Fuzzy α*- continuous mappings

By

S. Anumitha

Reg. No. 20PMA002

Supervisor

Dr. D. Jayanthi

Thesis submitted to

Avinashilingam Institute for Home Science and Higher Education forWomen,

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In Partial Fulfillment of the requirements for

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N. Balamani 19.05.2022
Signature of the Head of the Department

19/05/2022
Signature of the Supervisor

DECLARATION

I declare that the dissertation, entitled "**Fuzzy α^*-continuous mappings**" submitted by me for the Degree of Master of Science is a record of work carried out by me during the periodfrom December 2021 to May 2022 under the guidance of **Dr. D. Jayanthi**, Assistant Professor(SS),DepartmentofMathematics,AvinashilingamInstituteforHomeScienceandHigherEducation for Women, Coimbatore, and it has not formed the basis for the award of any Degree,Diploma,Associateship,Fellowshiporothersimilartitleinthis UniversityoranyotherUniversityor Institution of Higher Learning.

Signature of the Candidate

ACKNOWLEDGEMENT

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I humbly thank **GOD ALMIGHTY** who has showered his abundant grace on me and endowed me with wisdom, mental courage and good health throughout the period of my thesiswork.

I am extremely thankful to **Dr. P. R. KRISHNAKUMAR**, Former Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, forgiving me an opportunity to pursue my thesis in this esteemed institution.

I would like to thank **Dr. S. P. THYAGARAJAN**, Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for providing me the opportunity and exposure to the world of knowledge.

I wish to express my profound gratefulness to **Dr. PREMAVATHYVIJAYAN**, Former Vice Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her worthy encouragement and for providing all the necessary resources.

I express my sincere thanks to **Dr.V. BHARATHI HARISHANKAR**,Vice Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her constant encouragement throughout the thesis work.

I would like to thank **Dr. S. KOWSALYA**, Registrar, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for granting permission to carry out my research in this institution.

My humble gratitude to **Dr. G. PADMAVATHI**, Dean, School of Physical Sciences and Computational Sciences, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her excellent support and guidance during my thesis work.

My deep sense of gratitude to **Dr. P. JEYALAKSHMI**, Professor and Head (Rtd.), Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her guidance and every process involved in during the thesis work in a successful manner.

My Sincere thanks to **Dr. N. BALAMANI**, Assistant Professor (SS) and Head In-charge, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her commendable advice and guidance in each and every process involved in doing the thesis work in a successful manner.

I express my heart-felt thanks and sincere gratitude to my guide **Dr. D. JAYANTHI**, Assistant Professor (SS), Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her invaluable guidance, lively discussion, herpatience, enthusiastic support and sacrifice for the successful completion of my thesis. I render my indebtedness and great deal of heartfelt appreciation to my beloved guide for her keen interest, benevolent concern and untiring efforts without which this work would not have been shaped up and completed at all.

I extend my gratitude to all the **STAFF MEMBERS OF THE DEPARTMENT OF MATHEMATICS**, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for their help and encouragement and who were responsible for the good finish of this desertion.

Last but not least, I owe my special thanks to my beloved **FATHER, MOTHER, SISTER, FRIENDS** and **WELL WISHERS,** who helped me by providing full strength, support and encouragement to complete my thesis successfully. Without their relentless support, prayer and understanding throughout my thesis it would have been impossible for me to complete this thesis.

CONTENT

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INTRODUCTION

TOPOLOGY

Topology is the branch of mathematics that describes properties of spaces that remains unchanged under smooth deformation which allow no surfaces tearing or hole punching. It emerged through the development of concepts from geometry and set theory, such as space, dimension and transformation.

On the other hand, topology is a kind of geometry which deals with the positions of the particles and ignores their sizes. It concentrates on such qualitative questions whether a certain point is inside, or outside a certain closed curve or surface. The best way to appreciate what topology is about is by comparing it with geometry, the two being akin to each other in so many respects.

Topology is the study of sets on which one has a notation of "closeness" enough to decide which functions defined on it are continuous. Thus, it is a kind of generalized geometry or a kind of generalized analysis. The fundamental ideas in general topology are of convergence and continuity.

More formally, a topological space is a set X on which we have a topology, a collection of subsets of X which we call the "open" subsets of X. The only requirement are that both X itself and the empty subset must be among the open sets, that all unions of open sets are open, and that the intersection of two open set is open. However, stated in this generality, topological spaces can be quite bizarre. For example, in the most other disciplines of mathematics, the only topologies on finite sets are the discrete topologies (all subsets are open), but the definition permit many other. Thus a general theme in topology is to test the extent to which the axioms force the kind of structure one expects to use and then, as appropriate, introduce other axioms so as to better match the intended application.

Topology emerged as a well-defined mathematical discipline during the early year's twentieth century, but isolated instance of topological problems and precursors of the theory can be traced back several centuries. Gottfried Wilhelm Leibniz (1646-1716) was the first foresee a geometry in which position, rather than magnitude wasthe most important factor. The first practical application of topology was made in the year 1736 by the Swiss mathematician Leonhard Euler (1707-1783). Carl F. Gauss (1777-1855) predicted in 1833 that geometry of location would become a mathematical discipline of great importance. The word topology was first used by the German mathematician Joseph. Bernard Riemann (1826-1866) was the first mathematician toforesee topology in the generality which has achieved today.

Today, all the modern manufacturing enterprises are striving to develop best optimized reduced weight and cost effective products that meet the intended design functionally and reliability. In the last few years, topology optimization has emerged at the valuable tool to develop new design proposals especially in automobile and antigraft industries. Topology optimization calculates the optimal loads compatible deign, under specified boundary conditions and constraints. This result is an innovative design proposal irrespective of dependency of the designer experience and conventional design approaches.

Topology is used in many branches of mathematics, such as differentiable equations, dynamical systems, knot theory, and Riemann Surfaces in complex analysis. It is also used in string theory in physics, and for describing the space-time structure of universe.

Topological spaces and continuous functions between them are the primary objects of study in the field topology. For many years prior to the formalization of the field of topology, mathematicians used the concepts of an open set, a simple example of which is an open interval on the real line. But over time it was realized that many of the properties held by open sets on the real line could be said to hold for certain types of subsets in any set. Eventually the essential properties were distilled out and the concepts of a collection of open sets, called a topology.

The concepts of continuity of functions are one of the most important in topology. A topology on a set is s structure that establishes a notation of proximity on the set. Continuous functions between topological spaces preservers proximity, reflecting the idea that a continuous functions sends points that are close in one space to points that are close in order.

FUZZY TOPOLOGY

Fuzzy set theory provides us with a framework which is wider than that of classical set theory. Various mathematical structures, whose features emphasize the effects of ordered structure, can be developed on the theory. Fuzzy topology is one such branch, combining ordered structure with topological structure. One of the advantages of defining topology on a fuzzy set lies in the fact that subspace topologies can now be developed on fuzzy subsets of a fuzzy set.

Fuzzy sets provide a natural foundation for introducing new branch of mathematics namely fuzzy topology. The theory of fuzzy topological spaces has grown as an active field of mathematical research in recent times. The concept of fuzzy topological space was first introduced in 1968 by C.L. Chang he used the fuzzy set theory for defining and introducing fuzzy topological spaces, when he extended in a straight forward manner the concept of ordinary topological spaces to fuzzy topological spaces and gave some of the basic definitions and properties that are satisfied to fuzzy topological spaces. Later on many mathematicians contributed in the development of the theory of fuzzy topological spaces.

Fuzzy set theory offers us a new angle to observe and investigate the relation between sets and their elements other than traditional “Black or White” way. It tells us besides “belonging to” or “not belonging to”, other possibilities exist in the relation between an element and a set emerging in various practical processes. The point of view certainly offers us a new frame work of set theory, and then, in this new framework, we face the problem relating to mathematics, the study of which forms the contents of fuzzy mathematics.

The membership function is the crucial component of a fuzzy set. Therefore the operations with fuzzy sets are defined via their membership functions. We shall present the concepts suggested by Zadeh in 1965. They constitute a consistent framework for the theory of fuzzy sets. They are, however, not the only possible way to extend classical set theory consistently. Zadeh and other authors have suggested alternative or additional definitions for set theoretic operations.

The concept of fuzzy set is an extension of the concept of crisp set. Just as a crisp set on a universal set U is defined by its characteristic function from U to {0, 1}, is a fuzzy set on a domain U is defined by its membership function from U to [0,1].

Fuzzy mathematics is the branch of mathematics including fuzzy set theory and fuzzy logic that deals with partial inclusion of elements in a set on a spectrum, as opposed to simple binary "yes" or "no" (0 or 1) inclusion.

For example: Is Sandy honest?

Here we are applying the fuzzy concept,

Extremely honest = 1

Very honest = 0.90

Honest at time = 0.20

Extremely dishonest = 0.0

In this thesis the concept of fuzzy α*- continuous mapping in fuzzy topological spaces are introduced and studied. Some of the interesting remarks are proved with examples and their inter-relations with already existing fuzzy continuous mapping in fuzzy topological space are investigated and analyzed.

Chapter 1 begins with preliminary definitions and remarks followed by examplesrequired for fuzzy α*- continuous mapping.

Chapter 2 deals with the introduction of fuzzy α*-continuous mapping. Comparison between fuzzy α*- continuous mapping with already existing fuzzy continuous mappings, such as fuzzy semi-continuous mapping, fuzzy pre-continuous mapping, fuzzy α-continuous mapping, fuzzy β-continuous mapping, fuzzy γ-continuous mapping are examined and their inter-relation isgiven by a neat diagram.

Throughout this paper (X, τ) represent non-empty fuzzy topological space on which no separation axiom are assumed, unless otherwise explicitly mentioned. If A is a subset of a space (X, τ), cl(A) denotes the closure of A and int(A) denotes the interior of A respectively. The extension of the notions of union, intersection, contained in and contains in fuzzy topological spaces are denoted as ∨, ∧, ≤, ≥ respectively.

REVIEW OF LITERATURE

In 1965, L.A. Zadeh introduced the concept of fuzzy sets as a new approach for modelling uncertainties. The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. Topology provided the most natural framework for the concepts of fuzzy sets to flourish. The concept of fuzzy topological space was introduced by C.L.Chang in 1968.The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In 1970, N. Biwas introduced some mappings in topological spaces. In 1987, Supriti Saha introduced the idea of fuzzy δ-continuous mapping of a function in the fuzzy sets.

The concept of regular open and regular closed sets in topological spaces is introduced by Stone in 1937 and the concept of semi-open sets in topological spaces is introduced by Levine in 1963. Semi-closed set is introduced by Biwas in 1970 in topological spaces. Njastad (1965) has introduced the concept of α-open sets which are weaker than open sets in topological spaces. Andrijevic (1986) has introduced semi-pre-open sets (which is also known as β-open sets) in topological spaces. Abd El-Monsef, El-Deep and Mashhour introduced pre-open sets in 1982 in topological spaces. Abd El-Monsef et al.(1983) have introduced β-open sets and β-continuous mappings in topological spaces. Zadeh. L. A. (1987) introduced Fuzzy continuous mappings, Hatir and Noiri (1996) introduced α*-sets in topological spaces.

N. Levine (1963) introduced the concept of semi open sets and semi continuity in topological spaces, L. A. Zadeh (1965) introduced fuzzy sets, C. L. Chang (1968) introduced Fuzzy topological spaces, Warren R.H.(1978) introduced neighborhoods bases and continuity in fuzzy topological spaces Abdulla S. Bin Shahna (1991) introduced fuzzy strong semi continuity and fuzzy pre continuity, M.K. Singal and N. Parkash (1991) introduced fuzzy pre open sets and fuzzy pre separation axioms, M.K. Singal and N. Rajvansi (1992) introduced fuzzy alpha-sets and alpha-continuous maps, Jin Han Park, Jin Kuen Park (1988) discussed on regular generalized fuzzy closed sets and generalizations of fuzzy continuous functions,Thakur, S.S. and Singh, S (1998) introduced fuzzy semi-pre-open sets and fuzzy semi-pre-continuity, Dr. Hakeem and Ahmed Othman (2009) introduced new result of fuzzy alpha open sets alpha continuous mappings, S. Dang and A. Behera (1994) introduced fuzzy weakly semi-continuous functions, Azad K.K. (1981) introduced fuzzy semi-continuity, fuzzy almost continuity and fuzzy weekly continuity, Journal of Mathematical Analysis and Applications, Abd El -Monsef et.al. (1983) introduced β-open sets and β-continuous mappings, Supriti Saha (1987) introduced fuzzy continuous mapping, E. Hatir and Noitir (1996) introduced Decomposition of continuity and complete continuity, Balasubramanian and P. Sundaram (1997) introduced some generalizations of fuzzy continuous functions, Shi-Zhong Bai (1998) introduced fuzzy semi-preopen sets and semi pre-continuity, Hanafy, I. M (1999) introduced Fuzzy γ-open sets and fuzzy γ-continuity, Hanafy, I.M. and Al-Saadi, H.S. (2001) introducedStrong forms of continuity in fuzzy topological spaces, Erdal Ekici (2003) introduced on some types of continuous fuzzy functions, Palaniappan. L (2002) introduced fuzzy topology and Ahmed and Athar Kharal (1981) introduced fuzzy sets, fuzzy s-open and s-closed mappings, Ahmed Zahran, Abbas, S.E. (2010) introduced fuzzy δ-I-continuous mapping, Khaled M. A. et. al. (2010) introduced fuzzy α-continuous multi functions, Benchalli, S.S. and Jenifer Karnel, J. (2011) introduced Fuzzy gb -continuous maps in fuzzy topological spaces, M. Shukla (2012) introduced on fuzzy strongly α continuous mappings, Athar Kharal(2013) introduced fuzzy α continuous mappings, Zabidin Shelleh (2015) introduced on fuzzy θ semi- generalized continuous mappings, Thangarai and Dinakarn (2016) introduced on some simply * continuous functions and C. Sivashanmugaraja (2021) introduced fuzzy pre-γ* open and fuzzy pre* - γ continuity mappings in fuzzy topological spaces. Thangaraj and Senthil (2021) introduced A some - what fuzzy continuous functions. Annapoorani. K and Jayanthi. D (2021) introduced fuzzy t – open sets and Siva Sobhika. D and Jayanthi. D (2021) introduced fuzzy α*- open set.

1. SEMI- OPEN SETS AND SEMI – CONTINUITY IN TOPOLOGICAL SPACES

[Levine. N, 1963]

In this paper, the author investigated the notion of irresolute topological vector spaces. Irresolute topological vector spaces are defined by using semi open sets and irresolute mappings. The notion of irresolute topological vector spaces is analog to the notion of topological vector spaces, but mathematically it behaves differently.

2. FUZZY PRE* - γ -OPEN AND FUZZY PRE* - γ - CONTINUITY MAPPINGS IN FUZZY TOPOLOGICAL SPACES

[C. Sivashanmugaraja, 2021]

In this paper, the author has formulated a definition of fuzzy pre * - γ -open mapping and fuzzy super pre*- γ-open mapping in fuzzy topological spaces via pre*- γ-open fuzzy sets. Also he studied composite on these mappings. Moreover, he investigated relationships among thesemappings and also proved some properties and theorems.

3. FUZZY α- CONTINOUS MAPPINGS

[Athar Kharal, 2013]

In this paper, the author has generalized the notion of α-sets to fuzzy spaces. He also introduced the concepts of fuzzy α-continuous and fuzzy α-open mappings and discussed their relations with fuzzy continuous and other weaker forms of fuzzy continuous mappings. Counterexamples are given to show the non-coincidence of these mappings.

4. A NOTE ON FUZZY SEMI – PREOPEN SETS AND SEMI PRE-CONTINUITY

[Shi-Zhong Bai, 1998]

In this paper, the author has introduced the concepts of fuzzy semi-preopen sets and fuzzy semi-pre continuous mappings.

5. ON FUZZY WEAKLY SEMI-CONTINUOUS FUNCTIONS

[Dang. S and A. Behera, 1994]

This paper is devoted to the introduction and the study of fuzzy weakly semi-continuous functions between fuzzy topological spaces. Some properties of these functions are characterized in terms of quasi coincidence, quasi neighborhoods, θ-neighborhoods etc, Furthermore, some relations connecting fuzzy weakly semi-continuous functions and fuzzy retractions have been established.

6. ON FUZZY SEMI – PRE - OPEN SETS AND FUZZY SEMI – PRE - CONTINUITY

[S.S, Thakur and Surendar Singh, 1998]

In this paper, the author have introduced and studied the concept of fuzzy semi-pre-open sets and fuzzy semi-pre-continuous mappings in fuzzy topological spaces.

7. ON FUZZY θ - SEMI - GENERALIZED CONTINUOUS MAPPINGS

[Zabidin Salleh, 2015]

In this paper, the author has introduced the concept of fuzzy θ-semi-generalized continuous mapping by using a fuzzy θ- semi - generalized closed set that generalized the most forms and properties of fuzzy continuity. He also introduced the concept of fuzzy θ-semi-generalized closed (resp. fuzzy θ-semi-generalized open) mappings and obtained some of their characterizations. Several interesting results and counterexamples are obtained to establish their characterizations.

8. ON SOME GENERALIZATIONS OF FUZZY CONTINUOUS FUNCTIONS

[Balasubramanian and P. Sundaram, 1997]

In this paper, the authors defined and studied various generalizations of fuzzy continuous functions. In addition, they have also introduced and studied fuzzy generalized connectedness, generalized fuzzy extremely disconnectedness and fuzzy generalized compactness.

9. β - OPEN SETS AND β - CONTINUOUS MAPPINGS

[Abd El-Monsef et. al. 1983]

In this paper, the authors have introduced β-open sets in topological spaces. Further they introduced β-continuous mappings in topological spaces and studied in detail.

10. ON FUZZY STRONGLY α-CONTINUOUS MAPPINGS

[M. Shukla, 2012]

In this paper,the concept of strongly α-continuous mappings by Y. Beceren has been extended in topological spaces. He introduced and studied fuzzy strongly α-continuous maps on fuzzy topological spaces. Some of its properties have also been investigated. Relation between fuzzy strongly α-continuous and fuzzy α -continuous maphave also been established.

11. ON FUZZY STRONG SEMICONTINUITY AND FUZZY PRECONTINUITY

[Abdulla S.Bin Shahna, 1991]

In this paper, the author has introduced and studied the concept of semi open (semi closed) sets, semi-continuous mappings, almost continuous mappings and weakly continuous mappings in the fuzzy setting. In the same spirit, they have introduced and made a preliminary study of fuzzy strongly semi continuous, and fuzzy pre continuous mappings in this paper.

12. NEW RESULTS OF FUZZY ALPHA - OPEN SETS AND FUZZY ALPHA - CONTINUOUS MAPPINGS

[Dr.Hakeem and Ahmed Othman, 2009]

In this paper, the authors have extended the notion of semi α-open sets of general topology to fuzzy topology, and studied some notions based on this new concept in fuzzy topology. Also, they have studied the relations between fuzzy semi α-open sets and other types of fuzzy open sets. The authors also introduced the concept of fuzzy semi α-continuous mapping and other weaker forms of fuzzy semi α-continuous mapping and discussed their relations with fuzzy continuous mappings and other weaker forms of fuzzy continuous mappings.

13. FUZZY SETS, FUZZY S-OPEN AND S-CLOSED MAPPINGS

[Ahmad and AtharKharal, 1981]

In this paper, the authors fuzzified the findings and defined fuzzy semi-open mappings and fuzzy semi-closed mappings and established some interesting characterizations of these mappings by established several important fundamental identities and inequalities.

14. FUZZY SETS

[L. A. Zadeh, 1965]

In this paper, the author has introduced a new class of sets namely fuzzy sets which are characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. Further the author has provided the notions of inclusion, union, intersection, complement etc., with respect to the fuzzy sets.

15. FUZZY ALPHA-SETS AND ALPHA-CONTINUOUS MAPS

[M. K. Singal and N. Rajvansi, 1992]

In this paper, the authors have generalized the notion of α-sets to fuzzy spaces. They also introduced the concepts of fuzzy α-continuous and fuzzy α-open mappings and discussed their relations with fuzzy continuous and other weaker forms of fuzzy continuous mappings.

16. ON SOME TYPES OF CONTINUOUS FUZZY FUNCTIONS

[Erdal Ekici, 2003]

In this paper, the author introduced and proved, some characterizations and some properties of fuzzy continuous functions and its weaker and stronger forms including fuzzy weakly continuous, fuzzy θ-continuous, fuzzy strongly θ-continuous, fuzzy almost strongly θ-continuous, fuzzy weakly θ-continuous, fuzzy almost continuous, fuzzy super continuous, fuzzy δ-continuous.

17. FUZZY PRE - OPEN SETS AND FUZZY PRE SEPARATION AXIOMS

[M. K. Singal and N. Parkash, 1991]

In this paper, the authors have used the concept of pre-open sets due to A.S. Mashhour, M.E. Abd El-Monsef and S.N. El-Deep and introduced fuzzy pre-open sets. Further they introduced fuzzy separation axioms and investigated with the help of fuzzy pre-open sets.

18. DECOMPOSITION OF CONTINUITY AND COMPLETE CONTINUITY

[E. Hatir and T. Noiri, 1996]

In this paper, the authors first introduced the notions of α*-sets, B-sets and t-set and obtained decompositions of continuity and complete continuity.

19. FUZZY TOPOLOGICAL SPACES

[C. L. Chang, 1968]

In this article, the author has introduced fuzzy topological spaces. This concept is considered to be the generalization of general topological spaces. In brief, the basic concepts such as fuzzy open set, fuzzy closed set, fuzzy neighborhood, fuzzy continuity etc., are discussed in depth.

CHAPTER1

CHAPTER 1

PRELIMINARIES

Definition 1.1: [Zadeh, 1965]

Let X be a non-empty set. The **fuzzy set**A in X can be described in the form

$$A = \{\langle x, \mu_A(x)\rangle \ / \ x \in X\}$$

where the function μ_A: X → [0,1] is called the membership function and $\mu_A(x)$denotes the degree to which x ∈ A and $0 \leq \mu_A(x) \leq 1$ for each x ∈X.

A membership function assigned the values to the elements of the universal set fall within a specified range and indicates the membership grade of these elements in the set. Larger values denote higher degrees of set membership. A set defined by membership functions is a fuzzy set. The commonly used range of values of membership functions is the unit interval [0, 1].

Definition 1.2: [Zadeh, 1965]

Let A and B be two fuzzy sets of the form

$$A = \{\langle x, \mu_A(x)\rangle : x \in X\}$$

and

$$B = \{\langle x, \mu_B(x)\rangle : x \in X\}.$$

Then, inclusion, equal, complement, union and intersection etc., with respect to the fuzzy sets are

(i) $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ for all x ∈ X,

(ii) A = B if and only if $A \leq B$ and $A \geq B$,

(iii) $A^c = \{\langle x, (1- \mu_A(x)\rangle : x \in X\}$,

(iv) $A \vee B = \{\langle x, \mu_A(x) \vee \mu_B(x)\rangle : x \in X\}$,

(v) $A \wedge B = \{\langle x, \mu_A(x) \wedge \mu_B(x)\rangle : x \in X\}$.

The fuzzy sets $0_\sim = \langle x, 0\rangle$ and $1_\sim = \langle x, 1\rangle$ are respectively the empty set and the whole set of X

Definition 1.3: [Chang, 1968]

The **fuzzy topology** τ on X is a family of subsets in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \wedge G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\vee G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called the **fuzzy topological space** and any fuzzy set in τ is known as the **fuzzy open set** in X. The complement A^c of a fuzzy open set A in a fuzzy topological space (X, τ) is called the **fuzzy closed set** in X.

Definition 1.4: [Chang, 1968]

Let (X, τ) be the fuzzy topological space and $A = \langle x, \mu_A\rangle$ be a fuzzy set in X. Then the **fuzzy interior** and the **fuzzy closure** are defined by

$$\text{int}(A) = \vee \{G \,/\, G \text{ is a fuzzy open set in } X \text{ and } G \leq A\},$$

$$\text{cl}(A) = \wedge \{K \,/\, K \text{ is a fuzzy closed set in } X \text{ and } A \leq K\}.$$

It is to be noted that for any fuzzy set A in (X, τ), we have

$$\text{cl}(A^c) = (\text{int}(A))^c \text{ and } \text{int}(A^c) = (\text{cl}(A))^c$$

Definition 1.5: [Azad, 1981]

The fuzzy set A = ⟨x, μ_A⟩ in a fuzzy topological space (X, τ) is said to be a

(i) **fuzzy regular open set**if A = (int(cl(A))

(ii) **fuzzy regular closed set**if cl(int(A)) = A

Definition 1.6:

The fuzzy set A = ⟨x, μ_A⟩ in a fuzzy topological space (X, τ) is said to be a

(i) **fuzzy semi-open set** [Azad, 1981] if A ≤ cl(int(A))

(ii) **fuzzy pre-open set** [Abdulla S. Bin Shahna, 1991] if A ≤ int(cl(A))

(iii) **fuzzy α-open set** [Abdulla S. Bin Shahna, 1991] if A ≤ int(cl(int(A)))

(iv) **fuzzy β-open set** [Thakur and Singh, 1998] if A ≤ cl(int(cl(A)))

(v) **fuzzy γ-open set** [Hanafy, 1999] if A ≤ cl(int(A)) ∨ int(cl(A))

Definition 1.7:

The fuzzy set A = ⟨x, μ_A⟩ in a fuzzy topological space (X, τ) is said to be a

(i) **fuzzy semi-closed set** [Azad, 1981] if int(cl(A)) ≤ A

(ii) **fuzzy pre-closed set** [Abdulla S. Bin Shahna, 1991] if cl(int(A)) ≤ A

(iii) **fuzzy α-closed set** [Abdulla S. Bin Shahna, 1991] if cl(int(cl(A))) ≤ A

(iv) **fuzzy β-closed set** [Thakur and Singh, 1998] if int(cl(int(A))) ≤ A

(v) **fuzzy γ-closed set** [Hanafy, 1999] if int(cl(A)) ∧ cl(int(A))) ≤ A

Definition 1.8: [Chang, 1968]

Let (X, τ) be a fuzzy topological space and A, B be fuzzy sets in X. Then the following properties hold:

(i) $\text{int}(A) \le A$

(ii) $A \le \text{cl}(A)$

(iii) $A \le B \Rightarrow \text{int}(A) \le \text{int}(B)$

(iv) $A \le B \Rightarrow \text{cl}(A) \le \text{cl}(B)$

(v) $\text{int}(\text{int}(A)) = \text{int}(A)$

(vi) $\text{cl}(\text{cl}(A)) = \text{cl}(A)$

(vii) $\text{int}(A \wedge B) = \text{int}(A) \wedge \text{int}(B)$

(viii) $\text{cl}(A \vee B) = \text{cl}(A) \vee \text{cl}(B)$

(ix) $\text{int}(1_\sim) = 1_\sim$

(x) $\text{cl}(0_\sim) = 0_\sim$

Definition 1.9: [Siva Sobhika and Jayanthi, 2021]

A fuzzy set $A = \{ \langle x, \mu_A(x): x \in X\}$ in a fuzzy topological space X is said to be a

(i) **fuzzy α^*- open set** ($F\alpha^*$ OS) if $\text{int}(\text{cl}(\text{int}(A))) = \text{int}(A)$

(ii) **fuzzy α^*- closed set** ($F\alpha^*$ CS) if $\text{cl}(\text{int}(\text{cl}(A))) = \text{cl}(A)$

Definition 1.10: [Palaniappan, 2002]

Let X and Y be two non-empty sets and f: X→Y be a mapping. If $A = \{\langle x, \mu_A(x)\rangle: x \in X\}$ is afuzzy set in X, then the **image** of A under f, denoted by $f^{-1}(A)$, is the fuzzy set in Y defined by

$$f(A) = \{\langle y, f(\mu_A)(y) / y \in Y\rangle\}$$

Definition 1.11: [Palaniappan, 2002]

Let X and Y be two non-empty sets and f: X→Y be a mapping. If B = $\{\langle y, \mu_B(y)\rangle: y \in Y\}$ is afuzzy set in X, then the **preimage**of B under f, denoted by $f^{-1}(B)$, is the fuzzy set in X defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x) / x \in X\rangle\}$$

where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ for every $x \in X$.

Definition 1.12: [Chang, 1968]

Let f be a mapping from a fuzzy topological space (X, τ) into a fuzzy topological space (Y, σ). Then f is said to be a **fuzzy continuous mapping** if $f^{-1}(V)$ is a fuzzy open set in (X, τ) for every fuzzy open set V of (Y, σ).

Definition 1.13:

Let f be a mapping from a fuzzy topological spaces (X, τ) into a fuzzy topological spaces (Y, σ). Then f is said to be a

(i) **fuzzy semi continuous mapping** if $f^{-1}(V)$ isa fuzzy semi open set in (X, τ) for every fuzzy open set V of (Y, σ). [Azad, 1981]

(ii) **fuzzy α - continuous mapping** if $f^{-1}(V)$ is afuzzy α open set in (X, τ) for every fuzzy open set V of (Y, σ). [Abdulla S. Bin Shahna, 1991]

(iii) **fuzzy pre continuous mapping** if $f^{-1}(V)$ is a fuzzy pre open set in (X, τ) for every fuzzy open set V of (Y, σ). [Abdulla S. Bin Shahna, 1991]

(iv) **fuzzy β - continuous mapping** if $f^{-1}(V)$ is afuzzy β open set in (X, τ) for every fuzzy open set V of (Y, σ). [Abd El Monsef and Mohmoud, 1983]

(v) **fuzzy γ - continuous mapping** if $f^{-1}(V)$ is afuzzy γ open set in (X, τ) for every fuzzy open set V of (Y, σ). [Hanafy, 1999]

Definition 1.14: [Chang, 1968]

Let A, A_i ($i \in J$) be fuzzy set in X and B, B_j ($j \in K$) be fuzzy sets in Y and f: X →Y be a function. Then,

(i) $A_1 \subset A_2 \Rightarrow f(A_1) \subset f(A_2)$

(ii) $B_1 \subset B_2 \Rightarrow f(B_1) \subset f(B_2)$

(iii) $A \subset f^{-1}(f(A))$ [if f is injective, then $A = f^{-1}(f(A))$]

(iv) $f(f^{-1}(B)) \subset B$ [if f is surjective, then $B = f(f^{-1}(B))$]

(v) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

(vi) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

(vii) $\mathrm{f}^{-1}(0_{\sim}) = 0_{\sim}$

(viii) $\mathrm{f}^{-1}(1_{\sim}) = 1_{\sim}$

(ix) $f^{-1}(B^{C}) = (f^{-1}(B))^{c}$

Definition 1.15: [Siva Sobhika and Jayanthi, 2021]

Let A be a fuzzy set in a fuzzy topological space (X, τ) then the **α*- interior** and **α*- closure** of A are defined as

α*- int(A) = ∨ {G / G is a fuzzy α*open set in X and G ≤ A},

α*- cl(A) = ∧ {K / K is a fuzzy α* closed set in X and A ≤ K}.

CHAPTER 2

CHAPTER 2

Fuzzy α*- Continuous Mapping

2.1. Introduction

In 1965, L. A. Zadeh introduced the concept of fuzzy sets. Fuzzy Topology is used in all branches of mathematics. The concept of fuzzy topological space was introduced by C. L. Chang in 1968.Hatir and Noiri (1996) have introduced t-sets in topological spaces. Njastad (1965) has introduced the concept of α-open sets which are weaker than open sets in topological spaces. Hatir and Noiri (1996) introduced α*-sets in topological spaces. In 1969, N. Biwas introduced the concept of simply continuity by means of the notion of simply open sets and some properties. In 1987, Supriti Saha introduced the idea of fuzzy δ- continuous mapping of a function in the fuzzy sets.

2.2. α*- Continuous Mapping in Fuzzy Topological Spaces

In this section, we have introduced a new type of fuzzy continuous mapping called fuzzy α*- continuous mapping in fuzzy topological space. We have established the inter-relation between the newly introduced fuzzy α*- continuous mapping and already existing fuzzy pre continuous mapping, fuzzy semi continuous mapping, fuzzy α continuous mapping, fuzzy β continuous mapping and fuzzy γ continuous mapping. Some interesting remarks based on their mappings are established and examples are given whenever necessary.

Definition 2.2.1:

Let f : (X, τ) → (Y, σ) be a mapping from a fuzzy topological space (X, τ) into a fuzzy topological space (Y, σ), then f is said to be a **fuzzy α^*- continuous mapping** if $f^{-1}(V)$ is a fuzzy α^*- open set in (X, τ) for every fuzzy open set V of (Y, σ).

Example 2.2.2:

Let X = {a, b}, Y = {u, v}, and let τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are fuzzy topologieson X and Y respectively, where $G_1 = \langle x, (0.1_a, 0.3_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.5_v)\rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is a fuzzy α*- continuous mapping.

For, consider the fuzzy set $G_2 = \langle y, (0.4_u, 0.5_v)\rangle$ which is a fuzzy open set in Y.Then f is a fuzzy α*- continuous mapping as,$f^{-1}(G_2) = \langle x, (0.4_a, 0.5_b)\rangle$ is a fuzzy α*- open set in (X, τ), since

$$\begin{aligned} int(cl(int(f^{-1}(G_2)))) &= int(cl(G_1)) \\ &= int\,(G_1^{c}) \\ &= G_1 \end{aligned}$$

and $int(f^{-1}(G_2)) = G_1$

Therefore, $int(cl(int(f^{-1}(G_2))))= int(f^{-1}(G_2))$.

Hence, f is a fuzzy α*- continuous mapping.

Remark 2.2.3:

Every fuzzy continuous mapping is independent of every fuzzy α*- continuous mapping.

Example 2.2.4:

Let X = {a, b}, Y = {u, v}, and let τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are fuzzy topologieson X and Y respectively, where $G_1 = \langle x, (0.2_a, 0.3_b)\rangle$, $G_2 = \langle y, (0.2_u, 0.3_v)\rangle$.Then (X,$\tau$) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is a fuzzy α*- continuous mapping but not a fuzzy continuous mapping.

For, consider the fuzzy set $G_2 = \langle y, (0.2_u, 0.3_v)\rangle$ which is a fuzzy open set in Y. Then $f^{-1}(G_2) = \langle x, (0.2_a, 0.3_b)\rangle$is a fuzzy α*- open set in (X, τ) as,

$$\begin{aligned} int(cl(int(f^{-1}(G_2)))) &= int(cl(0_\sim)) \\ &= int\ (0_\sim) \\ &= 0_\sim \end{aligned}$$

$$\text{and} \quad int(f^{-1}(G_2)) = 0_\sim$$

Hence, $int(cl(int(f^{-1}(G_2)))) = int(f^{-1}(G_2))$ and hence f is a fuzzy α* - continuous mapping.

But, f is not a fuzzy continuous mapping as,

$$\begin{aligned} int(f^{-1}(G_2)) &= 0_\sim \\ &\neq f^{-1}(G_2) \end{aligned}$$

$$\text{Therefore, } int(f^{-1}(G_2)) \neq f^{-1}(G_1)$$

Hence, f is a fuzzy α*- continuous mapping but not a fuzzy continuous mapping.

Example 2.2.5:

Let X = {a, b}, Y = {u, v}, and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.4_a, 0.8_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.8_v)\rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is fuzzy continuous mapping but not a fuzzy α* - continuous mapping.

For, consider the fuzzy set $G_2 = \langle y, (0.2_u, 0.3_v)\rangle$ which is a fuzzy open set in Y, then $f^{-1}(G_2) = \langle x, (0.4_a, 0.8_b)\rangle$ is a fuzzy open set in (X, τ) as,

$$int(f^{-1}(G_2)) = G_1$$

$$= f^{-1}(G_2)$$

Therefore, $int(f^{-1}(G_2)) = f^{-1}(G_1)$

Therefore, f is a fuzzy continuous mapping.

But,f is not a fuzzy α*- continuous mapping as,

$$int(cl(int(f^{-1}(G_2)))) = int(cl(G_1))$$

$$= int\ (1_{\sim})$$

$$= 1_{\sim}$$

and $\quad int(f^{-1}(G_2)) = G_1$.

Therefore, $int(cl(int(f^{-1}(G_2))) \neq int(f^{-1}(G_2))$.

Hence, f is a fuzzy continuous mapping but not a fuzzy α*-continuous mapping.

Remark 2.2.6:

Every fuzzy semi continuous mapping is independent of every fuzzy α*- continuous mapping.

Example2.2.7:

Let X = {a, b}, Y = {u, v}. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ arefuzzy topologieson X and Y respectively, where $G_1 = \langle x, (0.4_a, 0.3_b)\rangle$, $G_2 = \langle y, (0.2_u, 0.1_v)\rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is fuzzy α*- continuous mapping but not a fuzzy semi continuous mapping.

For, consider the fuzzy open set $G_2 = \langle y, (0.2_u, 0.1_v)\rangle$in Y, then $f^{-1}(G_2) = \langle x, (0.2_a, 0.1_b)\rangle$ is a fuzzy α*- open set in (X, τ) as,

$$int(cl(int(f^{-1}(G_2)))) = int(cl(0_\sim))$$

$$= int\,(0_\sim)$$

$= 0_{\sim}$

and $\quad int(f^{-1}(G_2)) = 0_{\sim}$

Therefore, $int(cl(int(f^{-1}(G_2)))) = int(f^{-1}(G_2))$ and hence f is a fuzzy α*- continuous mapping.

But, f is not a fuzzy semi continuous mapping as,

$$cl(int(f^{-1}(G2)) = cl(0_{\sim})$$

$$= 0_{\sim}$$

and $\quad f^{-1}(G2) \nleq 0_{\sim}$

Therefore, $f^{-1}(G2) \nleq cl(int(f^{-1}(G_2))$ and hence $f^{-1}(G_2)$ is not a semi open set.

Hence, f is a fuzzy α*-continuous mapping but not a fuzzy semi continuous mapping.

Example 2.2.8:

Let X = {a, b}, Y = {u, v}. Then $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ are fuzzy topologieson X and Y respectively, where $G_1 = \langle x, (0.3_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.7_u, 0.9_v)\rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is a fuzzy semi continuous mapping but not a fuzzy α*- continuous mapping.

For, consider the fuzzy set $G_2 = \langle y, (0.7_u, 0.9_v)\rangle$which is a fuzzy open set in Y, then $f^{-1}(G_2) = \langle x, (0.7_a, 0.9_b)\rangle$is a fuzzy semi open set in (X, τ) as,

$$cl(int(f^{-1}(G_2)) = cl(G_1)$$

$$= 1_{\sim}$$

and $f^{-1}(G_2) \leq 1_{\sim}$

Therefore, $f^{-1}(G_2) \leq cl(int(f^{-1}(G_2))$ and $f^{-1}(G_2)$ is a fuzzy semi open set and hence f is a fuzzy semi continuous mapping.

But,f is a not fuzzy α*- continuous mapping as,

$$int(cl(int(f^{-1}(G_2)))) = int(cl(G_1)$$

$$= int\ (1_{\sim})$$

$$= 1_{\sim}$$

and $int(f^{-1}(G_2)) = G_1$

Therefore, $int(cl(int(f^{-1}(G_2)))) \neq int(f^{-1}(G_2))$. Hence $f^{-1}(G_2)$ is not a fuzzy α*- open set.

Hence, f is a fuzzy semi continuous mapping but not a fuzzy α*-continuous mapping.

Remark 2.2.9:

Every fuzzy pre continuous mapping is independent of every fuzzy α*- continuous mapping.

Example 2.2.10:

Let X = {a, b}, Y = {u, v}, Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are fuzzy topologieson X and Y respectively, where $G_1 = \langle x, (0.3_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.7_u, 0.2_v)\rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is a fuzzy α* - continuous mapping but not a fuzzy pre continuous mapping.

For, consider the fuzzy set $G_2 = \langle y, (0.7_u, 0.2_v)\rangle$ which is a fuzzy open setin Y, then $f^{-1}(G_2) = \langle x, (0.7_a, 0.2_b)\rangle$ is a fuzzy α*- open set in (X, τ) as,

$$\begin{aligned} int(cl(int(f^{-1}(G_2)))) &= int(cl(0_\sim)) \\ &= int\,(0_\sim) \\ &= 0_\sim \end{aligned}$$

$$\text{and } int(f^{-1}(G_2)) = 0_\sim$$

This implies that, $int(cl(int(f^{-1}(G_2)))) = int(f^{-1}(G_2))$. Therefore, f is a fuzzy α*- continuous mapping.

But, f is not a fuzzy precontinuous mapping as,

$$\begin{aligned} int(cl(f^{-1}(G_2)) &= int(G_1{}^c) \\ &= 0_\sim \end{aligned}$$

$$\text{and } f^{-1}(G_2) \nleq 0_\sim$$

This implies that, $f^{-1}(G_2) \nleq int(cl(f^{-1}(G_2))$ and $f^{-1}(G_2)$ is not a fuzzy pre open set in X.

Hence, f is a fuzzy α*- continuous mapping but not a fuzzy pre continuous mapping.

Example 2.2.11:

Let X = {a, b}, Y = {u,v}, Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are fuzzy topologieson X and Y respectively, where $G_1 = \langle x, (0.3_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.7_u, 0.9_v)\rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is fuzzy pre continuous mapping but not a fuzzy α*- continuous mapping.

For, consider the fuzzyset $G_2 = \langle y, (0.7_u, 0.9_v)\rangle$ which is a fuzzy open set in Y, then $f^{-1}(G_2) = \langle x, (0.7_a, 0.9_b)\rangle$ is a fuzzy pre open set in (X, τ) as,

$$int(cl(f^{-1}(G_2)) = int(1_\sim)$$

$$= 1_\sim$$

$$\text{and} \quad f^{-1}(G_2) \leq 1_\sim$$

This implies that, $f^{-1}(G_2) \leq int(cl(f^{-1}(G_2))$ and f is a fuzzy pre continuous mapping.

But, f is not a fuzzy α*- continuous mapping as, $f^{-1}(G_2)$ is not a fuzzy α*- open set in X, since

$$int(cl(int(f^{-1}(G_2)))) = int(cl(G_1))$$

$$= int\ (1_\sim)$$

$$= 1_\sim$$

$$\text{and} \quad int(f^{-1}(G_2)) = G_1$$

Therefore, $int(cl(int(f^{-1}(G_2)))) \neq int(f^{-1}(G_2))$, and hence f is not a fuzzyα*- continuous mapping.

Hence, f is a fuzzy pre - continuous mapping but not a fuzzy α*- continuous mapping.

Remark 2.2.12:

Every fuzzy α continuous mapping is independent of every fuzzy α*- continuous mapping.

Example 2.2.13:

Let X = {a, b}, Y = {u, v}, let $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.2_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.3_u, 0.5_v)\rangle$.Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is fuzzy α*- continuous mapping but not a αcontinuous mapping.

For, consider the fuzzy set $G_2 = \langle y, (0.3_u, 0.5_v)\rangle$ which is a fuzzy open set in Y, then $f^{-1}(G_2) = \langle x, (0.3_a, 0.5_b)\rangle$ is a fuzzy α*- open set in (X, τ) as,

$$int(cl(int(f^{-1}(G_2)))) = int(cl(G_1))$$

$$= int\ (G_1^c)$$

$$= G_1$$

and $$int(f^{-1}(G_2)) = G_1$$

Hence, $int(cl(int(f^{-1}(G_2)))) = int(f^{-1}(G_2))$ and f is a fuzzy α* - continuous mapping.

But, f is not a fuzzy α continuous mapping as, $f^{-1}(G_2)$ is not a fuzzy α*- open set in X, since

$$int(cl(int(f^{-1}(G_2))) = int(cl(G_1))$$

$$= int(G_1^c)$$

$$= G_1$$

$$\text{and } f^{-1}(G_2) \nleq G_1$$

This implies that, $f^{-1}(G_2 \nleq int(cl(int(f^{-1}(G_2))))$ and $f^{-1}(G_2)$ is not a fuzzy α- open set in X.

Hence, f is a fuzzy α*- continuous mapping but not a fuzzy α continuous mapping.

Example 2.2.14:

Let X = {a, b}, Y = {u, v}, let $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.6_a, 0.3_b)\rangle$, $G_2 = \langle y, (0.7_u, 0.9_v)\rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is fuzzy α continuous mapping but not a fuzzy α*- continuous mapping.

For, consider the fuzzyset $G_2 = \langle y, (0.7_u, 0.9_v)\rangle$ which is a fuzzy open set in Y, then $f^{-1}(G_2) = \langle x, (0.7_a, 0.9_b)\rangle$ is a fuzzy α open set in (X, τ), since

$$int(cl(int(f^{-1}(G_2)))) = int(cl(G_1))$$

$= int(1_{\sim})$

$= 1_{\sim}$

and $f^{-1}(G_2) \leq 1_{\sim}$

Therefore, $f^{-1}(G_2) \leq int(cl(int(f^{-1}(G_2))))$. Therefore, f is a fuzzy α continuous mapping.

But, f is not a fuzzy α^*- continuous mapping as, $f^{-1}(G_2)$ is not a fuzzy α^*- open set in X, since

$int(cl(int(f^{-1}(G_2))) = int(cl(G_1))$

$= int\ (1_{\sim})$

$= 1_{\sim}$

and $int(f^{-1}(G_2)) = G_1$

This implies that, $int(cl(int(f^{-1}(G_2)))) \neq int(f^{-1}(G_2))$.

Hence, f is a fuzzy α continuous mapping but not a fuzzy α^*- continuous mapping.

Remark 2.2.15:

Every fuzzy β continuous mapping is independent of every fuzzy α^*- continuous mapping.

Example 2.2.16:

Let $X = \{a, b\}$, $Y = \{u, v\}$, let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.2_a, 0.7_b)\rangle$, $G_2 = \langle y, (0.1_u, 0.3_v)\rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is a fuzzy α*- continuous mapping but not a fuzzy β continuous mapping.

For, consider the fuzzy set $G_2 = \langle y, (0.1_u, 0.3_v)\rangle$ which is a fuzzy open set in Y, then $f^{-1}(G_2) = \langle x, (0.1_a, 0.3_b)\rangle$ is a fuzzy α*- open set in (X, τ) as,

$$\begin{aligned} int(cl(int(f^{-1}(G_2)))) &= int(cl(0_\sim)) \\ &= int(0_\sim) \\ &= 0_\sim \end{aligned}$$

and $int(f^{-1}(G_2)) = 0_\sim$

This implies that, $int(cl(int(f^{-1}(G_2)))) = int(f^{-1}(G_2))$. Hence f is a fuzzy α*- continuous mapping.

But, f is not a fuzzy β continuous mapping as, $f^{-1}(G_2)$ is not a fuzzy β- open set in X, since

$$\begin{aligned} cl(int(cl(f^{-1}(G_2)))) &= cl(int(G_1^c)) \\ &= cl(0_\sim) \\ &= 0_\sim \end{aligned}$$

and $f^{-1}(G_2) \nleq 0_\sim$

Therefore, $f^{-1}(G_2) \nleq cl(int(cl(f^{-1}(G_2))$.

Hence, f is a fuzzy α*- continuous mapping but not a fuzzy β continuous mapping.

Example 2.2.17:

Let X = {a, b}, Y = {u, v}, Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.3_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.9_v)\rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is β continuous mapping but not a α*- continuous mapping.

For, consider the fuzzy set $G_2 = \langle y, (0.4_u, 0.9_v)\rangle$ which is a fuzzy open set in Y, then $f^{-1}(G_2) = \langle x, (0.4_a, 0.9_b)\rangle$ is a fuzzy β open set in (X, τ) as,

$$\begin{aligned} cl(int(cl(f^{-1}(G_2)))) &= cl(int(1_\sim)) \\ &= int(1_\sim) \\ &= 1_\sim \end{aligned}$$

$$\text{and} \qquad f^{-1}(G_2) \leq 1_\sim$$

$$\text{Therefore, } f^{-1}(G_2) \leq cl(int(cl(f^{-1}(G_2))))$$

Hence, f is a fuzzy β continuous mapping.

But, f is not a fuzzy α*- continuous mapping as, $f^{-1}(G_2)$ is not a fuzzy α*- open set in X, since

$$\begin{aligned} int(cl(int(f^{-1}(G_2)))) &= int(cl(G_1)) \\ &= int\,(1_\sim) \end{aligned}$$

$= 1_{\sim}$

and $int(f^{-1}(G_2)) = G_1$

Therefore, $int(cl(int(f^{-1}(G_2)))) \neq int(f^{-1}(G_2))$

Hence, f is a fuzzy β continuous mapping but not a fuzzy α*-continuous mapping.

Remark 2.2.18:

Every fuzzy γ - continuous mapping is independent of every fuzzy α*- continuous mapping.

Example 2.2.19:

Let X = {a, b}, Y = {u, v}, and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.3_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.4_u, 0.9_v)\rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is fuzzy γ continuous mapping but not a fuzzy α* continuous mapping.

For, consider the fuzzy set $G_2 = \langle y, (0.4_u, 0.9_v)\rangle$ which is a fuzzy open set in Y, then $f^{-1}(G_2) = \langle x, (0.4_a, 0.9_b)\rangle$ is a fuzzy γ- open set in (X, τ) as,

$cl(int(f^{-1}(G_2))) = cl(G_1)$

$= 1_{\sim}$

and $int(cl(f^{-1}(G_2)) = int(1_{\sim})$

$= 1_{\sim}$

Therefore, $cl(int(f^{-1}(G_2))) \vee int(cl(f^{-1}(G_2)))$

$= 1_{\sim} \vee 1_{\sim}$

$= 1_{\sim}$

and $f^{-1}(G_2) \leq 1_{\sim}$

Therefore, $f^{-1}(G_2) \leq cl(int(f^{-1}(G_2))) \vee int(cl(f^{-1}(G_2)))$.

Therefore, f is a fuzzy γ- continuous mapping.

But, f is not a fuzzy α*- continuous mapping as, $f^{-1}(G_2)$ is not a fuzzy α*- open set in X, since

$int(cl(int(f^{-1}(G_2)))) = int(cl(G_1))$

$= int\ (1_{\sim})$

$= 1_{\sim}$

and $int(f^{-1}(G_2)) = G_1$

Therefore, $int(cl(int(f^{-1}(G_2)))) \neq int(f^{-1}(G_2))$ and hence f is a fuzzy α*- continuous mapping.

Hence, f is a fuzzy γ continuous mapping but not a fuzzy α*- continuous mapping.

Example 2.2.20:

Let X = {a, b}, Y= {u, v}, and let $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.4_a, 0.1_b)\rangle$, $G_2 = \langle y, (0.2_u, 0.7_v)\rangle$. Then (X, τ) and (Y, σ) are fuzzy topological spaces.

Define a mapping f: $(X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is fuzzy α*- continuous mapping but not a fuzzy γ - continuous mapping.

For, consider the fuzzy set $G_2 = \langle y, (0.2_u, 0.7_v)\rangle$ which is a fuzzy open set in Y, then $f^{-1}(G_2) = \langle x, (0.2_a, 0.7_b)\rangle$ is a fuzzy α*- open set in (X, τ) as,

$$\begin{aligned} int(cl(int(f^{-1}(G_2)))) &= int(cl(0_\sim)) \\ &= int\,(0_\sim) \\ &= 0_\sim \end{aligned}$$

and $\quad int(f^{-1}(G_2)) = 0_\sim$

Therefore, $int(cl(int(f^{-1}(G_2)))) = int(f^{-1}(G_2))$ and hence, f is a fuzzy α*- continuous mapping.

But, f is not a fuzzy γ - continuous mapping, as $f^{-1}(G_2)$ is not a fuzzy γ- open set in X, since

$$\begin{aligned} cl\,(int(f^{-1}(G_2))) &= cl(0_\sim) \\ &= 0_\sim \end{aligned}$$

and $\quad int\,(cl(f^{-1}(G_2)) = int(G_1{}^c)$

$$= G_1$$

Therefore, $cl(int(f^{-1}(G_2))) \vee int(cl(f^{-1}(G_2)))$.

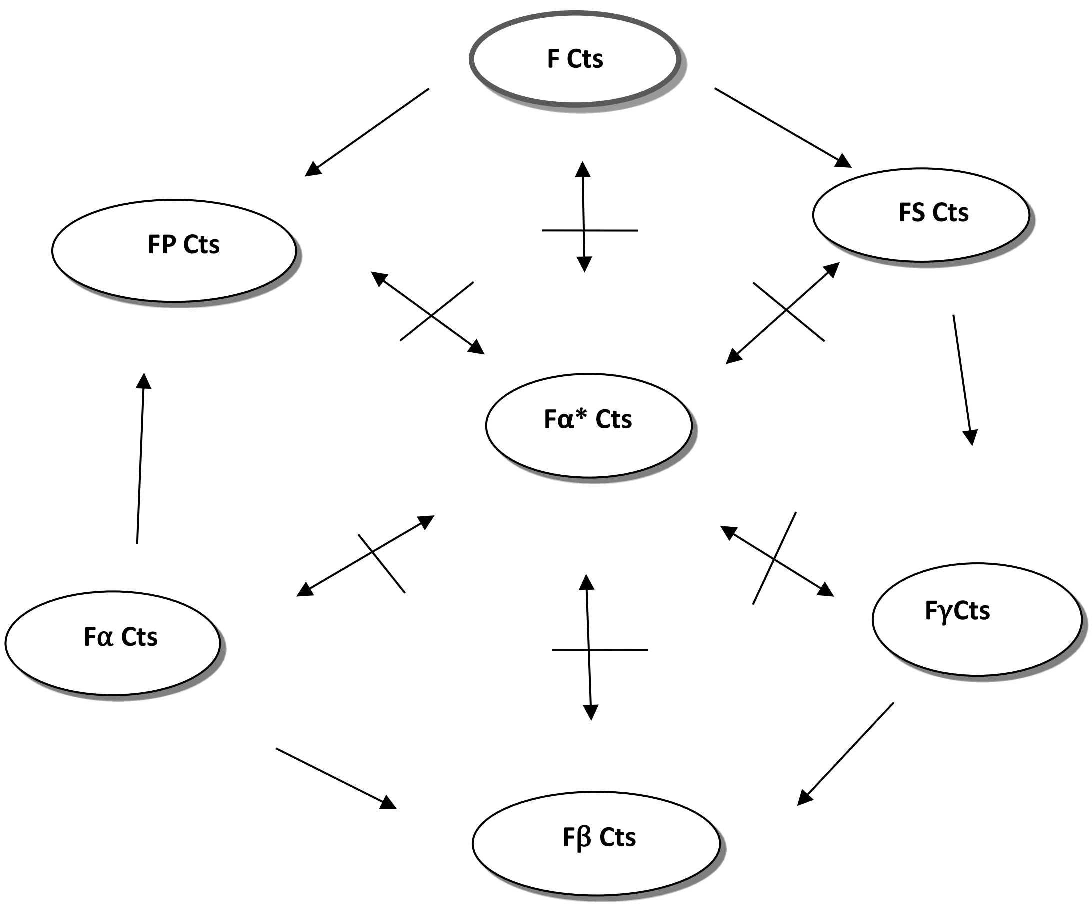
$$\begin{aligned} &= 0_\sim \vee G_1 \\ &= G_1 \end{aligned}$$

and $\quad f^{-1}(G_2) \not\leq G_1$

Therefore, $f^{-1}(G_2) \nleq cl(int(f^{-1}(G_2))) \vee int(cl(f^{-1}(G_2)))$.

Hence, f is a fuzzy α^*- continuous mapping but not a fuzzy γ - continuous mapping.

In the following diagram, the inter-relation between various types of fuzzy continuous mapping with fuzzy α^*- continuous mapping is established.



In this diagram no reverse implications are true in general.

Preposition 2.2.21:

A mapping f: (X, τ) → (Y, σ) is a fuzzy α*- continuous mapping if and only if the inverse image $f^{-1}(A)$ of each fuzzy closed set A in Y is a fuzzy α*- closed set in X.

Proof:

Necessity:

Let A be a fuzzy closed set in Y. This implies A^c is a fuzzy open set in Y. Then $f^{-1}(A^c)$ is fuzzy α*- open set in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a fuzzy α*- closed set in X. Hence f is a fuzzy α*- continuous mapping.

Sufficiency:

Let A be a fuzzy closed set in Y. Then A^c is a fuzzy open set in Y. By hypothesis, $f^{-1}(A^c)$ is fuzzy α*- open set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a fuzzy α*- closed set in X. Hence, f is a fuzzy α*- continuous mapping.

Preposition 2.2.22:

If f: (X, τ) →(Y, σ) is a fuzzy α*- continuous mapping and g: (Y, σ) → (Z, δ) is a fuzzy continuous mapping, then g o f :(X, τ) → (Z, δ) is a fuzzy α*- continuous mapping.

Proof:

Let V be a fuzzy open set in Z. Then $g^{-1}(V)$ is a fuzzy open set in Y, by hypothesis. Since f is a fuzzy α*- continuous mapping. $f^{-1}(g^{-1}(V))$ is a fuzzy α*- open set in X. Hence g o f is a fuzzy α*- continuous mapping.

Remark 2.2.23:

The composition of the two fuzzy α*- continuous mapping need not be a fuzzy α*- continuous mapping in general.

Example 2.2.24:

Let X = {a, b}, Y = {u, v}, and Z = {p, q} and let τ = $\{0_\sim, G_1, 1_\sim\}$ and σ = $\{0_\sim , G_2, 1_\sim\}$ and δ = $\{0_\sim, G_3, 1_\sim\}$ be fuzzy topologieson X ,Y and Z respectively, where $G_1 = \langle x, (0.3_a, 0.6_b)\rangle$, $G_2 = \langle y, (0.3_u, 0.4_v)\rangle$ $G_3 = \langle z, (0.4_p, 0.9_q)\rangle$.Then (X, τ), (Y, σ) and (Z, δ) are fuzzy topological spaces.

Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

Now, consider the fuzzy set $G_2 = \langle y, (0.3_u, 0.4_v)\rangle$ which is a fuzzy open set in Y, then $f^{-1}(G_2) = \langle x, (0.3_a, 0.4_b)\rangle$ is a fuzzy α*- open set in (X, τ) as,

$$int(cl(int(f^{-1}(G_2)))) = int(cl(0_\sim))$$

$$= int\ (0_\sim)$$

$$= 0_\sim$$

and $$int(f^{-1}(G_2)) = 0_\sim$$

Therefore, $int(cl(int(f^{-1}(G_2)))) = int(f^{-1}(G_2))$

Hence, f is a fuzzy α*- continuous mapping.

Define a mapping g: (Y, σ) → (Z, δ) byg(u) = p and g(v) = q.

Now, consider the fuzzy set $G_3 = \langle z, (0.4_p, 0.9_q)\rangle$ which is a fuzzy open set in Z, then $g^{-1}(G_3) = \langle y, (0.4_u, 0.9_v)\rangle$ is a fuzzy α*- open set in (X, τ) as,

$$int(cl(int(g^{-1}(G_3)))) = int(cl(G_2))$$

$$= int\ (G_2^c)$$

$$= G_2$$

and $$int(g^{-1}(G_3)) = G_2$$

Therefore, $int(cl(int(g^{-1}(G_3)))) = int(g^{-1}(G_3))$

Hence, g is a fuzzy α*- continuous mappings

Define a mapping g o f: (X, τ) → (Z, δ) by g(f(a)) = p and g(f(b)) = q which is a composite map of f and g.

Now, consider the fuzzy set $G_3 = \langle z, (0.4_p, 0.9_q)\rangle$ which is a fuzzy open set in Z, then $(g \circ f)^{-1}(G_3) = \langle x, (0.4_a, 0.9_b)\rangle$ is not a fuzzy α*- open set in (X, τ) as,

$$\begin{aligned} int(cl(int((g \circ f)^{-1}(G_3)))) &= int(cl(G_1)) \\ &= int\,(1_\sim) \\ &= 1_\sim \end{aligned}$$

and $int((g \circ f)^{-1}(G_3)) = G_1$

Therefore, $int(cl(int((g \circ f)^{-1}(G_3)))) \neq int((g \circ f)^{-1}(G_3))$.

Hence, g o f is not a fuzzy α*- continuous mapping and we conclude that the composition of two fuzzy α*- continuous mapping need not be a fuzzy α* - continuous mapping.

Proposition 2.2.25:

Let f: (X, τ) → (Y, σ) be a mapping from a fuzzy topological space X into a fuzzy topological space Y that satisfies $f^{-1}(B) = int(cl(int(f^{-1}(B))))$ for every B in Y, then f is a fuzzy α*- continuous mapping.

Proof:

Let B be a fuzzy open set in Y, then int(B) = B.

By hypothesis,

$$\begin{aligned} f^{-1}(int(B)) &= f^{-1}(B) \\ &= int(cl(int(f^{-1}(int(B))))) \end{aligned}$$

Therefore, $f^{-1}(B) = int(cl(int(f^{-1}(B)))$.

This implies $f^{-1}(B)$ is a fuzzy α* - open set in X.

Hence, f is a fuzzy α* - continuous mapping.

SUMMARY AND CONCLUSION

SUMMARY AND CONCLUSION

In **Chapter 1**, the preliminary definitions are discussed.

In **chapter 2,** we have introduced fuzzy α*-continuous mapping and made an attempt tocompare fuzzy α*- continuous mapping in fuzzy topological spaces with some of the existing fuzzy continuous mappings such as fuzzy continuous mapping, fuzzy pre-continuous mapping, fuzzy semi-continuous mapping, fuzzy α-continuous mapping, fuzzy β-continuous mapping, fuzzy γ-continuous mapping.

For future study, one can apply our results for the comparison of various fuzzy mappings indifferent number intervals especially through numerical examples. It can be extended to a next levelof connectedness and etc., I have done this thesis work wholeheartedly and of course I hope it will be satisfactory.

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