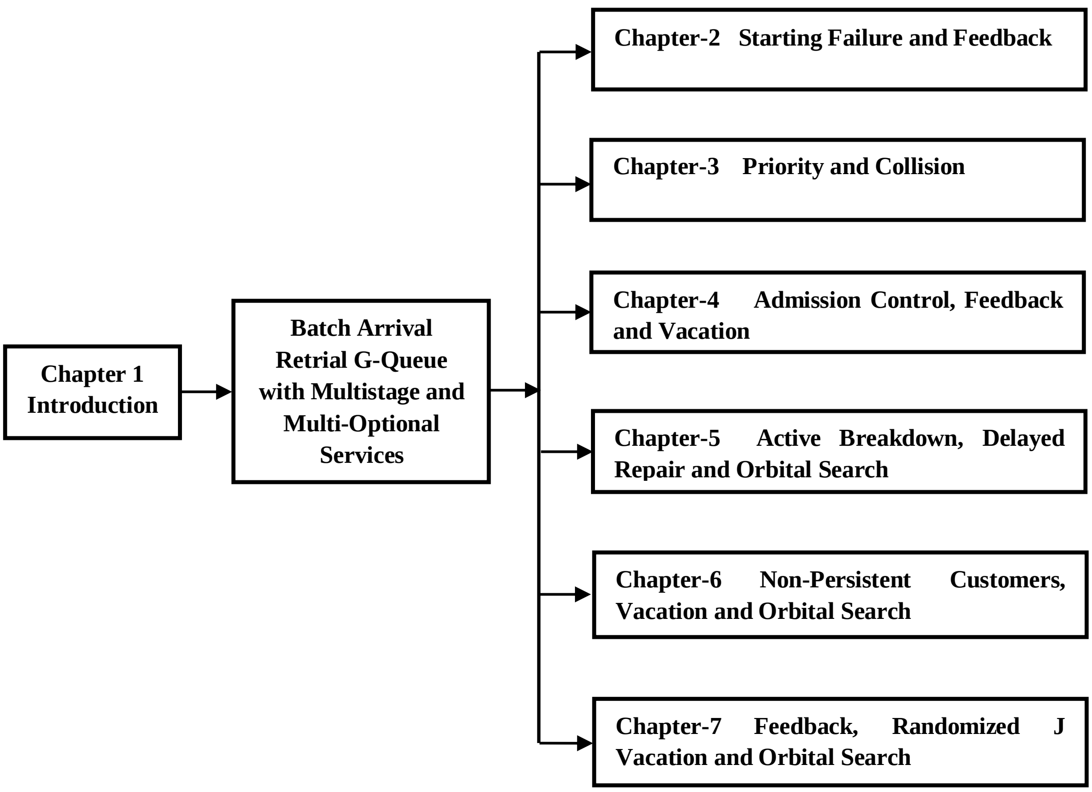

Fig. 1.3 Organisation of the Thesis

- Chapter 1 deals with the preliminary concepts of queueing theory and relevant survey of literature.

- Single server batch arrival retrial G-queue with multistage and multi-optional services, starting failure and feedback is considered in Chapter 2. Server provides service in two phases having first phase of essential service and second phase of multistages of service. In each stage of second phase, there are multi-optional heterogeneous services. The positive customers arrive in batches according to the Poisson process. Upon arrival, if the server is busy, all the customers join the orbit. Otherwise, one of the customers in the batch starts the essential service. If the server is started successfully, the customer gets the essential service immediately. Otherwise, the server is sent for repair instantaneously and the customer joins the orbit. After completing the essential service the customer moves to the second phase and opts any one of the services from stage 1, leaves the system or joins the orbit as feedback customer. After the completion of stage 1, the customer opts any one of the services from stage 2, leaves the system or feedback to the orbit. After the final stage service the customer either leaves the system or joins the orbit. The arrival of negative customers makes the server down and pushes out the positive customer being in service.

- In Chapter 3, batch arrival retrial G-queue with multistage and multi-optional services, priority and collision is analyzed. The server renders service in two phases with first phase of essential service and second phase of multistages of services with multi-options in each stage. If the server is free, one of the customers from the batch receives essential service immediately and others join the orbit. If the server is busy, the arriving batch either joins the orbit or collides with the customer in service resulting in all being shifted to the orbit or one of the customers in the batch interrupts the customer in service to get its own service. Priority or collision takes place when the server is busy at any phase of service and the interrupted customer resumes his service from the beginning.

- Chapter 4 deals with bulk arrival retrial G-queue with multistage and multi-optional services, admission control, feedback and vacation. Admission of positive customers to the system depends upon the state of the server. If the server is idle, one of the admitted customers enters the service immediately and the rest joins the orbit, otherwise all the admitted customers enter the orbit. After completion of essential or optional services if the customer is dissatisfied with the service he may join the orbit as a feedback customer. When the system becomes empty, the server may go for a vacation with certain probability.

- Batch arrival retrial G-queue with multistage and multi-optional services, active breakdown, delayed repair and orbital search is investigated in Chapter 5. The server becomes inactive due to random breakdown or due to negative arrival. In the case of breakdown due to negative arrival, the failed server is sent for repair immediately and the interrupted customer leaves the system forever. Whereas in the case of active breakdown, the repair of the failed server starts after some random time and the interrupted customer remains in the service position for the completion of the service. During the idle period, the server searches for customers in the orbit or remains idle.

- In Chapter 6, batch arrival retrial G-queue with multistage and multi-optional services, impatient customers, Bernoulli vacation and orbital search is discussed. Customers are allowed to balk and renege at a particular time. After service or repair completion if the system is empty, the server may take a single vacation with certain probability or remain idle in the system with the complementary probability. At the vacation completion epoch, the server may search for the customers in the orbit.

- Final chapter is devoted to study batch arrival multistage and multi-optional retrial G-queue with feedback, randomized J vacations and orbital search. Whenever the system becomes empty, the server goes for vacation and takes at most J vacations repeatedly until at least one customer is recorded in the orbit. At the end of J^{th} vacation, even if the orbit is empty, the server joins the system and remains idle. During the idle period in the non-empty system, the server may search for customers in the orbit.

Models are mathematically formulated and analyzed using the supplementary variable technique under steady state. The retrial time, service time, delay time, repair time and vacation time are assumed to follow general distribution. The steady state queue size distribution of the number of customers in the orbit, expected queue length and expected system size are derived explicitly. Stochastic decomposition law is verified. Reliability measures of the systems are obtained. Some special cases are deduced. Numerical results are added to demonstrate the influence of various parameters on the system behavior. Practical justifications are provided for the developed models.