CHAPTER IV

GENERALIZED INTERVAL VALUED NEUTROSOPHIC SOFT SETS

Definition:4.1

Let U be an initial universe set and E be a set of parameters.A non empty set $A \subset E$. Let $N(U)$ be the set of all neutrosophic sets of U. A **generalized neutrosphic soft set** $\widetilde{F}_A^{\mu}$ over U is defined by the set of all ordered pairs

$$\widetilde{F}_A^{\mu} = \{(\widetilde{F}_A(e), \mu(e)) : e \in E, \widetilde{F}_A(e) \in N(U), \mu(e) \in [0,1]\},$$

where $\widetilde{F}_A$ is a mapping given by $\widetilde{F}_A : E \to N(U) \times [0,1]$ and μ is a fuzzy set such that $\mu : E \to [0,1]$ here, $\widetilde{F}_A^{\mu}$ is a mapping defined as $\widetilde{F}_A^{\mu} : E \to N(U) \times [0,1]$. $\widetilde{F}_A^{\mu}$ is a parameterized family of neutrosophic sets of U, which has the degree of possibility of the approximate value set which is represented by $\mu(e)$ for each parameter e, so $\widetilde{F}_A^{\mu}$ can be expressed as follows:

$$\widetilde{F}_A^{\mu}(e)=\left\{\left(\frac{x_1}{\widetilde{F}_A(e)(x_1)},\frac{x_2}{\widetilde{F}_A(e)(x_2)},.....,\frac{x_n}{\widetilde{F}_A(e)(x_n)}\right),\mu(e)\right\}$$

Defenition:4.2

A binary operation $\otimes:[0,1]\times[0,1]\to[0,1]$ is continuous **t-norm** if $\otimes$ satisfies the following conditions:

1) $\otimes$ is commutative and associative,

2) $\otimes$ is continuous,

3) $a\otimes 1=a,\forall a\in[0,1]$,

4) $a\otimes b\le c\otimes d$ when ever $a\le c,b\le d$ and $a,b,c,d\in[0,1]$.

Defenition:4.3

A binary operation $\oplus:[0,1]\times[0,1]\to[0,1]$ is continuous **t-conorm** if $\oplus$ satisfies the following conditions:

1) $\oplus$ is commutative and associative,

2) $\oplus$ is continuous,

3) $a\oplus 0=a,\forall a\in[0,1]$,

4) $a\oplus b\le c\oplus d$ when ever $a\le c,b\le d$ and $a,b,c,d\in[0,1]$.

Definition:4.4

Let U be an initial universe set and E be a set of parameters. A non-empty set $A\subset E$. Let $IVN(U)$ denotes the set of all interval valued neutrosophic sets over U and

int[0,1] is the set of all closed subsets of [0,1]. A **generalized interval valued neutrosphic soft set** $\widetilde{F}_A^{\mu}$ over U is defined by the set of all ordered pairs

$$\widetilde{F}_A^{\mu} = \{(\widetilde{F}_A(e), \mu(e)) : e \in E, \widetilde{F}_A(e) \in IVN(U), \mu(e) \in [0,1]\},$$

where $\widetilde{F}_A$ is a mapping given by $\widetilde{F}_A : E \to IVN(U) \times [0,1]$and μ is a fuzzy set such that $\mu : E \to [0,1]$here, $\widetilde{F}_A^{\mu}$is a mapping defined by $\widetilde{F}_A^{\mu} : E \to IVN(U) \times [0,1]$

$\widetilde{F}_A^{\mu}$is a parameterized family of interval valued neutrosophic sets of U, which has the degree of possibility of the approximate value set which is represented by $\mu(e)$ for each parameter e, so $\widetilde{F}_A^{\mu}$ can be expressed as follows:

$$\widetilde{F}_A^{\mu}(e) = \left\{ \left(\frac{x_1}{\widetilde{F}_A(e)(x_1)}, \frac{x_2}{\widetilde{F}_A(e)(x_2)}, \ldots\ldots\ldots\ldots, \frac{x_n}{\widetilde{F}_A(e)(x_n)} \right), \mu(e) \right\}$$

Example:4.5

Consider two generalized interval valued neutrosophic soft sets $\widetilde{F}_A^{\mu}$and $\widetilde{G}_B^{\theta}$ over U. Suppose that $U = \{h_1, h_2, h_3\}$ be the set of three houses and $E = \{e_1, e_2, e_3\}$ be the set of parameters where e_1-cheap, e_2-moderate, e_3-comfortable. Suppose that $\widetilde{F}_A^{\mu}$and $\widetilde{G}_B^{\theta}$ are given as follows,

$$\widetilde{F}_A^{\mu}(e_1) = \left(\frac{h_1}{([0.2,0.3],[0.3,0.5],[0.2,0.3])}, \frac{h_2}{([0.3,0.4],[0.3,0.4],[0.5,0.6])}, \frac{h_3}{([0.5,0.6],[0.2,0.4],[0.5,0.7])} \right), (0.2)$$

$$\widetilde{F}_A^{\mu}(e_2) = \left(\frac{h_1}{([0.1,0.4],[0.5,0.6],[0.3,0.4])}, \frac{h_2}{([0.6,0.7],[0.4,0.5],[0.5,0.8])}, \frac{h_3}{([0.2,0.4],[0.3,0.6],[0.6,0.9])} \right), (0.5)$$

$$\widetilde{F}_A^{\mu}(e_3) = \left(\frac{h_1}{([0.2,0.6],[0.2,0.5],[0.1,0.5])}, \frac{h_2}{([0.3,0.5],[0.3,0.6],[0.4,0.5])}, \frac{h_3}{([0.6,0.8],[0.3,0.4],[0.2,0.3])} \right), (0.6)$$

and

$$\widetilde{G}_B^{\theta}(e_1)=\left(\frac{h_1}{([0.1,0.2],[0.1,0.2],[0.1,0.2])},\frac{h_2}{([0.4,0.5],[0.2,0.3],[0.3,0.5])},\frac{h_3}{([0.6,0.7],[0.1,0.3],[0.2,0.3])}\right),(0.4)$$

$$\widetilde{G}_B^{\theta}(e_2)=\left(\frac{h_1}{([0.2,0.5],[0.3,0.4],[0.2,0.3])},\frac{h_2}{([0.7,0.8],[0.3,0.4],[0.4,0.6])},\frac{h_3}{([0.3,0.6],[0.2,0.5],[0.4,0.6])}\right),(0.7)$$

$$\widetilde{G}_B^{\theta}(e_3)=\left(\frac{h_1}{([0.3,0.5],[0.1,0.3],[0.1,0.3])},\frac{h_2}{([0.4,0.5],[0.1,0.5],[0.2,0.3])},\frac{h_3}{([0.7,0.9],[0.2,0.3],[0.1,0.2])}\right),(0.8)$$

Definition:4.6

A generalized interval valued neutrosophic soft set $\widetilde{F}_A^{\mu}$ over U is said to be **empty generalized interval valued neutrosophic soft set**, denoted by ϕ^{μ} if such that $\phi^{\mu}(e)=\{\widetilde{F}_A(e),\mu(e)\}$, where $\widetilde{F}_A(e)=\{<x,([0,0],[1,1],[1,1])>\}$ and $\mu(e)=0$ for each $e\in E$ and $x\in U$

Definition:4.7

A generalized interval valued neutrosophic soft set $\widetilde{F}_A^{\mu}$ over U is said to be **universe generalized interval valued neutrosophic soft set** denoted by U^{μ} if $U^{\mu}:E\to IVN(U)\times[0,1]$ such that $U^{\mu}(e)=\{\widetilde{F}_A(e),\mu(e)\}$, where $\widetilde{F}_A(e)=\{<x,([1,1],[0,0],[0,0])>\}$ and $\mu(e)=1$ for each $e\in E$ and $x\in U$.

Definition:4.8

Let $\widetilde{F}_A^{\mu}$ be generalized interval valued neutrosophic soft set over U, where $\widetilde{F}_A^{\mu}(e)=\{\widetilde{F}_A(e),\mu(e)\}$ and

$\widetilde{F}_A(e) = \{< x, [\inf T_{\widetilde{F}_A(e)}(x), \sup T_{\widetilde{F}_A(e)}(x)], [\inf I_{\widetilde{F}_A(e)}(x), \sup I_{\widetilde{F}_A(e)}(x)], [\inf F_{\widetilde{F}_A(e)}(x), \sup F_{\widetilde{F}_A(e)}(x)] >: x \in U\}$ for all $e \in E$. Then, for $e_m \in E$ and $x_n \in U$

1. $\widetilde{F}_A^{\,*} = [\inf \widetilde{F}_A^{\,*}, \sup \widetilde{F}_A^{\,*}]$ is said to be interval valued **truth membership** part of $\widetilde{F}_A^{\,\mu}$

   where $\widetilde{F}_A^{\,*} = \{(\widetilde{F}_{mn}^{\,*}(e_m), \mu(e_m))\}$ and

   $\widetilde{F}_{mn}^{\,*}(e_m) = \{< x_n, [\inf T_{\widetilde{F}_A(e_m)}(x_n), \sup T_{\widetilde{F}_A(e_m)}(x_n)] >\}$,

2. $\widetilde{F}_A^{\,t} = [\inf \widetilde{F}_A^{\,t}, \sup \widetilde{F}_A^{\,t}]$ is said to be interval valued **indeterminacy membership** part of $\widetilde{F}_A^{\,\mu}$ where $\widetilde{F}_A^{\,t} = \{(\widetilde{F}_{mn}^{\,t}(e_m), \mu(e_m))\}$ and

   $\widetilde{F}_{mn}^{\,t}(e_m) = \{< x_n, [\inf I_{\widetilde{F}_A(e_m)}(x_n), \sup I_{\widetilde{F}_A(e_m)}(x_n)] >\}$,

3. $\widetilde{F}_A^{\,\Delta} = [\inf \widetilde{F}_A^{\,\Delta}, \sup \widetilde{F}_A^{\Delta}]$ is said to be interval valued **falsity membership** part of $\widetilde{F}_A^{\,\mu}$

   where $\widetilde{F}_A^{\,\Delta} = \{(\widetilde{F}_{mn}^{\,\Delta}(e_m), \mu(e_m))\}$ and

   $\widetilde{F}_{mn}^{\,\Delta}(e_m) = \{< x_n, [\inf F_{\widetilde{F}_A(e_m)}(x_n), \sup F_{\widetilde{F}_A(e_m)}(x_n)] >\}$

and every component of $\widetilde{F}_A^{\mu}$ is denoted by $\widetilde{F}_A^{\mu} = (\widetilde{F}_A^{*}, \widetilde{F}_A^{t}, \widetilde{F}_A^{\Delta})$ where

$$\widetilde{F}_{mn}^{\,*}(e_m) = \{< x_n, [\inf T_{\widetilde{F}_A(e_m)}(x_n), \sup T_{\widetilde{F}_A(e_m)}(x_n)] >\},$$
$$\widetilde{F}_{mn}^{\,t}(e_m) = \{< x_n, [\inf I_{\widetilde{F}_A(e_m)}(x_n), \sup I_{\widetilde{F}_A(e_m)}(x_n)] >\},$$
$$\widetilde{F}_{mn}^{\,\Delta}(e_m) = \{< x_n, [\inf F_{\widetilde{F}_A(e_m)}(x_n), \sup F_{\widetilde{F}_A(e_m)}(x_n)] >\}$$

are defined as the interval valued truth, interval valued indeterminacy and interval valued falsity membership values of n-th element according to the m-th parameter respectively.

Definition:4.9

Let U be an initial universe set and E be a set of parameters and let $A, B \subset E$ .Let $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ be two generalized interval valued neutrosophic soft sets over U. Then $\widetilde{F}_A^{\mu}$ is a **generalized interval valued neutrosophic soft subset** $\widetilde{G}_B^{\theta}$ if

1. μ is a subset of θ

2. $\widetilde{F}_A(e)$ is a generalized interval valued neutrosophic subset of $\widetilde{G}_B(e)$

   $\widetilde{F}_{mn}{}^{*}(e_m) \le \widetilde{G}_{mn}{}^{*}(e_m), \widetilde{F}_{mn}{}^{t}(e_m) \ge \widetilde{G}_{mn}{}^{t}(e_m)$ and $\widetilde{F}_{mn}{}^{\Delta}(e_m) \ge \widetilde{G}_{mn}{}^{\Delta}(e_m)$ where

   $\inf T_{\widetilde{F}_A(e_m)}(x_n) \le \inf T_{\widetilde{G}_B(e_m)}(x_n),\ \sup T_{\widetilde{F}_A(e_m)}(x_n) \le \sup T_{\widetilde{G}_B(e_m)}(x_n)$

   $\inf I_{\widetilde{F}_A(e_m)}(x_n) \ge \inf I_{\widetilde{G}_B(e_m)}(x_n),\ \sup I_{\widetilde{F}_A(e_m)}(x_n) \ge \sup I_{\widetilde{G}_B(e_m)}(x_n)$

   $\inf F_{\widetilde{F}_A(e_m)}(x_n) \ge \inf F_{\widetilde{G}_B(e_m)}(x_n),\ \sup F_{\widetilde{F}_A(e_m)}(x_n) \ge \sup F_{\widetilde{G}_B(e_m)}(x_n)$

   for all $e_m \in E, x_n \in U$

This relationship is denoted by $\widetilde{F}_A^{\mu} \subseteq \widetilde{G}_B^{\theta}$. Moreover if $\widetilde{G}_B^{\theta}$ is generalized interval valued neutrosophic soft subset of $\widetilde{F}_A^{\mu}$, then $\widetilde{F}_A^{\mu}$ is called a **generalized interval valued neutrosophic soft super set** of $\widetilde{G}_B^{\theta}$. This relationship is denoted by $\widetilde{F}_A^{\mu} \supseteq \widetilde{G}_B^{\theta}$.

Example:4.10

Consider two generalized interval valued neutrosophic soft sets $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ over U. Suppose that $U = \{h_1, h_2, h_3\}$ be set of three houses and $E = \{e_1, e_2, e_3\}$ be the set of parameters where e_1 -cheap, e_2 -moderate, e_3 -comfortable.

Suppose that $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ are as follows,

$$\widetilde{F}_A^{\mu}(e_1)=\left(\frac{h_1}{([0.1,0.2],[0.3,0.5],[0.2,0.3])},\frac{h_2}{([0.3,0.4],[0.3,0.4],[0.5,0.6])},\frac{h_3}{([0.5,0.6],[0.2,0.4],[0.5,0.7])}\right),(0.2)$$

$$\widetilde{F}_A^{\mu}(e_2)=\left(\frac{h_1}{([0.1,0.4],[0.5,0.6],[0.3,0.4])},\frac{h_2}{([0.6,0.7],[0.4,0.5],[0.5,0.8])},\frac{h_3}{([0.2,0.4],[0.3,0.6],[0.6,0.9])}\right),(0.5)$$

$$\widetilde{F}_A^{\mu}(e_3)=\left(\frac{h_1}{([0.2,0.6],[0.2,0.5],[0.1,0.5])},\frac{h_2}{([0.3,0.5],[0.3,0.6],[0.4,0.5])},\frac{h_3}{([0.6,0.8],[0.3,0.4],[0.2,0.3])}\right),(0.6)$$

and $$\widetilde{G}_B^{\theta}(e_1)=\left(\frac{h_1}{([0.2,0.3],[0.1,0.2],[0.1,0.2])},\frac{h_2}{([0.4,0.5],[0.2,0.3],[0.3,0.5])},\frac{h_3}{([0.6,0.7],[0.1,0.3],[0.2,0.3])}\right),(0.4)$$

$$\widetilde{G}_B^{\theta}(e_2)=\left(\frac{h_1}{([0.2,0.5],[0.3,0.4],[0.2,0.3])},\frac{h_2}{([0.7,0.8],[0.3,0.4],[0.4,0.6])},\frac{h_3}{([0.3,0.6],[0.2,0.5],[0.4,0.6])}\right),(0.7)$$

$$\widetilde{G}_B^{\theta}(e_3)=\left(\frac{h_1}{([0.3,0.7],[0.1,0.3],[0.1,0.3])},\frac{h_2}{([0.4,0.5],[0.1,0.5],[0.2,0.3])},\frac{h_3}{([0.7,0.9],[0.2,0.3],[0.1,0.2])}\right),(0.8)$$

$\widetilde{F}_A^{\mu}$ is called a generalized interval valued neutrosophic soft sub set of $\widetilde{G}_B^{\theta}$, that is $\widetilde{F}_A^{\mu}\subseteq\widetilde{G}_B^{\theta}$.

Definition:4.11

Let U be an initial universe set. Let $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ be two generalized interval valued neutrosophic soft sets over U. Then the **generalized interval valued neutrosophic soft union** of $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ is denoted by $\widetilde{H}_D^{\lambda}=\widetilde{F}_A^{\mu}\cup\widetilde{G}_B^{\theta}$ and is defined by $\widetilde{H}_D^{\lambda}=([\inf\widetilde{H}_D^{*},\sup\widetilde{H}_D^{*}],[\inf\widetilde{H}_D^{t},\sup\widetilde{H}_D^{t}],[\inf\widetilde{H}_D^{\Delta},\sup\widetilde{H}_D^{\Delta}])$ where

$\lambda(e_m)=\mu(e_m)\oplus\theta(e_m)$,

$$\inf\widetilde{H}_{mn}^{*}=\inf\widetilde{F}_{mn}^{*}(e_m)\oplus\inf\widetilde{G}_{mn}^{*}(e_m)$$

$$\inf\widetilde{H}_{mn}^{t}=\inf\widetilde{F}_{mn}^{t}(e_m)\otimes\inf\widetilde{G}_{mn}^{t}(e_m)$$

$$\inf \widetilde{H}_{mn}^{\Delta} = \inf \widetilde{F}_{mn}^{\Delta}(e_m) \otimes \inf \widetilde{G}_{mn}^{\Delta}(e_m)$$

$$\text{and} \sup \widetilde{H}_{mn}^{*} = \sup \widetilde{F}_{mn}^{*}(e_m) \oplus \sup \widetilde{G}_{mn}^{*}(e_m)$$

$$\sup \widetilde{H}_{mn}^{t} = \sup \widetilde{F}_{mn}^{t}(e_m) \otimes \sup \widetilde{G}_{mn}^{t}(e_m)$$

$$\sup \widetilde{H}_{mn}^{\Delta} = \sup \widetilde{F}_{mn}^{\Delta}(e_m) \otimes \sup \widetilde{G}_{mn}^{\Delta}(e_m) \text{ for all } e_m \in E \text{ and } m,n \in \Lambda.$$

Definition:4.12

Let U be an initial universe set. Let $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ be two generalized interval valued neutrosophic soft sets over U. Then the **generalized interval valued neutrosophic soft intersection** of $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ is denoted by $\widetilde{K}_D^{\varepsilon} = \widetilde{F}_A^{\mu} \cap \widetilde{G}_B^{\theta}$ and is defined by

$$\widetilde{K}_D^{\varepsilon} = ([\inf \widetilde{K}_D^{*}, \sup \widetilde{K}_D^{*}], [\inf \widetilde{K}_D^{t}, \sup \widetilde{K}_D^{t}], [\inf \widetilde{K}_D^{\Delta}, \sup \widetilde{K}_D^{\Delta}]) \quad \text{where}$$

$$\varepsilon(e_m) = \mu(e_m) \otimes \theta(e_m),$$

$$\inf \widetilde{K}_{mn}^{*} = \inf \widetilde{F}_{mn}^{*}(e_m) \otimes \inf \widetilde{G}_{mn}^{*}(e_m)$$

$$\inf \widetilde{K}_{mn}^{t} = \inf \widetilde{F}_{mn}^{t}(e_m) \oplus \inf \widetilde{G}_{mn}^{t}(e_m)$$

$$\inf \widetilde{K}_{mn}^{\Delta} = \inf \widetilde{F}_{mn}^{\Delta}(e_m) \oplus \inf \widetilde{G}_{mn}^{\Delta}(e_m)$$

$$\text{and} \sup \widetilde{K}_{mn}^{*} = \sup \widetilde{F}_{mn}^{*}(e_m) \otimes \sup \widetilde{G}_{mn}^{*}(e_m)$$

$$\sup \widetilde{K}_{mn}^{t} = \sup \widetilde{F}_{mn}^{t}(e_m) \oplus \sup \widetilde{G}_{mn}^{t}(e_m)$$

$$\sup \widetilde{K}_{mn}^{\Delta} = \sup \widetilde{F}_{mn}^{\Delta}(e_m) \oplus \sup \widetilde{G}_{mn}^{\Delta}(e_m) \text{ for all } e_m \in E \text{ and } m,n \in \Lambda.$$

Example:4.13

Let us consider the two generalized interval valued neutrosophic soft sets $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ is defined in example 4.5. Suppose that the t-co norm is defined by $\oplus(a,b)=\max\{a,b\}$ and the t-norm is defined by $\otimes(a,b)=\min\{a,b\}$ for $a,b\in[0,1]$. Then $\widetilde{H}_D^{\lambda}=\widetilde{F}_A^{\mu}\cup\widetilde{G}_B^{\theta}$ is defined as follows:

$$\widetilde{H}_D(e_1)=\left(\frac{h_1}{([0.2,0.3],[0.1,0.2],[0.1,0.2])},\frac{h_2}{([0.4,0.5],[0.2,0.3],[0.3,0.5])},\frac{h_3}{([0.6,0.7],[0.1,0.3],[0.2,0.3])}\right),(0.4)$$

$$\widetilde{H}_D(e_2)=\left(\frac{h_1}{([0.2,0.5],[0.3,0.4],[0.2,0.3])},\frac{h_2}{([0.7,0.8],[0.3,0.4],[0.4,0.6])},\frac{h_3}{([0.3,0.6],[0.2,0.5],[0.4,0.6])}\right),(0.7)$$

$$\widetilde{H}_D(e_3)=\left(\frac{h_1}{([0.3,0.6],[0.1,0.3],[0.1,0.3])},\frac{h_2}{([0.4,0.5],[0.1,0.5],[0.2,0.3])},\frac{h_3}{([0.7,0.9],[0.2,0.3],[0.1,0.2])}\right),(0.8)$$

Example:4.14

Let us consider the two generalized interval valued neutrosophic soft sets $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ is defined in example 4.5. Suppose that the t-conorm is defined by $\oplus(a,b)=\max\{a,b\}$ and the t-norm is defined by $\otimes(a,b)=\min\{a,b\}$ for $a,b\in[0,1]$. Then $\widetilde{K}_D^{\varepsilon}=\widetilde{F}_A^{\mu}\cap\widetilde{G}_B^{\theta}$ is defined as follows:

$$\widetilde{K}_D(e_1)=\left(\frac{h_1}{([0.1,0.2],[0.3,0.5],[0.2,0.3])},\frac{h_2}{([0.3,0.4],[0.3,0.4],[0.5,0.6])},\frac{h_3}{([0.5,0.6],[0.2,0.4],[0.5,0.7])}\right),(0.2)$$

$$\widetilde{K}_D(e_2)=\left(\frac{h_1}{([0.1,0.4],[0.5,0.6],[0.3,0.4])},\frac{h_2}{([0.6,0.7],[0.4,0.5],[0.5,0.8])},\frac{h_3}{([0.2,0.4],[0.3,0.6],[0.6,0.9])}\right),(0.5)$$

$$\widetilde{K}_D(e_3)=\left(\frac{h_1}{([0.2,0.5],[0.2,0.5],[0.1,0.5])},\frac{h_2}{([0.3,0.5],[0.3,0.6],[0.4,0.5])},\frac{h_3}{([0.6,0.8],[0.3,0.4],[0.2,0.3])}\right),(0.6)$$

Theorem:4.15

Let $\widetilde{F}_A^{\mu}$, $\widetilde{G}_B^{\theta}$ and $\widetilde{H}_D^{\lambda}$ be three generalized interval valued neutrosophic soft sets over U. Then

1. $\widetilde{F}_A^{\mu}\cup\widetilde{G}_B^{\theta}=\widetilde{G}_B^{\theta}\cup\widetilde{F}_A^{\mu}$
2. $\widetilde{F}_A^{\mu}\cap\widetilde{G}_B^{\theta}=\widetilde{G}_B^{\theta}\cap\widetilde{F}_A^{\mu}$

3. $(\widetilde{F}_A^{\mu} \cup \widetilde{G}_B^{\theta}) \cup \widetilde{H}_D^{\lambda} \cong \widetilde{F}_A^{\mu} \cup (\widetilde{G}_B^{\theta} \cup \widetilde{H}_D^{\lambda})$

4. $(\widetilde{F}_A^{\mu} \cap \widetilde{G}_B^{\theta}) \cap \widetilde{H}_D^{\lambda} \cong \widetilde{F}_A^{\mu} \cap (\widetilde{G}_B^{\theta} \cap \widetilde{H}_D^{\lambda})$

Theorem:4.16

Let $\widetilde{F}_A^{\mu}, \widetilde{G}_B^{\theta}$ and $\widetilde{H}_D^{\lambda}$ be three generalized interval valued neutrosophic soft sets over U. Suppose that the t-conorm is defined by $\oplus(a,b) = \max\{a,b\}$ and the t-norm is defined by $\otimes(a,b) = \min\{a,b\}$ for $a,b \in [0,1]$. Then

1. $\widetilde{H}_D^{\lambda} \cap (\widetilde{F}_A^{\mu} \cup \widetilde{G}_B^{\theta}) = (\widetilde{H}_D^{\lambda} \cap \widetilde{F}_A^{\mu}) \cup (\widetilde{H}_D^{\lambda} \cap \widetilde{G}_B^{\theta})$

2. $\widetilde{H}_D^{\lambda} \cup (\widetilde{F}_A^{\mu} \cap \widetilde{G}_B^{\theta}) = (\widetilde{H}_D^{\lambda} \cup \widetilde{F}_A^{\mu}) \cap (\widetilde{H}_D^{\lambda} \cup \widetilde{G}_B^{\theta})$

Definition:4.17

The **complement of generalized interval valued neutrosophic soft set** $\widetilde{F}_A^{\mu}$ over U, denoted by $\widetilde{F}_A^{\mu(C)}$ and is defined by

$\widetilde{F}_A^{\mu(C)} = ([\inf \widetilde{F}_A^{*(C)}, \sup \widetilde{F}_A^{*(C)}], [\inf \widetilde{F}_A^{\iota(C)}, \sup \widetilde{F}_A^{\iota(C)}], [\inf \widetilde{F}_A^{\Delta(C)}, \sup \widetilde{F}_A^{\Delta(C)}])$

where $\mu^{(C)}(e_m) = 1 - \mu(e_m)$ and

$\inf \widetilde{F}_{mn}^{*(C)} = \inf \widetilde{F}_{mn}^{\Delta(C)}$; $\inf \widetilde{F}_{mn}^{\iota(C)} = 1 - \sup \widetilde{F}_{mn}^{\iota(C)}$; $\inf \widetilde{F}_{mn}^{\Delta(C)} = \inf \widetilde{F}_{mn}^{*(C)}$

$\sup \widetilde{F}_{mn}^{*(C)} = \sup \widetilde{F}_{mn}^{\Delta(C)}$; $\sup \widetilde{F}_{mn}^{\iota(C)} = 1 - \inf \widetilde{F}_{mn}^{\iota(C)}$; $\sup \widetilde{F}_{mn}^{\Delta(C)} = \sup \widetilde{F}_{mn}^{*(C)}$

Example:4.18

Let us consider the two generalized interval valued neutrosophic soft sets $\widetilde{F}_A^{\mu}$ is defined in example 4.5.Then the complement of $\widetilde{F}_A^{\mu}$ is defined by $\widetilde{F}_A^{\mu(C)}$ is as follows:

$$\widetilde{F}_A^{\mu(C)}(e_1)=\left(\frac{h_1}{([0.2,0.3],[0.5,0.7],[0.2,0.3])},\frac{h_2}{([0.5,0.6],[0.6,0.7],[0.3,0.4])},\frac{h_3}{([0.5,0.7],[0.6,0.8],[0.5,0.6])}\right),(0.8)$$

$$\widetilde{F}_A^{\mu(C)}(e_2)=\left(\frac{h_1}{([0.3,0.4],[0.4,0.5],[0.1,0.4])},\frac{h_2}{([0.5,0.8],[0.5,0.6],[0.6,0.7])},\frac{h_3}{([0.6,0.9],[0.4,0.7],[0.2,0.4])}\right),(0.5)$$

$$\widetilde{F}_A^{\mu(C)}(e_3)=\left(\frac{h_1}{([0.1,0.5],[0.8,0.5],[0.2,0.5])},\frac{h_2}{([0.4,0.5],[0.4,0.7],[0.3,0.5])},\frac{h_3}{([0.2,0.3],[0.6,0.7],[0.6,0.8])}\right),(0.4)$$

Theorem:4.19

Let $\widetilde{F}_A^{\mu}$ be a generalized interval neutrosophic soft sets over U. Then

1. $\widetilde{F}_A^{\mu}$ is a generalized interval neutrosophic soft subset of $\widetilde{F}_A^{\mu}\cup\widetilde{F}_A^{\mu(C)}$
2. $\widetilde{F}_A^{\mu}\cap\widetilde{F}_A^{\mu(C)}$ is a generalized interval neutrosophic soft subset of $\widetilde{F}_A^{\mu}$.

Definition:4.20

Let U be an initial universe set. Let $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ be two generalized interval valued neutrosophic soft sets over U. Then the '**AND**' operation of $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ is denoted by $\widetilde{H}_D^{\lambda}=\widetilde{F}_A^{\mu}\wedge\widetilde{G}_B^{\theta}$ is the mapping $\widetilde{H}_D^{\lambda}:D\to IN(U)\times[0,1]$ and is defined as $\widetilde{H}_D^{\lambda}=([\inf\widetilde{H}_D^{*},\sup\widetilde{H}_D^{*}],[\inf\widetilde{H}_D^{t},\sup\widetilde{H}_D^{t}],[\inf\widetilde{H}_D^{\Delta},\sup\widetilde{H}_D^{\Delta}])$ where

$\lambda(e_m)=\min(\mu(e_k),\theta(e_h))$ and $\inf\widetilde{H}^{*}(e_m)=\min\{\inf\widetilde{F}^{*}(e_{kn}),\inf\widetilde{G}^{*}(e_{hn})\}$

$$\inf\widetilde{H}^{t}(e_m)=\max\{\inf\widetilde{F}^{t}(e_{kn}),\inf\widetilde{G}^{t}(e_{hn})\}$$

$\inf\widetilde{H}^{\Delta}(e_m)=\max\{\inf\widetilde{F}^{\Delta}(e_{kn}),\inf\widetilde{G}^{\Delta}(e_{hn})\}$ $\sup\widetilde{H}^{*}(e_m)=\min\{\sup\widetilde{F}^{*}(e_{kn}),\sup\widetilde{G}^{*}(e_{hn})\}$

$\sup\widetilde{H}^{t}(e_m)=\max\{\sup\widetilde{F}^{t}(e_{kn}),\sup\widetilde{G}^{t}(e_{hn})\}$ $\sup\widetilde{H}^{\Delta}(e_m)=\max\{\sup\widetilde{F}^{\Delta}(e_{kn}),\sup\widetilde{G}^{\Delta}(e_{hn})\}$

$$\text{for all } e_m=(e_k,e_h)\in D\subseteq E\times E \text{ and } m,n,k,h\in\wedge.$$

Definition:4.21

Let U be an initial universe set. Let $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ be two generalized interval valued neutrosophic soft sets over U. Then the '**OR**' operation of $\widetilde{F}_A^{\mu}$ and $\widetilde{G}_B^{\theta}$ is denoted by $\widetilde{K}_D^{\varepsilon} = \widetilde{F}_A^{\mu} \vee \widetilde{G}_B^{\theta}$ is the mapping $\widetilde{K}_D^{\varepsilon} : D \to IN(U) \times [0,1]$ and is defined as $\widetilde{K}_D^{\varepsilon} = ([\inf \widetilde{K}_D^{*}, \sup \widetilde{K}_D^{*}], [\inf \widetilde{K}_D^{\iota}, \sup \widetilde{K}_D^{\iota}], [\inf \widetilde{K}_D^{\Delta}, \sup \widetilde{K}_D^{\Delta}])$

where $\varepsilon(e_m) = \max(\mu(e_k), \theta(e_h))$ and $\inf \widetilde{K}^{*}(e_m) = \max\{\inf \widetilde{F}^{*}(e_{kn}), \inf \widetilde{G}^{*}(e_{hn})\}$

$$\inf K^{\iota}(e_m) = \min\{\inf \widetilde{F}^{\iota}(e_{kn}), \inf \widetilde{G}^{\iota}(e_{hn})\}$$

$$\inf \widetilde{K}^{\Delta}(e_m) = \min\{\inf \widetilde{F}^{\Delta}(e_{kn}), \inf \widetilde{G}^{\Delta}(e_{hn})\}$$

$$\sup \widetilde{K}^{*}(e_m) = \max\{\sup \widetilde{F}^{*}(e_{kn}), \sup \widetilde{G}^{*}(e_{hn})\}$$

$$\sup \widetilde{K}^{\iota}(e_m) = \min\{\sup \widetilde{F}^{\iota}(e_{kn}), \sup \widetilde{G}^{\iota}(e_{hn})\} \quad \sup \widetilde{K}^{\Delta}(e_m) = \min\{\sup \widetilde{F}^{\Delta}(e_{kn}), \sup \widetilde{G}^{\Delta}(e_{hn})\}$$

$$\text{for } e_m = (e_k, e_h) \in D \subseteq E \times E \text{ and } m, n, k, h \in \Lambda.$$