
Review of Literature

1. Fuzzy Sets and their generalizations:

More often than not, the classes of objects encountered in the real physical world do not have a precisely defined criteria of membership. This ambiguity of objects of the classes gave birth to a concept which may be of use in dealing with these imprecise classes of objects. Yet, the fact remains that such imprecisely defined classes play an important role in human thinking, particularly in the domains of pattern recognition, communication of information and abstraction. Among the various paradigmatic changes in Science and Mathematics, one such change concerns the concept of uncertainty. Identification of this important role of uncertainty by some researchers began the stage of transition from the traditional view to modern view of uncertainty and such transition is characterized by the emergence of several new theories of uncertainty from probability theory.

An important point in the evaluation of the modern concept of uncertainty was the paper by Zadeh [73]. In his paper, Zadeh introduced a theory whose objects - fuzzy sets, are sets with boundaries that are not precise. The membership in a fuzzy is not a matter of affirmation or denial, but rather a matter of degree. This concept is being used and is also found to be more appropriate in solving problems of all disciplines such as Economics, Expert System, Artificial Intelligence, Computer Science, Control Engineering, Psychology, Management Science, Operations Research, Pattern recognition, Robotics, Washing Machines, Topological Spaces, Algebraic Structures etc.,

In the classical set theory a subset A of a set X can be defined by its characteristic function $\chi_A : X \rightarrow \{0,1\}$ such that $\chi_A(x) = 0$, if $x \notin A$ and $\chi_A(x) = 1$, if $x \in A$. The mapping may be represented by a set of ordered pairs $\{(x, \chi_A(x)) \mid x \in A\}$ with exactly one ordered pair corresponding to each element of $A \subseteq X$. The first element of the ordered pair is an element of the set X and the second is its value in $\{0,1\}$. The value 0 is used to represent a non-membership and the value 1 is used to represent the membership of the element in $A \subseteq X$. The second element in the ordered pair is 1 if the statement is true and is 0 if it is false.

On the other hand, a fuzzy set A of a set X can be defined as a set of ordered pairs $\{(x, \mu_A(x)) \mid x \in X, \mu_A(x) \in [0,1]\}$. This defines a mapping $\mu_A : X \rightarrow [0,1]$. The value 0 is used for representing the complete non-membership and 1 is used for complete membership and values in between are used to represent the intermediate membership values of the elements in X . The set X is referred to as the universe of discourse for the fuzzy set A . Frequently the mapping μ_A is described as a membership function of A , the degree to which the statement x is in A is true, determined by the value $\mu_A(x)$.

Goguen [21] generalized the notion of fuzzy sets into the notion of L -fuzzy sets. An L -fuzzy set, A on X is a mapping from $A : X \rightarrow L$, where L is a complete lattice. In 1986, Atanassov [8] introduced the notion of intuitionistic fuzzy set as an extension of fuzzy set which not only a membership degree is given, but also a non-membership degree is involved. After the concept of intuitionistic fuzzy set was introduced, several papers have been published by mathematicians to extend the classical mathematical concepts and fuzzy mathematical concepts to the case of intuitionistic fuzzy mathematics. After a long research, Atanassov and Stoeva [11] introduced the concept of Intuitionistic L -fuzzy sets in 1984. Here he defined both membership and non-membership functions from the Universe of discourse X to the set L , where L is a complete lattice.

The notion of interval-valued fuzzy sets was first introduced by Zadeh [74] in 1975 as an extension of fuzzy sets. An interval-valued fuzzy set is a fuzzy set whose membership function is many-valued and forms an interval in the membership scale. This idea gives the simplest method to capture the imprecision of the membership grade for a fuzzy set.

In 1989, Atanassov and Gargov [10] combined intuitionistic fuzzy set with interval-valued fuzzy set and introduced the notion of interval-valued intuitionistic fuzzy set whose membership degree and non-membership degree were intervals rather than real numbers.

Moreover, Jun et al. [36] introduced the concept of cubic sets, as a generalization of fuzzy set and interval-valued fuzzy set. Jun et al. [34] applied the notion of cubic sets to a group and introduced the notion of cubic subgroups.

2. Algebra related with Propositional Calculi:

Algebraic structures play an important role in Mathematics with wide range of applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory etc. Several algebraic structure have been introduced in the recent past. In 1966, Imai and Iseki [25,26] introduced two classes of abstract algebras: BCK-algebras and BCI-algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In 1983, Hu and Li [24] introduced a wide class of abstract algebra: BCH-algebras. They have shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. Neggers et al.[59,58,60] introduced the notions of d-algebras in 1999, Q-algebras in 2001 and B-algebras in 2002 as a generalization of BCK/BCI-algebras. In 2010, Megalai and Tamilarasi [47] introduced TM-algebra. During 2011, Keawrahun and Leerawat [39] introduced a new structured algebra called SU-algebra. Prabayah and Leerawat [63] discussed ideals and congruences in KU-algebras. With all these ideas, Chandramouleeswaran et al. [18] introduced the notion of Z-algebras which is not a generalization of BCK/BCI-algebras.

3. Fuzzy Algebraic Structures:

The study of fuzzy sets and their application to mathematical concepts has reached to what is commonly called as fuzzy mathematics. Fuzzy algebra is an important branch of fuzzy mathematics. The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy subgroups in 1971 by Rosenfeld [65]. During 1985, Bhattacharya and Mukherji [13] introduced fuzzy structure in the form of fuzzy relations and fuzzy groups. In 1991, Xi [71] applied the concept of fuzzy sets to BCK algebras. In 1992, Meng and Xin [48] introduced the concept of implicative ideals in BCI-algebras. In 1994, Jun and Meng[32] introduced fuzzy p-ideals in BCI-algebras. In 1997, Meng et al. [49] and Mostafa [50] fuzzified the concept of implicative ideals in BCK-algebras, independently. In 1999, Jun et al.[33,38] studied the concepts of fuzzy ideals, fuzzy implicative ideals and constructed a fuzzy characteristic implicative ideal in BCK-algebras. In the same year, Khalid and Ahmad[40] introduced fuzzy H-ideals in BCI-algebras. In 2004, Fuzzy BG-subalgebras of BG-algebras

were developed by Ahn and Lee [7]. Abu Ayub Ansari and Chandramouleeswaran [2,3] investigated fuzzy β -Subalgebras of β -algebras in 2013 and fuzzy β -ideals in β -algebras in 2014. Christopher Jefferson and Chandramouleeswaran [19] applied fuzzy algebraic structures in BP-algebras in 2015. In 2016, Abbass and Neamah [1] discussed fuzzy implicative ideal of a BH-algebra and gave some properties of this fuzzy ideal and link it with other types of fuzzy ideals of a BH-algebra. In 2018, Mostafa and Ola Wageeh Abd El-Baseer[51a] introduced the notion of fuzzy implicative ideals of KU-algebras and several basic properties are investigated. In 2019, Pongsumpao et al.[62a] studied the notions of fuzzy UP-subalgebras and fuzzy UP-ideals of UP-algebras in terms of level subsets of a fuzzy set and investigated some of their properties.

In 2009, the concept of fuzzy translations and fuzzy multiplications of fuzzy Subalgebras in BCK/ BCI-algebras have been discussed by Lee et al.[44]. In 2013, similar concept have been discussed in BF-algebras by Chandramouleeswaran et al.[17]. In 2014, Abu Ayub Ansari and Chandramouleeswaran [4,5] discussed the concepts of fuzzy translations, fuzzy extensions and fuzzy multiplications of fuzzy β -subalgebras and fuzzy β -ideals in β -algebras. In 2016, Sithar Selvam and Nagalakshmi [69] discussed the notions of homomorphism and cartesian product on fuzzy α -translations and fuzzy α -multiplications of PMS-algebras. In 2017, the ideas of fuzzy translation and fuzzy multiplication in CI-algebras have been discussed by Hameed and Mohammed [23]. In the same year, Hameed and Alkurdi [22] investigated fuzzy translation, fuzzy extension and fuzzy multiplication of fuzzy QS-subalgebras and fuzzy QS-ideals in QS-algebras. In 2020, Prasanna et al. [62b] investigated some algebraic properties of ω -fuzzy translations and ω -fuzzy multiplications in BP-algebras.

In 1986, as a generalization of Zadeh's fuzzy set, the notion of intuitionistic fuzzy set was introduced by Atanassov[8]. After the concept of intuitionistic fuzzy set was introduced, several articles have been published by mathematicians to extend the classical mathematical concepts and fuzzy mathematical concepts to the case of intuitionistic fuzzy mathematics. In 2000, Jun and Kim [31] established the intuitionistic fuzzification of the concepts of subalgebras and ideals in BCK-algebras. In 2005, Zarandi and Borumand Saeid [76] studied the intuitionistic fuzzification of the concept of subalgebras and ideals of BG-algebras.

In 2011, Mostafa et al.[51] established the concepts of intuitionistic fuzzification of KU-ideals and the image (pre-image) of KU-ideals in KU-algebras. In 2012, Peng [62] introduced the notion of intuitionistic fuzzy subalgebras of B-algebras which is related to several classes of algebras such as BCK/BCI-algebras. In 2015, Jana et al. [29] studied the intuitionistic fuzzification of G-Subalgebras in G-algebras.

In 1967, Goguen [21] generalized the notion of fuzzy sets into the notion of L-fuzzy sets where L is a complete lattice. After a long research, Atanassov and Stoeva [11] introduced the concept of Intuitionistic L-fuzzy sets in 1984. Muralikrishna and Chandramouleeswaran [52,53,54] studied intuitionistic L-fuzzy Subalgebras / L-fuzzy ideals of BF/BG-algebras in 2010 and in 2011, they [55,56] studied Cartesian product of intuitionistic L-fuzzy BF-Ideals in BF-algebras and the images(inverse images) of intuitionistic L-fuzzy BF-ideals under BF-homomorphism of BF-algebras. In 2017, Christopher Jefferson and Chandramouleeswaran [19a] investigated the notion of intuitionistic L-fuzzy structures on BP-algebras.

Extending the concept of fuzzy sets, many scholars introduced various notions of higher order fuzzy sets. Among them, interval-valued fuzzy sets provides a flexible mathematical framework to cope with imperfect and imprecise information. In 1994, Biswas[15] defined interval-valued fuzzy subgroups (that is, i-v fuzzy subgroups) of Rosenfeld's nature and investigated some elementary properties. In 2000, Jun [30] defined the notion of an interval-valued fuzzy subalgebras (ideals) of a BCK-algebra and studied their properties. In 2006, Borumand Saeid [16] introduced the notion of interval-valued fuzzy B-Subalgebras, the level and strong level B-subalgebras of a B-algebra and studied some of their properties. Zarandi and Borumand Saeid [75] introduced the notion of interval-valued fuzzy BF-Subalgebras in 2009 and investigated their properties. In 2018, Kaviyarasu et al.[38a] introduced the concept of interval-valued fuzzy INK-ideals in INK-algebras and studied some of their properties.

In 2012, using a fuzzy set and an interval-valued fuzzy set, Jun et al. [36] introduced a new notion called a cubic set and investigated several properties. Meanwhile, in 2010, Jun et al.[35] introduced the notion of cubic subalgebras/cubic ideals in BCK/BCI-algebras and then they investigated several properties and in 2011, Jun et al.[34]

applied the notion of cubic sets to a group and introduced the notion of cubic subgroup. Subsequently, the theory of cubic sets attracted by several mathematicians. In 2013, Yaqoob [72] defined the notion of cubic KU-ideals of KU-algebras and studied the homomorphic image and inverse image of cubic KU-ideals of KU-algebras. In 2015, Senapati et al. [68] introduced the concept of cubic set to subalgebras, ideals and closed ideal of B-algebras. In 2018, Hameed and Badia Hassan Hadi[22a] introduced the notions of cubic AT-ideals and cubic AT-subalgebras in AT-algebras and several properties are investigated. In 2019, El-Gendy[20a] introduced the notions of cubic BOI-subalgebras and cubic BOI-ideals in BOI-algebras and their properties are investigated.

As a generalization of both intuitionistic fuzzy sets and interval-valued fuzzy sets, Atanassov and Gargov [10] introduced the concept of interval-valued intuitionistic fuzzy set in 1989. In 2012, Satyanarayana et al.[66] applied the concept of interval-valued intuitionistic fuzzy ideals to BF-algebras and investigated some interesting properties. In 2012, Senapati et al.[67] introduced and studied the concept of interval-valued intuitionistic fuzzy subalgebras of BG-algebra. In 2015, Narciso C.Gonzaga Jr and Jocelyn P.Vilela [57] investigated the concept of interval-valued intuitionistic fuzzy B-Subalgebras of a B-algebra and some of their properties are explored. In 2018, Indhira et al. [25a] introduced and studied the concept of interval-valued intuitionistic fuzzy INK-ideals in INK-algebras. In the same year, Hemavathi et al. [22b] introduced the notion of interval valued intuitionistic fuzzy subalgebras of β -algebra and investigated some of the related results.

Motivated by the study of algebras arising from propositional calculus and the theory of fuzzy sets in our work, we have studied the fuzzy structures in Z-algebras : fuzzy Z-Subalgebras, fuzzy Z-ideals, fuzzy H-ideals, fuzzy p-ideals, fuzzy implicative ideals, fuzzy sub-implicative ideals, fuzzy α -translations, fuzzy β -multiplications, intuitionistic fuzzy Z-Subalgebras, intuitionistic fuzzy Z-ideals, intuitionistic L-fuzzy Z-Subalgebras, intuitionistic L-fuzzy Z-ideals, interval-valued fuzzy Z-Subalgebras, interval-valued fuzzy Z-ideals, interval-valued intuitionistic fuzzy Z-Subalgebras , interval-valued intuitionistic fuzzy Z-ideals, cubic Z-Subalgebras and cubic Z-ideals in Z-algebras.