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n-FUZZY PROXIMITY-I : *n*-FUZZY PROXIMITY AND *n*-FUZZY UNIFORMITY

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In this paper, the concept of n-fuzzy proximity and n-fuzzy uniformity are introduced. Some interesting results connecting these concepts are obtained.

KEY WORDS : Fuzzy proximity, fuzzy uniformity, n-fuzzy proximity, n-fuzzy uniformity, I_n – valued fuzzy sets, n^{th} upper approximation of a fuzzy set, n^{th} lower approximation of a fuzzy set.

Mathematics Subject Classification : 54 E 05, 54 E 15.

INTRODUCTION

The concept of fuzzy topology was first introduced by Chang, C.L. [1]. The notions of proximity and uniformity are important concepts in general topology. The problem of generalization of these notions to fuzzy topological spaces has been intensively discussed over the past 30 years. In 1979 Katsaras [4] introduced the first definition of fuzzy proximity. He proved that, with every Lowen fuzzy uniform structure, a fuzzy proximity structure can be associated. In this paper a new approach is followed in the study of fuzzy proximity by considering finite valued fuzzy sets. Given a set X, a fuzzy proximity ρ_{n*} is defined for I_n-valued fuzzy sets. Then it is extended to I^X as $E_x(\rho_{n*})$. The concept of Hutton uniformity [3] is defined for I_n-valued fuzzy sets and then it is extended to all fuzzy sets. It is here shown that every n-fuzzy uniformity $\mathcal{U}_{n*}$ induces an n-fuzzy proximity $\rho_{\mathcal{U}_{n*}}$. An important result proved here is that the extended fuzzy proximity $E_x(\rho_{\mathcal{U}_{n*}})$ of $\rho_{\mathcal{U}_{n*}}$ coincides with the fuzzy proximity $\rho_{\mathcal{U}}$ induced by the extended fuzzy uniformity $\mathcal{U}$ of $\mathcal{U}_{n*}$.

PRELIMINARY RESULTS

Definition : 2.1 [4]

A binary relation ρ on I^X is called a **fuzzy proximity on X** if ρ satisfies the following axioms.

For any $f, g, h \in I^X$.

(KFP1) $f \rho g$ implies $g \rho f$

(KFP2) $(f \vee h) \rho g$ iff $f \rho g$ or $h \rho g$

(KFP3) $f \rho g$ implies $f \neq \mathbf{0}$ and $g \neq \mathbf{0}$

(KFP4) $f \bar{\rho} g$ implies that there exists $A \subseteq X$ such that $f \bar{\rho} \chi_A$ and $(1 - \chi_A) \bar{\rho} g$. [$\bar{\rho}$ is the negation of ρ)

(KFP5) $f \wedge g \neq \mathbf{0}$ implies $f \rho g$.

Here (X, ρ) is called a **fuzzy proximity space**.

Note : The axiom KFP4 is a modified version of Katsaras fuzzy proximity [4].

Definition [Hutton, 3] : 2.2

A fuzzy uniformity $\mathcal{U}$ on a set X is a collection of maps $\mu : I^X \to I^X$ satisfying the following conditions :

(HU1) $\mathcal{U} \neq \phi$

(HU2) $f \leq \mu(f)$ for all $f \in I^X$ and $\mu(\mathbf{0}) = \mathbf{0}$

(HU3) $\mu(\vee_{\lambda \in \Omega} f_\lambda) = \vee_{\lambda \in \Omega} \mu(f_\lambda)$ for $\{f_\lambda \mid \lambda \in \Omega\} \subseteq I^X$

(HU4) $\mu \in \mathcal{U}, \mu \leq \mu_1 \Rightarrow \mu_1 \in \mathcal{U}$

(HU5) $\mu_1, \mu_2 \in \mathcal{U} \Rightarrow \mu_1 \wedge \mu_2 \in \mathcal{U}$

(HU6) $\mu \in \mathcal{U} \Rightarrow$ there exists $\nu \in \mathcal{U}$ such that $\nu . \nu \leq \mu$ where $\nu . \nu$ denotes the composition of mappings.

(HU7) $\mu \in \mathcal{U} \Rightarrow \mu^{-1} \in \mathcal{U}$ where

$$\mu^{-1}(g) = \wedge \{h \in I^X \mid \mu(1-h) \leq (1-g)\}$$

The pair $(X, \mathcal{U})$ is called a fuzzy uniform space.

Remark : 2.3

A collection of maps $\mu : I^X \to I^X$ satisfying HU1, HU2, HU3, HU5, HU6 and HU7 is a base $B(\mathcal{U})$ for a fuzzy uniformity $\mathcal{U}$. The associated fuzzy uniformity $\mathcal{U}$ is given by $\mathcal{U} = \{\mu \mid$ there exists a $\mu_1 \in B(\mathcal{U})$ such that $\mu_1 \leq \mu\}$.

Definition : 2.4 [2]

Let $I_n = \{0, 1/n, 2/n, \ldots, 1\}$. *A* **$I_n$-valued fuzzy set** on X is an element of the set I_n^X of all functions from X to I_n.

In [2] the definition and properties of n^{th} order approximations are given as follows :

Definition : 2.5 [2]

With every fuzzy set f defined on a set X and with every positive integer n, a finite fuzzy set $^n f$ with values in I_n is associated as follows :

For $x \in X$

(i) if $f(x) = 0$, define $^n f(x) = 0$.

(ii) if $1/n < f(x) \leq (i+1)/n$ define $^n f(x) = (i+1)/n$, for $i = 0, 1, 2, \ldots, n-1$.

$^n f$ is called the ***n*th upper approximation of *f*.**

Proposition : 2.6 [2]

(i) $f(x) = i/n \Rightarrow {}^n f(x) = i/n$ for $i = 1, 2, \ldots, n$

(ii) $f \leq {}^n f$ for all n.

(iii) $f \leq g \Rightarrow {}^n f \leq {}^n g$

(iv) $f \leq {}^n g \Rightarrow {}^n f \leq {}^n g$

(v) ${}^n({}^n f) = {}^n f$

(vi) ${}^n(\vee f_\lambda) = \vee ({}^n f_\lambda)$

(vii) ${}^n(\overset{m}{\underset{k=1}{\wedge}} f_k) = \overset{m}{\underset{k=1}{\wedge}} ({}^n f_k)$

Definition : 2.7 [2]

For each fuzzy set f on a set X, the ***n*th lower approximation** ${}_n f$ is defined as follows :

For $x \in X$,

(i) if $f(x) = 1$ define ${}_n f(x) = 1$

(ii) if $i/n \leq f(x) < (i+1)/n$, define ${}_n f(x) = i/n$ for $i = 0, 1, 2, \ldots, n-1$.

Proposition : 2.8 [2]

(i) If $f(x) = i/n$ then ${}_n f(x) = i/n$, for $i = 0, 1, \ldots, n-1$

(ii) ${}_n f \leq f$ for all n.

(iii) $f \leq g \Rightarrow {}_n f \leq {}_n g$.

(iv) ${}_n f \leq g \Rightarrow {}_n f \leq {}_n g$

(v) ${}_n({}_n f) = {}_n f$

(vi) ${}_n(\wedge f_\lambda) = \wedge ({}_n f_\lambda)$

(vii) ${}_n(\overset{m}{\underset{k=1}{\vee}} f_k) = \overset{m}{\underset{k=1}{\vee}} ({}_n f_k)$

Proposition : 2.9 [2]

(i) ${}_n(1-f) = 1 - {}^n f$ and ${}^n(1-f) = 1 - {}_n f$

(ii) ${}^n f \leq g \Rightarrow {}^n f \leq {}_n g$

(iii) $f \leq {}_n g \Rightarrow {}^n f \leq {}_n g$, $f \leq {}^n g \Rightarrow {}_n f \leq g \leq {}^n g$

(iv) ${}_n({}^n f) = {}^n f$

(v) ${}^n({}_n f) = {}_n f$

(vi) ${}^n f \neq \mathbf{0} \Rightarrow f \neq \mathbf{0}$

Proposition : 2.10 [2]

(i) If $f \in I_n^X$ then ${}_n f = f = {}^n f$.

(ii) For $A \subseteq X$, $\chi_A = {}^n\chi_A = {}_n\chi_A$

Proposition : 2.11 [2]

Let $\theta : X \to Y$ be a function then

(i) For all $f \in I^X$, ${}^n(\theta(f)) = \theta({}^n f)$

(ii) For given $f \in I^X$, ${}^n(\theta^{-1}(f)) = \theta^{-1}({}^n f)$

n-FUZZY PROXIMITY AND *n*-FUZZY UNIFORMITY

Definition : 3.1

A binary relation ρ_{n*} on I_n^X is called an ***n*-fuzzy proximity on *X*** if ρ_{n*} satisfies the following axioms.

For any $f, g, h \in I_n^X$.

$(FP_{n*}1)$ $f\,\rho_{n*}\,g$ implies $g\,\rho_{n*}\,f$

$(FP_{n*}2)$ $(f \vee h)\ \rho_{n*}\ g$ iff $f\,\rho_{n*}\,g$ or $h\,\rho_{n*}\,g$

$(FP_{n*}3)$ $f\,\rho_{n*}\,g$ implies $f \neq \mathbf{0}$ and $g \neq \mathbf{0}$

$(FP_{n*}4)$ $f\,\bar{\rho}_{n*}\,g$ implies that there exists an $A \subseteq X$ such that $f\,\bar{\rho}_{n*}\,\chi_A$ and $(1-\chi_A)\ \bar{\rho}_{n*}\ g$

$(FP_{n*}5)$ $f \wedge g \neq \mathbf{0}$ implies $f\,\rho_{n*}\,g$

The pair (X, ρ_{n*}) is called an ***n*-fuzzy proximity space**.

Definition : 3.2

Given an n-fuzzy proximity ρ_{n*}, it is extended to a binary relation $E_x(\rho_{n*})$ as follows:

$f(E_x(\rho_{n*}))\,g \Leftrightarrow {}^nf\ \rho_{n*}\ {}^ng$. Here $E_x(\rho_{n*})$ is called **the extension of** ρ_{n*}.

Theorem : 3.3

If ρ_{n*} is an n-fuzzy proximity, then its extension $E_x(\rho_{n*})$ is a fuzzy proximity.

Proof : For $f, g, h \in I^X$,

$(FP1)$ $f(E_x(\rho_{n*}))\,g \Leftrightarrow {}^nf\ \rho_{n*}\ {}^ng$

$\Leftrightarrow {}^ng\ \rho_{n*}\ {}^nf \Leftrightarrow g\,E_x(\rho_{n*})\,f$

$(FP2)$ $(f \vee h)\,(E_x(\rho_{n*}))\,g \Leftrightarrow {}^n(f \vee h)\ \rho_{n*}\ {}^ng$

$\Leftrightarrow ({}^nf \vee {}^nh)\ \rho_{n*}\ {}^ng$ $[\because {}^n(f \vee h) = {}^nf \vee {}^nh]$

$\Leftrightarrow {}^nf\,\rho_{n*}\ {}^ng$ or ${}^nh\ \rho_{n*}\ {}^ng$

$\Leftrightarrow f(E_x(\rho_{n*}))\,g$ or $h(E_x(\rho_{n*}))\,g$

$(FP3)$ $f(E_x(\rho_{n*}))\,g \Rightarrow {}^nf\ \rho_{n*}\ {}^ng$

$\Rightarrow {}^nf \neq \mathbf{0}$ and ${}^ng \neq \mathbf{0}$

$\Rightarrow f \neq \mathbf{0}$ and $g \neq \mathbf{0}$ [Proposition 2.9 (vi)]

$(FP4)$ $f\ \overline{(E_x\ (\rho_{n*}))}\ g \Rightarrow {}^nf\ \bar{\rho}_{n*}\ {}^ng$

$\Rightarrow$ there exists $\chi_A \in I^X$ s.t. ${}^nf\ \bar{\rho}_{n*}\ \chi_A$ and $(1-\chi_A)\ \bar{\rho}_{n*}\ {}^ng$

$\Rightarrow {}^nf\ \bar{\rho}_{n*}\ {}^n\chi_A$ and ${}^n(1-\chi_A)\ \bar{\rho}_{n*}\ {}^ng$ $[\because {}^n\chi_A = \chi_A]$

$\Rightarrow f(E_x(\rho_{n*}))\,\chi_A$ and $(1-\chi_A)\,(E_x\,(\rho_{n*}))\,g$

$(FP5)$ $f \wedge g \neq \mathbf{0}$ $\Rightarrow {}^nf \wedge {}^ng \neq \mathbf{0}$

$\Rightarrow {}^nf\ \rho_{n*}\ {}^ng \Rightarrow f(E_x(\rho_{n*}))\,g$

$\therefore$ $E_x(\rho_{n*})$ is a fuzzy proximity.

Definition : 3.4

A collection $\mathcal{U}_{n*}$ of maps $\mu_{n*} : I_n^X \to I_n^X$ is called **an n-fuzzy uniformity** on X if $\mathcal{U}_{n*}$ satisfies the following axioms.

(HU_{n*} 1) $\mathcal{U}_{n*} \neq \phi$

(HU_{n*} 2) $f \leq \mu_{n*}(f)$ for all $f \in I_n^X$ and $\mu_{n*}(\mathbf{0}) = \mathbf{0}$

(HU_{n*} 3) $\mu_{n*}\left(\bigvee_{\alpha \in \Omega} f_\alpha\right) = \bigvee_{\alpha \in \Omega} (\mu_{n*}(f_\alpha))$, for $\{f_\alpha / \alpha \in \Omega\} \subseteq I_n^X$

(HU_{n*} 4) $\mu_{n*} \in \mathcal{U}_{n*}, \mu_{n*} \leq \mu'_{n*} \Rightarrow \mu'_{n*} \in \mathcal{U}_{n*}$

(HU_{n*} 5) $\mu_{n*}, \lambda_{n*} \in \mathcal{U}_{n*} \Rightarrow \mu_{n*} \wedge \lambda_{n*} \in \mathcal{U}_{n*}$

(HU_{n*} 6) $\mu_{n*} \in \mathcal{U}_{n*} \Rightarrow$ there exists $\nu_{n*} \in \mathcal{U}_{n*}$ such that $\nu_{n*} . \nu_{n*} \leq \mu_{n*}$

(HU_{n*} 7) $\mu_{n*} \in \mathcal{U}_{n*} \Rightarrow \mu_{n*}^{-1} \in \mathcal{U}_{n*}$ where

$\mu_{n*}^{-1}(g) = \wedge \{h \in I_n^X \mid \mu_{n*}(1-h) \leq 1-g\}$, for $g \in I_n^X$.

The pair $(X, \mathcal{U}_{n*})$ is called an **n-fuzzy uniform space**.

Definition : 3.5

Given $\mu_{n*} : I_n^X \to I_n^X$ define $\mu : I^X \to I^X$ such that $\mu(f) = \mu_{n*}({}^n f)$. Then μ is called the **extension of** μ_{n*}.

Remark : 3.6

The map $\mu_{n*} \to \mu$ is one-one.

Theorem : 3.7

Let $\mathcal{U}_{n*}$ be an n-fuzzy uniformity on X. Given $\mu_{n*} \in \mathcal{U}_{n*}$ let μ be the extension of μ_{n*}. The collection $B(U) = \{\mu \mid \mu_{n*} \in \mathcal{U}_{n*}\}$ is a base for a fuzzy uniformity $\mathcal{U}$ on X.

Proof :

($HU1$) $\mathcal{U}_{n*} \neq \phi \Rightarrow B(\mathcal{U}) \neq \phi$

($HU2$) Let $f \in I^X$ and $\mu \in B(\mathcal{U})$

Then $\mu_{n*} \in \mathcal{U}_{n*}$ Now $f \leq {}^n f \leq \mu_{n*}({}^n f) = \mu(f)$

$\mu(\mathbf{0}) = \mu_{n*}({}^n\mathbf{0}) = \mu_{n*}(\mathbf{0}) = \mathbf{0}$ [Definition : 2.5]

($HU3$) Let $g = \bigvee_{\lambda \in \Delta} f_\lambda$

Then ${}^n g = \bigvee_{\lambda \in \Delta} ({}^n f_\lambda)$ $[\because {}^n(\vee f_\lambda) = \vee({}^n f_\lambda)]$

Then $\mu(g) = \mu_{n*}({}^n g) = \mu_{n*}(\vee {}^n f_\lambda) = \vee(\mu_{n*}({}^n f_\lambda)) = \vee \mu(f_\lambda)$

($HU5$) Let $\mu_1, \mu_2 \in B(\mathcal{U})$

For $f \in I^X$

$$\begin{aligned}(\mu_1 \wedge \mu_2)(f) &= \mu_1(f) \wedge \mu_2(f)\\ &= (\mu_1)_{n*}({}^n f) \wedge (\mu_2)_{n*}({}^n f)\\ &= ((\mu_1)_{n*} \wedge (\mu_2)_{n*})({}^n f)\end{aligned}$$

Now $\mu_1, \mu_2 \in B(\mathcal{U}) \Rightarrow (\mu_1)_{n*}, (\mu_2)_{n*} \in \mathcal{U}_{n*}$

Then $(\mu_1)_{n^*} \wedge (\mu_2)_{n^*} \in \mathcal{U}_{n^*}$. This implies $\mu_1 \wedge \mu_2 \in \mathcal{U}$.

(*HU*6) Let $\mu \in B(\mathcal{U})$ be the extension of $\mu_{n^*} \in \mathcal{U}_{n^*}$. There exists $\lambda_{n^*} \in \mathcal{U}_{n^*}$ such that $\lambda_{n^*} . \lambda_{n^*} \leq \mu_{n^*}$. Let λ be the extension of λ_{n^*}.

Now
$$
\begin{aligned}
(\lambda . \lambda)(f) &= \lambda(\lambda(f)) \\
&= \lambda(\lambda_{n^*}({}^n f)) \\
&= \lambda_{n^*}({}^n\lambda_{n^*}({}^n f)) \\
&= \lambda_{n^*}(\lambda_{n^*}({}^n f)) && [\because {}^n(\lambda_{n^*}({}^n f)) = \lambda_{n^*}({}^n f)] \\
&= (\lambda_{n^*} . \lambda_{n^*})({}^n f) \\
&\leq \mu_{n^*}({}^n f) \leq \mu(f)
\end{aligned}
$$

(*HU*7) Let $\mu \in \mathcal{U}$ be the extension of $\mu_{n^*} \in \mathcal{U}_{n^*}$. Let $\nu_{n^*} = (\mu_{n^*})^{-1}$.

Let ν be the extension of ν_{n^*}.

Claim : $\nu = \mu^{-1}$

By definition
$$
\begin{aligned}
\nu(f) = \nu_{n^*}({}^n f) &= (\mu_{n^*})^{-1}({}^n f) \\
&= \wedge \{h \in I_n^X \mid \mu_{n^*}(1-h) \leq 1 - {}^n f\}
\end{aligned}
$$

Proposition 2.10 (i) $\Rightarrow$ for any element $h \in I_n^X$, $h = {}_n h = {}^n h$.

$$\therefore \quad \nu(f) = \wedge \{{}_n h \in I_n^X \mid \mu_{n^*}(1 - {}_n h) \leq 1 - {}^n f\}$$

Now
$$
\begin{aligned}
\mu^{-1}(f) &= \wedge \{h \in I^X \mid \mu(1-h) \leq 1 - f\} \\
&= \wedge \{h \in I^X \mid \mu_{n^*}({}^n(1-h) \leq 1 - f\} \\
&= \wedge \{h \in I^X \mid \mu_{n^*}(1 - {}_n h) \leq 1 - f\} \\
& \qquad [\because {}^n(1-h) = 1 - {}_n h \text{ from } 2.9\text{ (i)}] \\
&= \wedge \{{}_n h \in I^X \mid \mu_{n^*}(1 - {}_n h) \leq 1 - f\} && [\because {}_n h < h]
\end{aligned}
$$

Define
$$
\begin{aligned}
A &= \{{}_n h \in I_n^X \mid \mu_{n^*}(1 - {}_n h) \leq 1 - {}^n f\} \\
B &= \{{}_n h \in I_n^X \mid \mu_{n^*}(1 - {}_n h) \leq 1 - f\}
\end{aligned}
$$

Then $A \subseteq B$ since $1 - {}^n f \leq 1 - f$.

Now $\quad {}_n h \in B \Rightarrow \mu_{n^*}(1 - {}_n h) \leq (1 - f) \quad \ldots (1)$

Let $\mu_{n^*}(1 - {}_n h) = g$. Since $\mu_{n^*}(1 - {}_n h) \in I_n^X$, $g = {}_n g$

Now $\quad {}_n g = g \leq 1 - f$

$$
\begin{aligned}
\therefore \quad {}_n g &\leq {}_n(1 - f) && \text{[Proposition 2.8 (iii)]} \\
&= 1 - {}^n f && \ldots (2)
\end{aligned}
$$

(1) and (2) $\Rightarrow \mu_{n^*}(1 - {}_n h) \leq 1 - {}^n f$

$\therefore \quad B \subseteq A$

$\therefore \quad A = B \Rightarrow \nu(f) = \mu^{-1}(f), \forall f.$

$\therefore$ $B(\mathcal{U})$ is a base for a fuzzy uniformity $\mathcal{U}$ on X.

Definition : 3.8

The uniformity $\mathcal{U}$ induced from the base $B(\mathcal{U}) = \{\mu \mid \mu_{n^*} \in B_{n^*}\}$ is called the **extension of $\mathcal{U}_{n^*}$**.

Definition [Katsaras, 6] : 3.9

Let $\mathcal{U}$ be a uniformity on X. Define a binary relation ρ_U on I^X such that $f\,\overline{\rho_U}\,g \Leftrightarrow$ there exists $\mu \in \mathcal{U}$ such that $\mu(f) \leq 1 - g$.

Then $\rho_{\mathcal{U}}$ is proved to be a fuzzy proximity. It is called **the fuzzy proximity induced by the uniformity $\mathcal{U}$.**

Definition : 3.10

Given an n-fuzzy uniformity $\mathcal{U}_{n^*}$ on X for $f, g \in I_n^X$ define $\rho_{\mathcal{U}_{n^*}}$ such that $f\,\overline{\rho_{U_{n^*}}}\,g \Leftrightarrow$ there exists $\mu_{n^*} \in \mathcal{U}_{n^*}$ such that $\mu_{n^*}(f) \leq 1 - g$.

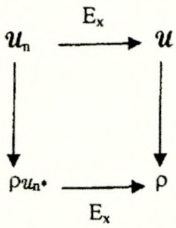
Remark : 3.11

$\rho_{\mathcal{U}_{n^*}}$ is an n-fuzzy proximity on X and it is called the ***n*-fuzzy proximity induced by $\mathcal{U}_{n^*}$.**

In the following theorem, we prove that the extended fuzzy proximity of the n-fuzzy proximity induced by the n-fuzzy uniformity is the same as the fuzzy proximity induced by the extended fuzzy uniformity of the n-fuzzy uniformity.

Theorem : 3.12

Let $\mathcal{U}_{n^*}$ be an n-fuzzy uniformity on X. Let $\mathcal{U}$ be the extended fuzzy uniformity on X. Let $\rho_{\mathcal{U}_{n^*}}$ be the fuzzy proximity induced by $\mathcal{U}_{n^*}$. Then the extended fuzzy proximity $E_X(\rho_{\mathcal{U}_{n^*}})$ of $\rho_{\mathcal{U}_{n^*}}$ coincides with the fuzzy proximity $\rho_{\mathcal{U}}$ induced by the fuzzy uniformity $\mathcal{U}$. (*i.e.*) the following diagram is commutative



$\rho_1 = \rho_2$ where $\rho_1 = E_x(\rho_{\mathcal{U}_{n^*}})$ and $\rho_2 = \rho_{\mathcal{U}}$

Proof : Let ρ_1 be the extended fuzzy proximity of $\rho_{U_{n^*}}$. Let ρ_2 be the fuzzy proximity induced by the extended fuzzy uniformity $\mathcal{U}$ of $\mathcal{U}_{n^*}$.

Now for $f, g \in I^X$

$$f\,\overline{\rho_1}\,g \Leftrightarrow f\,\overline{E_x(\rho_{U_{n^*}})}\,g \Leftrightarrow {}^n f\,\overline{\rho_{U_{n^*}}}\,{}^n g$$

$$\Leftrightarrow \text{there exists } \mu_{n^*} \in \mathcal{U}_{n^*} \text{ such that } \mu_{n^*}({}^n f) \leq 1 - {}^n g$$

$$f\,\overline{\rho_2}\,g \Leftrightarrow \text{there exists } \mu \in \mathcal{U} \text{ s.t. } \mu(f) \leq 1 - g$$

$$\Leftrightarrow \text{there exists } \mu_{n^*} \in \mathcal{U}_{n^*} \text{ s.t. } \mu_{n^*}({}^n f) \leq 1 - g$$

$$\Leftrightarrow {}_n(\mu_{n^*}({}^n f)) \leq {}_n(1 - g) \qquad \text{[Proposition 2.8 (iii)]}$$

$$\Leftrightarrow \mu_{n^*}({}^n f)) \leq {}_n(1 - g) \qquad \text{[Proposition 2.10 (i)]}$$

$$\Leftrightarrow \mu_{n^*}({}^n f) \leq 1 - {}^n g \qquad \text{[Proposition 2.9 (ii)]}$$

The last implication follows from the fact that $\mu_{n^*}({}^n f) \in I_n^X$

$\therefore \qquad \rho_1 = \rho_2.$

Hence the proximity ρ_1 obtained by extending $\rho_{\mathcal{U}_{n^*}}$ coincides with the proximity ρ_2 induced by the fuzzy uniformity $\mathcal{U}$ obtained by extending the n-fuzzy uniformity $\mathcal{U}_{n^*}$ (*i.e.*) $E_x(\rho_{\mathcal{U}_{n^*}}) = \rho_{\mathcal{U}}$.

REFERENCES

1. Chang, C.L., Fuzzy Topological Spaces, *J. Math. Anal. Appl.*, **24**, 182-190 (1968).
2. Jeyalakshmi, P., A New Approach to the Study of Fuzzy Topological Spaces, *J. Analysis*, **7**, 83-88 (1999).
3. Hutton, B., Uniformities on Fuzzy Topological Spaces, *J. Math. Anal. Appl.*, **58**, 559-571 (1977).
4. Katsaras, A.K., Fuzzy Proximity Spaces, *J. Math. Anal. Appl.*, **68**, 100-110 (1979).
5. Katsaras, A.K., On Fuzzy Proximity Spaces, *J. Math. Anal. Appl.*, **75**, 571-583 (1980).
6. Katsaras, A.K., On Fuzzy Uniform Spaces, *J. Math. Anal. Appl.*, **101**, 97-113 (1984).

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