



Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category 'A' by MHRD (now MoE)

Re-accredited with 'A++' Grade by NAAC. CGPA 3.65/4, Category I by UGC

Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination - May 2025

IV Semester

Class : II PG
Major : Mathematics

Time: 3 Hours
Max. Marks: 100

23MMAC21 Topology - II

Course Outcomes:

CO1: Categorize the separation axioms:

CO2: Appreciate the relation between metric spaces and topological spaces.

CO3: Prove standard theorems in topology.

CO4: Demonstrate the concept of complete metric space.

CO5: Categorize the notion of convergent in topological spaces.

Part A

10 x 1 = 10

Choose the Correct Answer

1. A space that has a countable basis at each of its points is to satisfy _____ CO1K2
a. Second countable b. First countable c. Finite countable d. Separable
2. A subspace of a regular space is regular, a product of regular spaces is _____ CO1K1
a. Hausdorff b. Normal c. Separable d. Regular
3. A countable space is _____ CO2K2
a. Hausdorff b. Normal c. Regular d. Lindelof
4. Every topological group is _____ CO2K4
a. Completely regular b. Hausdorff
c. Compact Hausdorff space d. Perfectly normal
5. A collection \mathcal{A} of subsets of X has the _____ if every countable intersection of CO3K1
elements of \mathcal{A} is nonempty.
a. Finite intersection b. Countable intersection
c. Countable d. Finite
6. An arbitrary product of compact spaces is compact in the _____ CO3K2
a. Minimal element b. Product topology
c. Lindelof space d. Finite intersection
7. If every Cauchy sequence in X converges then its _____ CO4K3
a. Convergent subsequence b. Complete
c. Euclidean d. Uniform metric
8. The metric space (X, d) is compact iff it is _____ CO4K4
a. Totally bounded b. Complete
c. Bounded d. Complete and totally bounded
9. If X is locally compact then X is _____ CO5K3
a. Compactly generated b. Compact
c. Compact convergence d. Point open topology
10. If X is locally compact and second countable, it is _____ CO5K4
a. σ -compact b. Compact Hausdorff
c. Compact convergence d. Uniform

Part B
Answer ALL questions

5 x 6 = 30

Each answer should not exceed 400 words or two pages

11. a. Prove that a subspace of a Hausdorff space is Hausdorff, a product of Hausdorff spaces is Hausdorff. CO1K2
- (or)
11. b. Prove that every metrizable space is normal. CO1K1
12. a. State and prove Imbedding theorem. CO2K4
- (or)
12. b. Prove that a product of a completely regular space is completely regular. CO2K2
13. a. Let $A \subset X$. Let $f : A \rightarrow Z$ be a continuous map of A into the Hausdorff space Z , then prove there is at most one extension of f to a continuous function $g : \bar{A} \rightarrow Z$. CO3K3
- (or)
13. b. Let X be a completely regular space. If Y_1 and Y_2 are two compactifications of X satisfying the extension property, then prove Y_1 and Y_2 are equivalent. CO3K1
14. a. Prove that a metric space X is complete if every Cauchy sequence in X has a convergent subsequence. CO4K2
- (or)
14. b. Let X be a space. Let (Y, d) be a metric space. If the subset \mathcal{F} of $\mathcal{C}(X, Y)$ is totally bounded under the uniform metric corresponding to d , then prove \mathcal{F} is equicontinuous under d . CO4K6
15. a. Let X be a space and let (Y, d) be a metric space. On the set $\mathcal{C}(X, Y)$, prove that the compact open topology and the topology of compact convergence coincide. CO5K5
- (or)
15. b. Let X be a compactly generated space. Let (Y, d) be a metric space. Then prove $\mathcal{C}(X, Y)$ is closed in Y^X in the topology of compact convergence. CO5K1

Part C

5 x 12 = 60

Answer ALL questions

Each answer should not exceed 800 words or four pages

16. a. Suppose that X has a countable basis. Then prove (a) every open covering of X contains a countable sub collection covering X . (b) there exists a countable subset of X that is dense in X . CO1K2
- (or)
16. b. Prove that every regular space with a countable basis is normal. CO1K3
17. a. State and Prove Urysohn Lemma. CO2K1
- (or)
17. b. State and Prove Tietze extension theorem. CO2K1
18. a. State and prove Tychonoff theorem. CO3K1
- (or)
18. b. Let X be a completely regular space, then prove that there exists a compactification Y of X having the property that every bounded continuous map $f : X \rightarrow \mathbb{R}$ extends uniquely to a continuous map of Y into \mathbb{R} . CO3K4
19. a. Let (X, d) be a metric space, then prove that there is an isometric imbedding of X into a complete metric space. CO4K5
- (or)
19. b. Prove that a metric space (X, d) is compact if and only if it is complete and totally bounded. CO4K2
20. a. Let X and Y be spaces, given $\mathcal{C}(X, Y)$ the compact open topology, then prove that if $f : X \times Z \rightarrow Y$ is continuous, then so is the induced function $F : Z \rightarrow \mathcal{C}(X, Y)$. Also the converse holds if X is locally compact Hausdorff. CO5K2
- (or)
20. b. State and Prove Ascoli's theorem. CO5K3
