# Chapter IX

# CHAPTER – IX

# APPLICATION OF TRIANGULAR FUZZY NUMBERS IN SOLVING FUZZY LINEAR PROGRAMMING PROBLEM

In standard arithmetic operators, the addition and subtraction (resp. multiplication and division) of fuzzy numbers are not reciprocal operations. According to this statement, it is not possible to solve inverse problems exactly using the standard fuzzy arithmetic operators. To overcome this in function principle operation of triangular fuzzy number a new operation is proposed by Nagoor Gani, A and Mohamed Assarudeen, S.N. [39] that allows exact inversion.

These new fuzzy arithmetic operations on triangular fuzzy numbers and its properties are discussed in this chapter with examples. An application of this operator is also discussed.

For example consider the two triangular fuzzy numbers $\tilde{A}$ = (2, 4, 6) and $\tilde{B}$ = (1, 2, 3). Then

(i) $\tilde{A} + \tilde{B} = (3, 6, 9)$

(ii) $\tilde{A} - \tilde{B} = (-1, 2, 5)$

(iii) $\tilde{A} \times \tilde{B} = (2, 8, 18)$

(iv) $\frac{\tilde{A}}{\tilde{B}} = \left(\frac{2}{3}, \frac{4}{2}, \frac{6}{1}\right) = (0.66, 2, 6)$

(v) $\tilde{A} - \tilde{A} = (-4, 0, 4)$

(vi) $\frac{\tilde{A}}{\tilde{A}} = \left(\frac{2}{6}, \frac{4}{4}, \frac{6}{2}\right) = (0.33, 1, 3)$

Here, $\tilde{A} - \tilde{A} \neq 0$, $\frac{\tilde{A}}{\tilde{A}} \neq 1$, where 0 and 1 are singletons whose fuzzy representation is (0, 0, 0) and (1, 1, 1). It is follows that the $\tilde{C}$ solution of the fuzzy linear equation $\tilde{A} + \tilde{B} = \tilde{C}$ is not as we would expect, $\tilde{B} = \tilde{C} - \tilde{A}$.

For example, $\tilde{A} + \tilde{B} = (2, 4, 6) + (1, 2, 3) = (3, 6, 9) = \tilde{C}$

But (1, 2, 3) = (3, 6, 9) – (2, 4, 6) = (– 3, 2, 6) $\neq \tilde{B}$.

The same annoyance appears when solving the fuzzy equation $\tilde{A} \times \tilde{B} = \tilde{C}$ whose solution is not given by $\tilde{B} = \dfrac{\tilde{C}}{\tilde{A}}$

For example, $\tilde{A} \times \tilde{B}$ = (2, 8, 18) = $\tilde{C}$

But $\tilde{B} = \dfrac{(2,8,18)}{(2,4,6)} = \left(\dfrac{2}{6}, \dfrac{8}{4}, \dfrac{18}{2}\right)$ = (0.33, 2, 9) $\neq \tilde{B}$.

Therefore, the addition and subtraction (resp. multiplication and division) of fuzzy numbers are not reciprocal operations. According to this statement, it is not possible to solve inverse problems exactly using the standard fuzzy arithmetic operators.

To overcome this in function principle operation of triangular fuzzy number a new operation is proposed that allows exact inversion.

**New Operation for Subtraction and Division on Triangular Fuzzy Number**

**Definition : 9.1**

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ then, $\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$. The new subtraction operation exist only if the following condition is satisfied $DP(\tilde{A}) \geq DP(\tilde{B})$ where $DP(\tilde{A}) = \dfrac{a_3 - a_1}{2}$ and $DP(\tilde{B}) = \dfrac{b_3 - b_1}{2}$ where DP denotes difference point of a Triangular fuzzy number.

**Properties of Subtraction Operator**

**Definition : 9.2**

(i) Inverse operator of + : $\tilde{B} + (\tilde{A} - \tilde{B}) = (\tilde{A} - \tilde{B}) + \tilde{B}$

(ii) Multiplication by a scalar : $w(\tilde{A} - \tilde{B}) = (w\tilde{A} - w\tilde{B})$

(iii) Neutral element : $(\tilde{A} - 0) = \tilde{A}$

(iv) Associativity : $\tilde{A} - (\tilde{B} - \tilde{C}) = (\tilde{A} - \tilde{B}) - \tilde{C}$

(v) Inverse element : Any Fuzzy Number is its own inverse under the modified subtraction.

i.e., $\tilde{A} - \tilde{A} = 0$.

(vi) Regularity : $\tilde{A} - \tilde{B} = \tilde{A} - \tilde{C} \Rightarrow \tilde{B} = \tilde{C}$

(vii) Psedu-distributivity with regard to + :

$$(\tilde{A} + \tilde{B}) - (\tilde{C} + \tilde{D}) = (\tilde{A} - \tilde{C}) + (\tilde{B} - \tilde{D})$$

**Definition : 9.3**

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ then, $\frac{\tilde{A}}{\tilde{B}} = \left(\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}\right)$. The new division operation exists only if the following conditions are satisfied $\left|\frac{DP(\tilde{A})}{MP(\tilde{A})}\right| \geq \left|\frac{DP(\tilde{B})}{MP(\tilde{B})}\right|$ and the negative triangular fuzzy number should be changed into negative multiplication of positive number.

Where $MP(\tilde{A}) = \frac{a_3 + a_1}{2}$ ; $DP(\tilde{A}) = \frac{a_3 - a_1}{2}$ ;

$MP(\tilde{B}) = \frac{b_3 + b_1}{2}$ and $DP(\tilde{B}) = \frac{b_3 - b_1}{2}$ ; where MP denotes midpoint and DP denotes difference point of a triangular fuzzy number.

**Properties of Division Operator**

**Definition : 9.4**

(i) Inverse operator of x : $\tilde{B} \times \frac{\tilde{A}}{\tilde{B}} = \frac{\tilde{A}}{\tilde{B}} \times \tilde{B} = \tilde{A}$

(ii) Neutral element : The singleton $\tilde{1} = (1, 1, 1)$ defined by a constant profile equal to $\tilde{1}$ is a right neutral element of division.

$$\frac{\tilde{A}}{\tilde{1}} = \left(\frac{a_1}{1}, \frac{a_2}{1}, \frac{a_3}{1}\right) = (a_1, a_2, a_3) = \tilde{A}.$$

(iii) Inverse element : Any fuzzy number is its own inverse under the modified division operator

$$\frac{\tilde{A}}{\tilde{A}} = \left(\frac{a_1}{a_1}, \frac{a_2}{a_2}, \frac{a_3}{a_3}\right) = (1, 1, 1)$$

(iv) Regularity : $\frac{\tilde{A}}{\tilde{B}} = \frac{\tilde{A}}{\tilde{C}} \Rightarrow \tilde{B} = \tilde{C}$.

(v) Distributivity with regard to + : $\frac{(\tilde{A}+\tilde{B})}{\tilde{C}} = \frac{\tilde{A}}{\tilde{C}} + \frac{\tilde{B}}{\tilde{C}}$

**Example : 9.5**

**(a)** $\tilde{A}$ = (5, 6, 7)

$$MP(\tilde{A}) = \frac{a_3 + a_1}{2} = \frac{7+5}{2} = 6;$$

$$DP(\tilde{A}) = \frac{a_3 - a_1}{2} = \frac{7-5}{2} = 1.$$

**(i) Subtraction**

Now $DP(\tilde{A})$ = 1 therefore $DP(\tilde{A}) = DP(\tilde{A})$. Hence $\tilde{A}-\tilde{A}$ is satisfying the condition $\tilde{A} - \tilde{A} = (a_1 - a_1, a_2 - a_2, a_3 - a_3)$ = (5 – 5, 6 – 6, 7 – 7) = (0, 0, 0)

**(ii) Division**

Now $\left|\frac{DP(\tilde{A})}{MP(\tilde{A})}\right| = \left|\frac{1}{6}\right|$ = 0.167 therefore $\left|\frac{DP(\tilde{A})}{MP(\tilde{A})}\right| = \left|\frac{DP(\tilde{A})}{MP(\tilde{A})}\right|$. Hence $\frac{\tilde{A}}{\tilde{A}}$ satisfying the above condition. $\frac{\tilde{A}}{\tilde{A}} = \left(\frac{5}{5}, \frac{6}{6}, \frac{7}{7}\right)$ = (1, 1, 1)

**(b)** $\tilde{B}$ = (– 5, – 3, – 2)

$$MP(\tilde{B}) = \frac{a_3 + a_1}{2} = \frac{-2-5}{2} = -3.5;$$

$$DP(\tilde{B}) = \frac{a_3 - a_1}{2} = \frac{-2+5}{2} = 1.5;$$

**(i) Subtraction**

Now $DP(\tilde{B})$ = 1.5 therefore $DP(\tilde{B}) = DP(\tilde{B})$. Hence $\tilde{B} - \tilde{B}$ satisfying the condition $\tilde{B} - \tilde{B}$ = (– 5 + 5, – 3 + 3, – 2 + 2) = (0, 0, 0).

**(ii) Division**

Now $\left|\frac{DP(\tilde{B})}{MP(\tilde{B})}\right| = \left|\frac{1.5}{-3.5}\right|$ = 0.428 therefore $\left|\frac{DP(\tilde{B})}{MP(\tilde{B})}\right| = \left|\frac{DP(\tilde{B})}{MP(\tilde{B})}\right|$.

Hence $\frac{\tilde{B}}{\tilde{B}}$ satisfying the condition. $\frac{\tilde{A}}{\tilde{A}} = \left(\frac{-5}{-5}, \frac{-3}{-3}, \frac{-2}{-2}\right)$ = (1, 1, 1).

## 9.6 Application of the New Operators

The new operators are used to solve the following fuzzy linear programming problem using simplex algorithm :

Max $\tilde{z}$ = (5, 7, 9) $\tilde{x}_1$ + (6, 8, 10) $\tilde{x}_2$

Subject to constraint

(1, 2, 3) $\tilde{x}_1$ + (2, 3, 4) $\tilde{x}_2$ $\leq$ (4, 6, 8)

(4, 5, 6) $\tilde{x}_1$ + (3, 4, 5) $\tilde{x}_2$ $\leq$ (8, 10, 12)

**Solution**

The standard form is

Max $\tilde{z}$ = (5, 7, 9) $\tilde{x}_1$+ (6, 8, 10) $\tilde{x}_2$ + (0, 0, 0) $\tilde{x}_3$ + (0, 0, 0) $\tilde{x}_4$

Subject to constraint

(1, 2, 3) $\tilde{x}_1$ + (2, 3, 4) $\tilde{x}_2$ + (1, 1, 1) $\tilde{x}_3$ = (4, 6, 8)

(4, 5, 6) $\tilde{x}_1$ + (3, 4, 5) $\tilde{x}_2$+ (1, 1, 1) $\tilde{x}_4$ = (8, 10, 12)

**Simplex Table**

| | $\tilde{x}_1$ | $\tilde{x}_2$ | $\tilde{x}_3$ | $\tilde{x}_4$ | RHS |
|---|---|---|---|---|---|
| $\tilde{x}_3$ | (1, 2, 3) | **(2, 3, 4)** | (1, 1, 1) | (0, 0, 0) | (4, 6, 8) |
| $\tilde{x}_4$ | (4, 5, 6) | (3, 4, 5) | (0, 0, 0) | (1, 1, 1) | (8, 10, 12) |
| $\tilde{z}$ | – (5, 7, 9) | – (6, 8, 10) | (0, 0, 0) | (0, 0, 0) | (0, 0, 0) |
| $\tilde{x}_2$ | **(0.5, 0.66, 0.75)** | (1, 1, 1) | (0.25, 0.33, 0.5) | (0, 0, 0) | (1, 2, 4) |
| $\tilde{x}_4$ | (0.25, 2.36, 4.5) | (1, 1, 1) | – (0.75, 1.32, 2.5) | (1, 1, 1) | (– 12, 2, 9) |
| $\tilde{z}$ | (– 2, – 1.72, – 1.5) | (0, 0, 0) | (1.5, 2.64, 5) | (0, 0, 0) | (6, 16, 40) |
| $\tilde{x}_1$ | (1, 1, 1) | (1.33, 1.52, 2) | (0.5, 0.5, 0.66) | (0, 0, 0) | (1.33, 3.03, 8) |
| $\tilde{x}_4$ | (0, 0, 0) | – (0.33, 3.59, 9) | – (0.88, 2.5, 5.47) | (1, 1, 1) | (– 48, – 5.15, 8.67) |
| $\tilde{z}$ | (0, 0, 0) | (1.99, 2.61, 4) | (2.25, 3.5, 18.86) | (0, 0, 0) | (7.99, 21.21, 56) |

Hence $\tilde{x}_1$ = (1.33, 3.03, 8) and

$\tilde{x}_2$ = (0, 0, 0) then

$\tilde{z}$ = (7.99, 21.21, 56).