Introduction

Topology is broadly considered to be a part of Mathematics through the examination started by the incredible mathematician Henri Poincare in the nineteenth century. Topology has been created as a field of study with the rudimentary knowledge of geometry and set theory. The topological structures are appropriate for the quantitative information as well as for the subjective information. So the ideas of sets and capacities in topological spaces are exceptionally evolved and utilized in numerous pure and applied mathematics.

General topology normally considers local properties of spaces and is closely related to analysis. It generalizes the concept of continuity to define topological spaces, in which limits of sequences can be considered.

In the fields of Computer Aided Geometric Design and Engineering (briefly, CAGD), Information systems, Artificial Intelligence and Image processing, one can visualize the existence of topological structures and the usage of topological properties. The concept of a topological space is concerned with generalizing the structure of sets in Euclidean spaces. Having Open sets as a powerful tool for defining topological spaces, Stone (1937) defined the notion of Regular open sets in his novel paper which related the theory of Boolean Rings to General Topology.

The concept of closed sets is a fundamental object in general topology.In general topology repeated applications of interior and closure operators give rise to several different kinds of new classes of sets. Levine (1970) initiated the study of generalized closed sets in order to extend many of the important properties of closed sets to a larger family. Moreover, he characterized them as well as determines their behavior relative to unions, intersections and subspaces. Also it was shown that compactness, normality and completeness in a uniform space are inherited by g-closed subsets. He defined that the complement of g-closed set is a g-open set. Then the images and inverse images of g-closed and g-open sets under continuous closed transformations were explored by him. Dunham (1982) has established a generalized closure using Levine's generalized closed sets as Cl*.

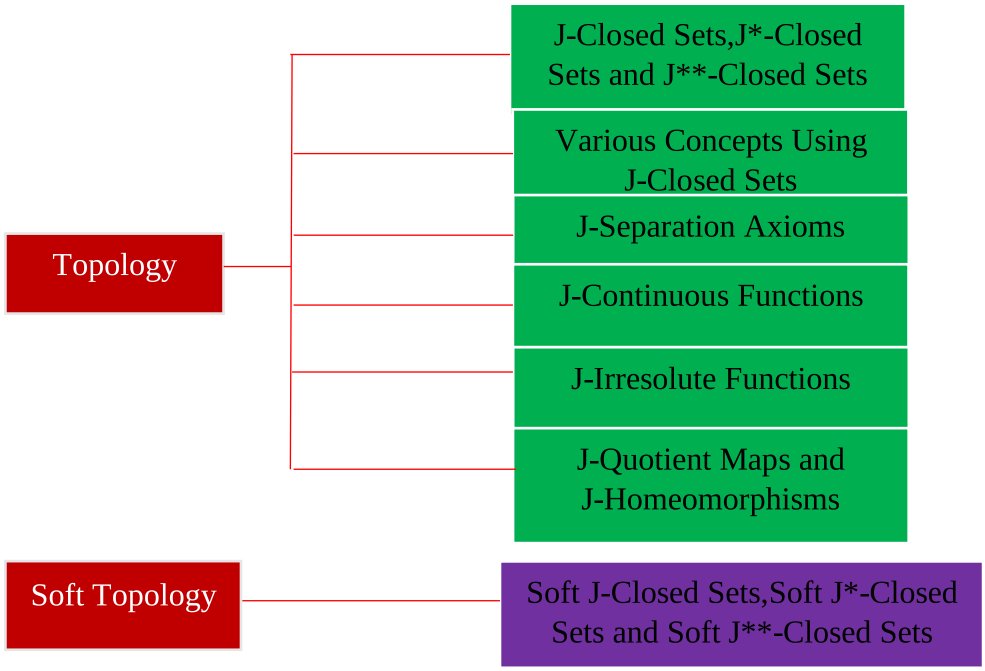
The notion of continuity is an important concept in general topology as well as all branches of mathematics of course its weak forms and strong forms of continuity are important,too.

In 1972, Crossley and Hildebrand presented the thought of irresoluteness. Different fascinating issues emerge when one thinks about irresoluteness.

In the year 1999, Molodtsov initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainty problems. Soft systems provide a general framework with the involvement of parameters. In recent years the development in the field of soft set theory and its application has been taking place in a rapid pace. In 2011, Shabir and Naz introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. In 2012, Kannan introduced soft g-closed sets in soft topological spaces. Intensive research on the field of soft g-closed sets was done as the theory developed by many mathematicians in soft topological spaces in the successive years.

The endeavor of the present work is to initiate the concept of η*-open sets using regular*-open sets introduced by Annalakshmi (2016) and to introduce the three types of g-closed sets namely J-closed sets, J*-closed sets and J-closed sets in topological spaces, to investigate their relations with other similar concepts, to study the properties of the newly introduced concepts and to characterize them in various spaces.**

The deliberations in the research work include the following topics.



OBJECTIVES:

- To define η*-closure operator using Cl* operator.
- To introduce new g-closed sets using η*-closure operator.
- To enhance the characterizations of J, J* and J**-closed sets in various spaces.
- To extend the theory of J, J* and J**-closed sets to soft topology.

METHODOLOGY:

The study of J, J* and J**-closed sets has been done by the following methods.

- Analytical method of comparing J, J* and J**-closed sets with other existing g-closed sets.
- Obtaining counter examples wherever necessary to substantiate the result.
- Interpreting the results as diagrams.
- Analysis of preservation of topological properties by J,J* and J**-closed sets.
- Obtaining Characterization theorems in many spaces like $T_{1/2}$, almost weakly Hausdorff, T_δ, semi-regular, T_b, αT_b, partition space, R_1 -spaces respectively.

NOTIONS:

Throughout the thesis, the following notations are used.

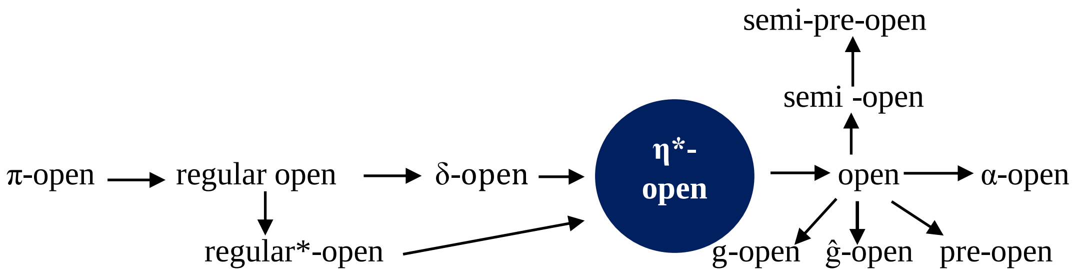
- ✓ (Y,ζ), (Z,σ) and (P,μ) denote non-empty topological spaces on which no separation axioms are mentioned, unless it is stated specifically.
- ✓ The triplet (Y,ζ,E) is a soft topological space over Y.
- ✓ In all the diagrams, A $\rightarrow$ B represents A implies B but not conversely and A $\nleftrightarrow$ B represents A and B are independent of each other.

Chapter I:

Chapter I deals with the study of the preliminary definitions and results in topological spaces and soft topological spaces which are used to accomplish the research work.

Chapter II:

In **Chapter II**, the concept of a class of new sets namely η*-open sets which is placed between the classes of δ-open set and open set. The basic properties are procured and the concepts of η*-cluster point, η*-adherent point and a η*-derived set are introduced and studied. The following relation is established.



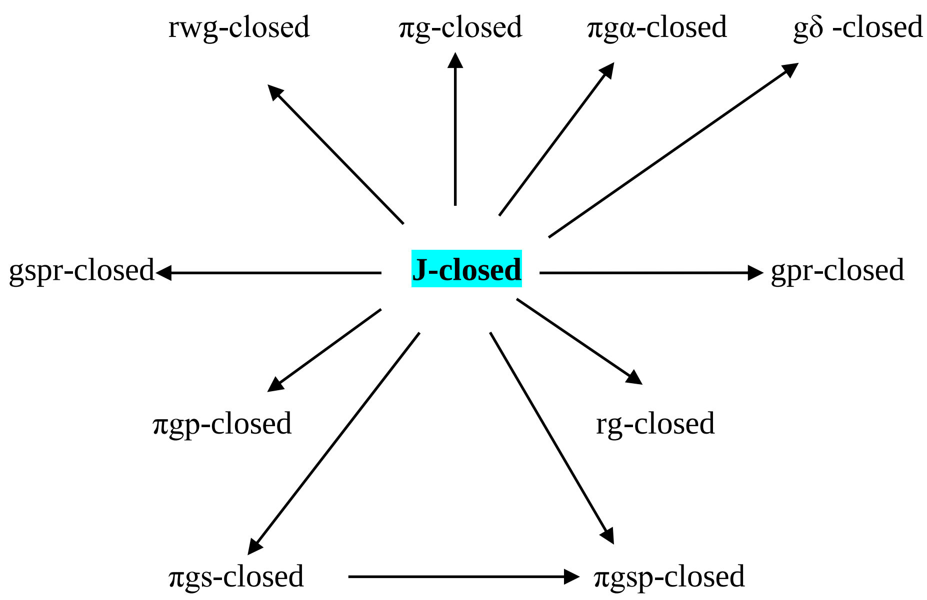
In this Chapter, a new class of sets namely J-closed sets is initiated using η*-open sets in topological spaces. This new concept is weaker than closedness and infact it is weaker than g-closedness but stronger than gδ-closedness.

The following definitions are introduced in this chapter:

- A subset D of a topological space (Y,ζ) is called **η*-open set** if it is a union of regular*-open sets (r*-open sets).We denote the set of all η*-open sets in (Y,ζ) by η*O(Y,ζ) or η*O(Y).
- The complement of a η*-open set is called a **η*-closed set**. We denote η*-closed sets in (Y,ζ) by η*C(Y,ζ) or η*C(Y).
- For a subset D of a topological space (Y,ζ), **η*-Interior of D** is the union of all η*-open sets of Y contained in D. We denote by the symbol η*-Int(D).
- The intersection of all η*-closed sets of Y containing D is called as the **η*-Closure of D** (η*-Cl(D)).
- The intersection of all η*-open subsets of Y containing D is called the **η*- kernel of D** and is denoted by η*-ker(D).
- Let C be a subset of the topological space (Y,ζ),then
- an element x∈Y is called a **η*-adherent point of C** if every η*-open set in Y containing x intersects C.
- a point x∈Y is called a **η*-cluster point of C** if for every η*-open set V containing x intersects C in a point different from x.
- the set of all η*-cluster points of C is denoted by **η*-D(C)**.
- the subset M of C is **η*-open in C** if C = M ∩ V where V is η*-open in Y.
- for a subset M of C, **η*-Closure of M in C** denoted by $\eta^*\text{-}Cl_C(M)$ = η*-Cl(M) ∩ C.

- A subset D of a topological space (Y,ζ) is said to be
  - **J-closed set** if Cl(D)$\subseteq$M whenever D$\subseteq$M, where M is η*-open in (Y,ζ). The class of all J -closed sets of (Y,ζ) is denoted by JC(Y,ζ).
  - **J-open** if its complement D^c is J-closed in (Y,ζ). The collection of all J-open sets in (Y,ζ) is denoted by JO(Y,ζ).
  - **J*-closed set** if η*-Cl(D)$\subseteq$M whenever D$\subseteq$M, M∈ ζ**.** The family of all J*-closed sets of (Y,ζ) is denoted by J*C(Y,ζ).
  - **J**-closed set** if η*-Cl(D)$\subseteq$M whenever D$\subseteq$M, M is η*-open in (Y,ζ).The class of all J**-closed sets of (Y,ζ) is denoted by J**C(Y,ζ).
- Let B $\subseteq$ A $\subseteq$ Y. Then
  - B is **J-closed relative to A** if $Cl_A(B)\subseteq$M, whenever B $\subseteq$ M, M is η*-open in A.
  - B is **J*-closed relative to A** if η*-$Cl_A(B)\subseteq$M whenever B $\subseteq$ M, M ∈ ζ in A.
  - B is **J**-closed relative to A** if η*-$Cl_A(B)\subseteq$M, whenever B $\subseteq$ M, M is η*-open in A**.**

The following diagram explains that of J-closed set is stronger than the following existing generalized closed sets.



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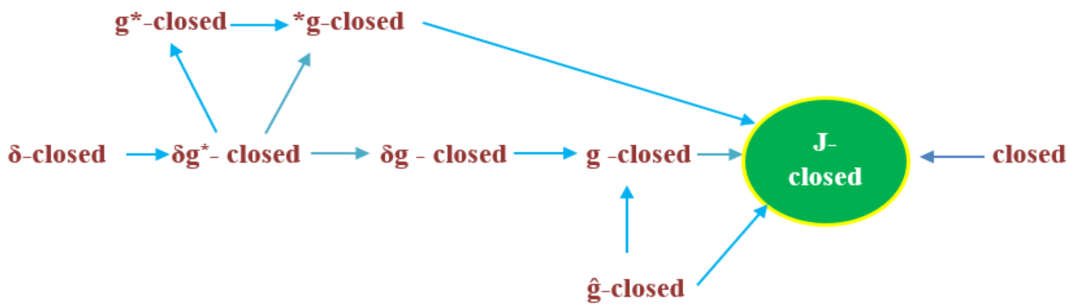

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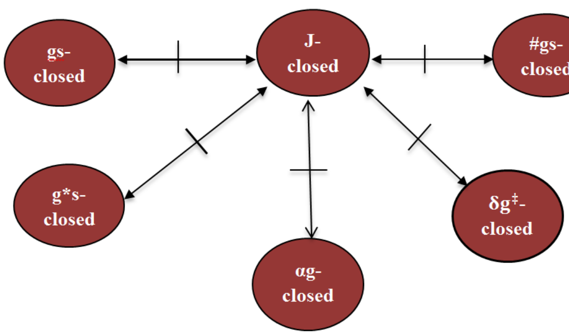

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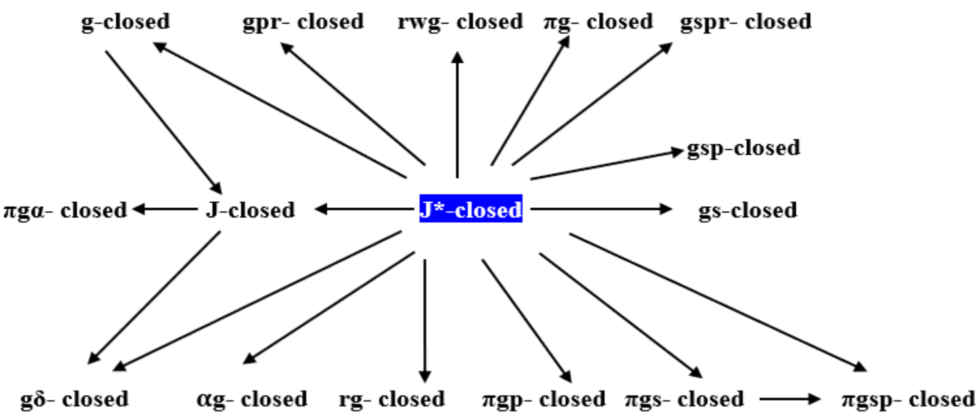

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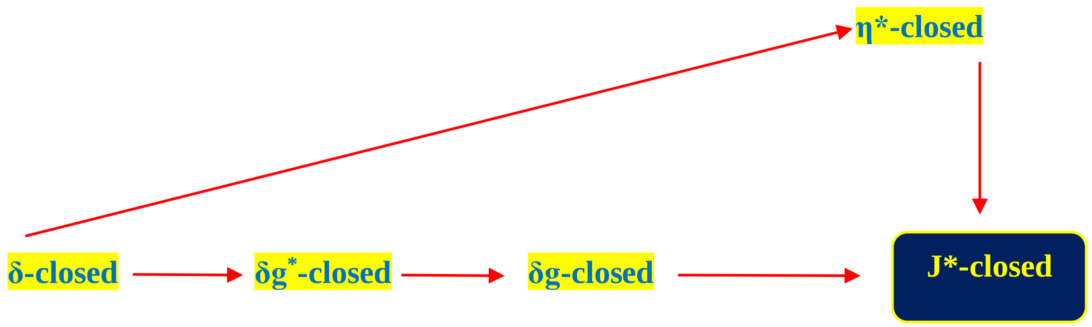

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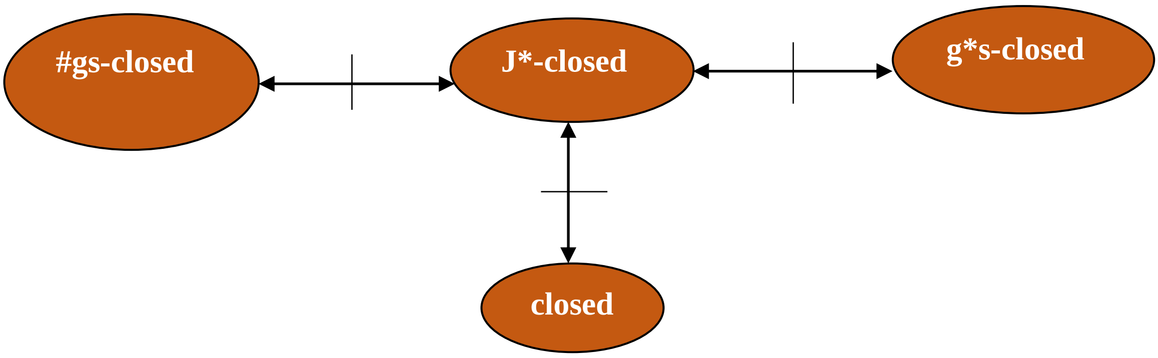

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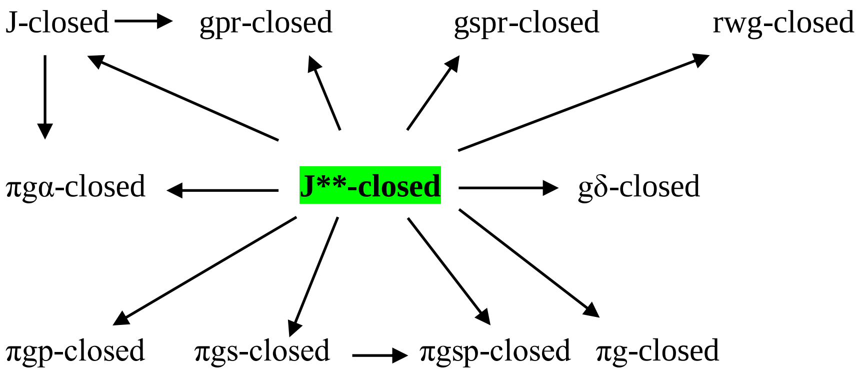

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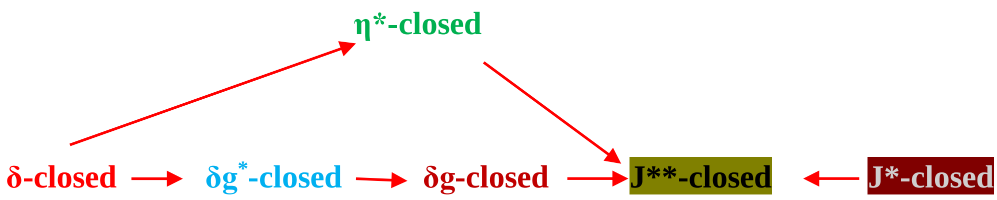

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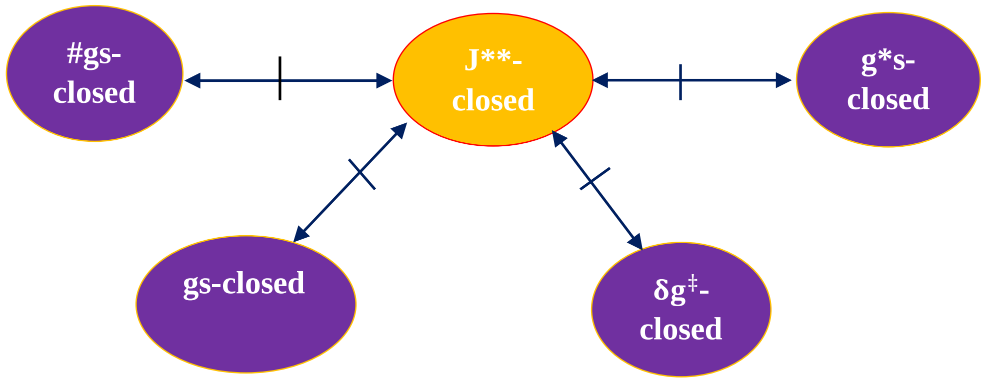


Significant properties of η*-open sets and J-closed sets:

- ❖ D is η*-open iff $D = \eta^*\text{-Int}(D)$.
- ❖ In any topological space (Y,ζ), if D and E are subsets of Y then we get the following:

- $\eta^*\text{-Int}(\phi) = \phi$
- $\eta^*\text{-Int}(Y) = Y$
- $\eta^*\text{-Int}(D) \subseteq D$
- $D \subseteq E \Rightarrow \eta^*\text{-Int}(D) \subseteq \eta^*\text{-Int}(E)$
- $\text{Int}(\eta^*\text{-Int}(D)) \subseteq \text{Int}(D)$
- $\delta\text{Int}(D) \subseteq \eta^*\text{-Int}(D) \subseteq \text{Int}(D) \subseteq D$
- $\eta^*\text{-Int}(\cup_{i\in\varepsilon}\{D_i\}) = \cup_{i\in\varepsilon}\eta^* - \text{Int}(\{D_i\})$
- $\eta^*\text{-Int}(D \cap E) = \eta^*\text{-Int}(D) \cap \eta^*\text{-Int}(E)$.

- ❖ In any topological space (Y,ζ), we get the following result hold:

- $\eta^*\text{-Cl}(\emptyset) = \emptyset$

- η*-Cl(Y) = Y
- D⊆ η*-Cl(D)
- D⊆ E ⇒ η*-Cl(D) ⊆ η*-Cl(E),if D and E are subsets of Y
- D⊆ Cl(D) ⊆ η*-Cl(D) ⊆ δCl(D)
- η*-Cl(D ∪ E) = η*-Cl(D) ∪ η*-Cl(E),if D and E are subsets of Y
- η*-Cl($\cap_{i\in\varepsilon}$ $\{D_i\}$) = $\cap_{i\in\varepsilon}$ η*-Cl ($\{D_i\}$)
- Cl(D) ⊆ Cl(η*-Cl(D)).

❖ (η* -Cl(D))^c = η* -Int(D^c).

❖ (η*-Int(D))^c = η* -Cl(D^c).

❖ The finite union of J-closed sets is J-closed.

❖ The finite intersection of J-closed sets need not be J-closed.

❖ The intersection of a J-closed set and a δ-closed set is always J-closed.

❖ If D is a J-closed set of (Y,ζ), then Cl(D) – D does not contain a non-empty η*-closed set.

❖ If D is a J-closed set of (Y,ζ), then D is closed iff Cl(D) – D is η*-closed.

❖ If D is a η*-open set and a J-closed set of (Y,ζ), then D is a closed set of Y.

❖ If D is J-closed and η*-open and F is closed in (Y,ζ), then D∩ F is closed.

❖ If D is a J-closed set in a space (Y,ζ) and D ⊆ B ⊆ Cl(D) then B is also a J-closed set.

❖ Let B ⊆ A ⊆ Y and suppose that B is J-closed in Y, then B is J-closed relative to A. The converse is true if A is closed in Y.

❖ A subset D of a topological space (Y,ζ) is J-open if and only if G ⊆ int(D) whenever G ⊆ D and G is η*-closed.

❖ If A and B are J -open sets in (Y,ζ), then A ∩ B is J -open in (Y,ζ).

❖ If D is J-open in Y then the only η*-open set containing int(D) ∪ D^c is Y.

❖ Every singleton set is either η*-closed or J-open in (Y,ζ).

❖ If a subset D is J-closed in (Y,ζ), then Cl(D) −D is J-open.

Similar properties of J*-closed sets and J**-closed sets are obtained.

Important Characterizations:

1. In a $T_{1/2}$-space, η*-closed sets coincide with δ-closed sets. Moreover in this space, the family of J-closed sets is equivalent with that of gδ-closed sets and the family of J*-closed sets coincide with the family of δg-closed sets and the family of J**-closed sets coincide with the family of $\delta g^{\ddagger}$-closed sets.

2. In a semi-regular space, δ-closed sets, η*-closed sets and closed sets coincide. So δ-closure, η*-closure, closure of any subset coincide. Therefore from definitions J**-closed sets, J*-closed sets,J-closed sets,g-closed sets, δg-closed sets, gδ-closed sets and $\delta g^{\ddagger}$-closed sets coincide.

3. (a) In a $T_{1/2}$-space and a semi-regular space, J-closed sets are closed.

   (b) In T_b (resp. αT_b) -spaces,J*-closed sets are closed.

4. Regarding partition spaces we have obtained following results.

   (a) Every subset of a partition space is J-closed. The result fails for J*-closed sets and J**-closed sets.

   (b) If every subset of a space Y which is both $T_{1/2}$ and semi-regular is J-closed, then Y is also a partition space.

   (c) In a T_b-space (resp. αT_b-space) Y if every subset is J*-closed, then Y is a partition space.

   (d) Let D be a subset of the partition space (Y,ζ). Then the following conditions are equivalent:

   (i) D is δg-closed

   (ii) D is J*-closed

   (iii) D is g-closed.

5. (a) If a compact subset D of a R_1-topological space (Y,ζ) is J-closed (resp.J*-closed, J**-closed), then D is a $\delta g^{\ddagger}$-closed set.

   (b) The converse is true only when

   (i) Y is $T_{1/2}$ in the case of J-closed sets.

(ii) Y is semi-regular in the case of J*-closed sets.

(iii) Y is $T_{1/2}$ (resp.semi-regular) in the case of J**-closed sets.

6. In Hausdroff spaces, a finite set D is one of these three types namely J-closed set, J*-closed set or J**-closed set,then D is $\delta g^{\ddagger}$-closed sets.

7. In a T_{δ}-space, δg-closed, J**-closed, J*-closed, g-closed, J-closed,gδ-closed sets are equivalent.

8. In an almost weakly Hausdorff space (Y,ζ),the g-closed sets of (Y,ζ_S) are δ-closed sets in (Y,ζ) and thus δg*-closed sets, δg-closed sets,J**-closed sets,J*-closed sets respectively.

9. If D is a pre-open subset of a topological space (Y,ζ).Then the following conditions are equivalent:

(a) D is δg-closed
(b) D is J*-closed
(c) D is g-closed.

CHAPTER III:

In this **Chapter**, J-Closure and J-interior operators are introduced. Also J-Neighbourhood of a point and J-Neighbourhood of a subset, J-Limit point, J-Derived set, J-Frontier, J-Border, J-Exterior of a subset are analysed using the concept of J-closed sets. Some exciting features of these notions are obtained. Moreover, the interrelations between them are established.

The following definitions are introduced in this chapter:

- The **J-closure of D** (briefly **JCl(D)**) of a topological space (Y,ζ) is defined as follows.

  JCl(D) =∩{F⊆ Y:D⊆Fand F∈JC(Y,ζ)}
- A subset M of a topological space (Y,ζ) is said to be a **J-Neighbourhood of** x∈Y if there exists a J-open set D such that x ∈ D ⊆ M.The set of all J-Neighbourhoods of x is denoted by **JNr(x)**.
- A subset M is said to be a **J-Neighbourhood of N** ⊆ Y if there exists a J-open set A such that N ⊆ A⊆ M.The set of all J-Neighbourhoods of N is denoted by **JNr(N)**.

- Let D ⊆Y and a point y ∈ Y is known as a **J-limit point** of D if every J-Neighbourhood of y intersects D in some point other than y itself. The set of all J-limit points of D ⊆Y is called **J-Derived set** of D and is denoted by **JDr(D)**.
- Let D be a subset of Y.A point y∈ D is said to be **J-interior point** of D if D is a J-Neighbourhood of y. The set of all J-interior points of D is called the **J-interior of D** and is denoted by **Jint(D)**.
- A subset D of a topological space (Y,ζ) is said to be **J-saturated set** if JCl({x}) ⊆ D for each x ∈ D. The set of all J-saturated sets in (Y,ζ) is denoted by **JSr(Y)**.
- Let D ⊆ Y. A subset D of (Y,ζ) is known as the **J-Frontier of D** is defined as JCl(D)−Jint(D) and is denoted by **JFr(D)**.
- Let D be a subset of a topological space (Y,ζ). The **J-Border of D** is defined as D −Jint(D) and is denoted by **JBr(D)**.
- Let D be a subset of a topological space (Y,ζ). The **J-Exterior** of D is defined as Y−JCl(D) and is denoted by **JEr(D).**

Significant Properties of J-closure operator:

- Let D be any subset of (Y,ζ). If D is J -closed in (Y,ζ), then JCl(D)=D.
- For a subset D of (Y,ζ), D⊆ JCl(D) ⊆ Cl(D).
- Let D and B be subsets of (Y,ζ). Then the following statements are true:
  - JCl(∅) = ∅ and JCl(Y)=Y.
  - If D⊆ B, then JCl(D)⊆JCl(B).
  - D⊆JCl(D).
  - JCl(D)∪JCl(B) = JCl(D∪B).
  - JCl(D∩B)⊆ JCl(D)∩JCl(B).
  - JCl(JCl(D)) = JCl(D).
- For each y∈ Y, y∈JCl(D) if and only if U ∩ D ≠ ϕ for every J -open set U in (Y,ζ) containing y.

Vital Properties of J-Neighbourhood operator :

- A J-open set N is a J-Neighbourhood of each of its points.

- If M is a J-closed subset of a topological space (Y,ζ) and x ∈Y–M, then there exists a J-Neighbourhood N of x such that N ∩M =∅.
- In a topological space (Y,ζ) with x ∈ Y, the following results are true.
  - JNr(x)≠ ∅
  - If M ∈JNr(x), then x ∈ M
  - If M ∈JNr(x) and M⊆ N, then N ∈JNr(x)
  - If M ∈JNr(x) and N ∈JNr(x), then M ∩ N∈JNr(x)
  - If M ∈JNr(x) then there exists a N ∈JNr(x) such that N⊆ M and N∈ JNr(y), for every y ∈ N
  - Nr(x) ⊆ JNr(x).
- JNr(x) satisfies
  - M ∈ JNr(x) such that x ∈ M
  - N,M ∈ JNr(x) implies N ∩M ∈ JNr(x) then $\mathcal{B}$ = {∅} ∪ {G ⊆ Y / x ∈ G implies N ∈ JNr(x) such that x∈ N ⊆ G} forms a basis for the topological space (Y,ζ).

Notable Properties of J-Derived set:

- Let A,B⊆Y.Then the following statements are valid in (Y,ζ).
  - JDr(∅) = ∅
  - JDr(A)⊆Dr(A)
  - If A ⊆ B,then JDr(A)⊆JDr(B)
  - JDr(A∪B) = JDr(A) ∪ JDr(B)
  - JDr(A∩B)⊆JDr(A)∩JDr(B)
  - [JDr(JDr(A))]– A ⊆JDr(A)
  - JDr(A∪JDr(A))⊆A∪JDr(A).
- Let A⊆ Y.If A is J-closed then JDr(A) ⊆A.
- For any subset A of a topological space (Y,ζ),JCl(A) = A ∪ JDr(A).
- JCl(A) – A ⊆JDr(A).

Substanial Properties of J-Interior operator:

- If D is a subset of Y, then
  - Jint(D) = ∪{G:G ⊆ D and G∈JO((Y,ζ))}

- int(D) ⊆ Jint(D)

❖ For any two subsets D and B of (Y,ζ), the following statements are true:

- Jint(Y) = Y and Jint(ϕ) = ϕ
- Jint(D) ⊆ D
- If B is any J-open set contained in D, then B ⊆ Jint(D)
- If D⊆ B, then Jint(D)⊆ Jint(B)
- Jint(D) ∪Jint(B) ⊆Jint(D∪B)
- Jint(D∩B) = Jint(D) ∩Jint(B)

❖ Jint(D) = D – JDr(Y –D) .

❖ Let D be any subset of (Y,ζ), the following statements are true:

- If D is J -open in (Y,ζ)then Jint(D) = D.
- $(\text{Jint}(D))^c = \text{JCl}(D^c)$
- $(\text{JCl}(D))^c = \text{Jint}(D^c)$.

❖ Every J-closed set is a J-Saturated set but not conversely.

Remarkable Properties of J-Frontier operator:

❖ Let D ⊆ Y. Then the upcoming results hold good.

- JFr(D) ⊆Fr(D)
- JCl(D) = Jint(D) ∪ JFr(D)
- Jint(D) ∩ JFr(D) = ∅
- If D is a J-open set then JFr(D) ⊆ JDr(D)
- JFr(D) = JCl(D) ∩ JCl(Y− D)
- JFr(D) is J-closed
- JFr(D) = JFr(Y−D)
- JFr(JFr(D)) ⊆ JFr(D)
- JFr(Jint(D)) ⊆ JFr(D)
- JFr(JCl(D)) ⊆ JFr(D)
- Jint(D) = D– JFr(D).

❖ Let A⊆B and Jint(B) = ∅ then JFr(A)⊆JFr(B).

Phenomenal Properties of J-Border:

❖ Let A be a subset of a topological space (Y,ζ).Then the following results hold:

- JBr(A) ⊆Br(A)
- JBr(∅) = ∅
- JBr(Y) = ∅
- JBr(A) ⊆ A
- A = Jint(A) ∪JBr(A)
- If A is J-open, then JBr(A) = ∅
- Jint(A) ∩JBr(A) = ∅
- JBr(Jint(A)) = ∅
- Jint(JBr(A)) = ∅
- JBr(JBr(A)) = JBr(A)
- JBr(A) = A ∩ JCl(Y−A)
- JBr(A) = JDr(Y –A)
- Jint(A) = A –JBr(A)
- JBr(A) ⊆ JFr(A)

Incredible Properties of J-Exterior operator:

❖ Let A be a subset of a topological space (Y,ζ). Then the following results hold:

- JEr(A) ⊆Er(A)
- JEr(Y) = ∅
- JEr(∅) = Y
- JEr(A) = Jint(Y –A) = Y –JCl(A)
- If A ⊆B then JEr(A) ⊇ JEr(B)
- JEr(A∪B) = JEr(A) ∩JEr(B)
- JEr(A∩B) ⊇JEr(A) ∩JEr(B)
- JEr(JEr(A)) = Jint(JCl(A))
- JEr(A) = JEr(Y – JEr(A))
- Y = Jint(A) ∪ JEr(A) ∪ JFr(A)
- Jint(A) ⊆ JEr(JEr(A))
- JEr(A) ∪ JEr(B) ⊆JEr(A ∩ B).

❖ JEr(A) need not be J-open.

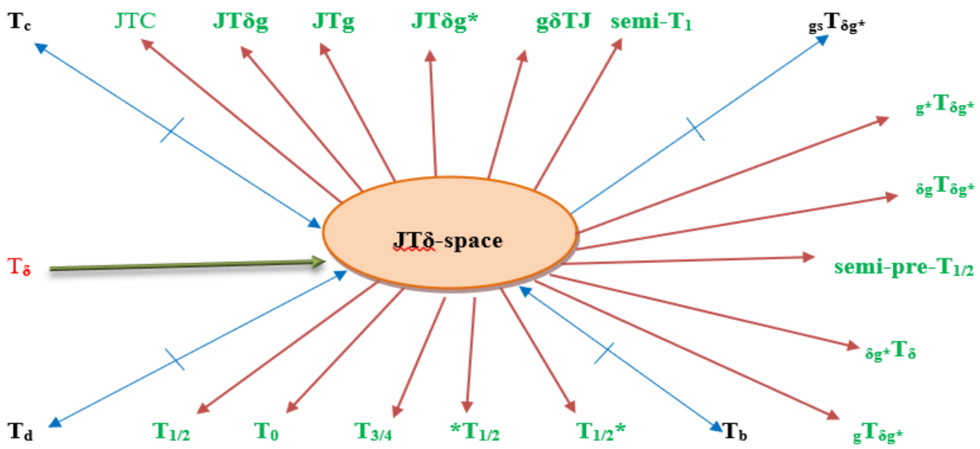
CHAPTER IV:

In this **Chapter**, six interesting new spaces namely JTδ-space, JTg-space, JTC-space, JTδg*-space,JTδg-space and gδTJ-space are introduced and the dependence of these new spaces with other existing separation axioms are analysed. Interrelations of these new six spaces are also investigated.

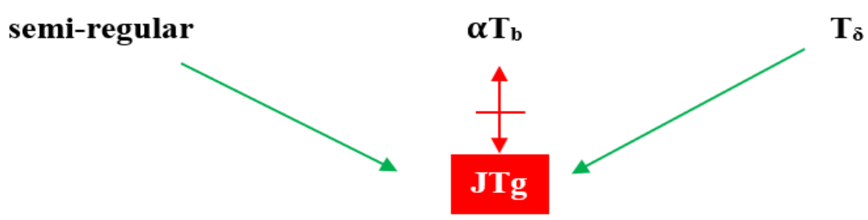
The following definitions are introduced in this chapter:

- A topological space (Y,ζ) is known as
  - **JTδ-space** when each J-closed set is a δ-closed set in (Y,ζ).
  - **JTg- space** when each J-closed set is a g-closed set in (Y,ζ).
  - **JTC-space** when each J-closed set is a closed set in (Y,ζ).
  - **JTδg*-space** when each J-closed set is a δg*-closed set in (Y,ζ).
  - **JTδg-space** when each J-closed set is a δg-closed set in (Y,ζ).
  - **gδTJ- space** when each gδ-closed set is a J-closed set in (Y,ζ).

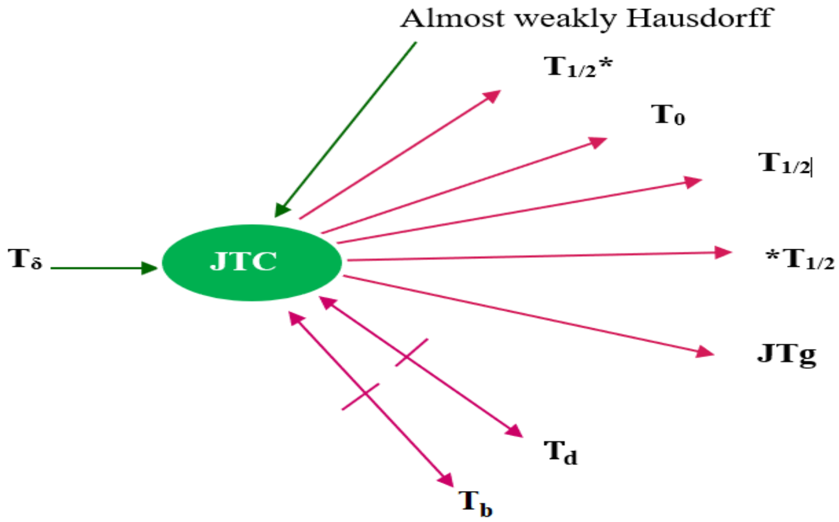
The following figure depicts the relations of JTδ-space with other spaces.



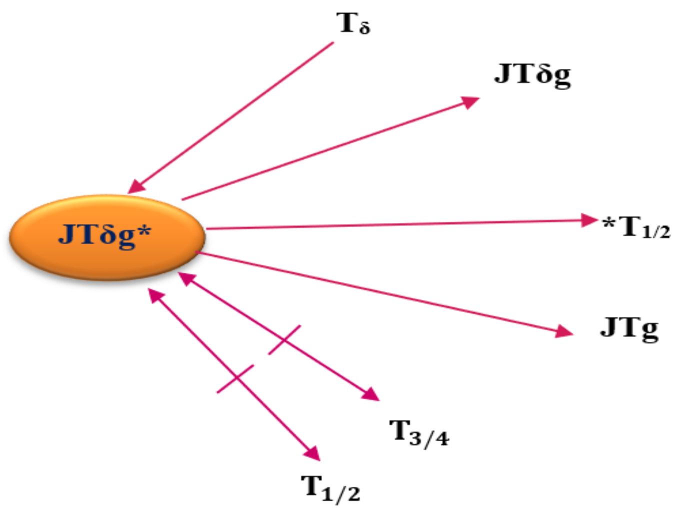
The following figure depicts the relations of JTg-space with other spaces.



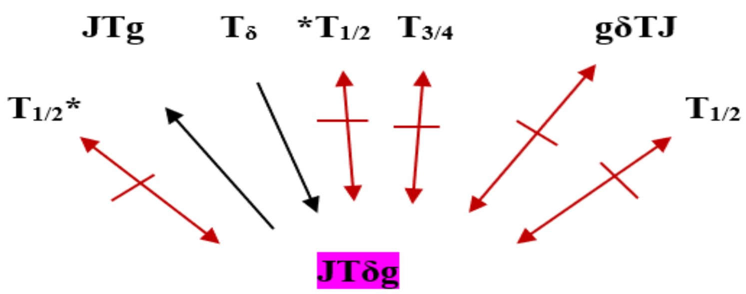
The following figure depicts the relations of JTC-space with other spaces.



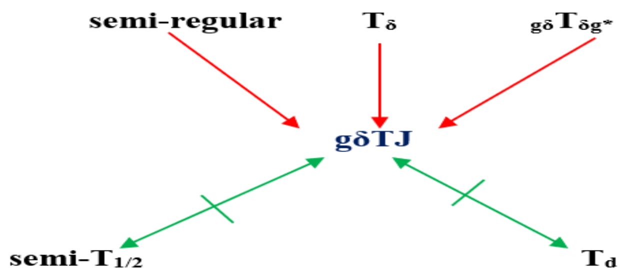
The following figure depicts the relations of JTδg*-space with other spaces.



The following figure depicts the relations of JTδg-space with other spaces.



The following figure depicts the relations of gδTJ-space with other spaces.



The interesting properties and results proved in this chapter are given below:

- If (Y,ζ) is a $T_{3/4}$ and a JTδg-space, then it becomes a JTδ-space and a JTg-space respectively.
- If (Y,ζ) is both a JTg-space and a $T_{1/2}$-space, then it becomes a JTC-space(resp. a $T_{1/2}$*-space).
- If (Y,ζ) is both a T_d-space and a JTg-space (resp.a T_b-space and a JTg-space) then the following closed sets are equivalent:
- A gs-closed set.
- A J-closed set.
- If a given space (Y,ζ) is a T_b-space and a JTC-space, then gs-closed sets coincide with J-closed sets.
- In an almost weakly Hausdorff space (Y,ζ), the following statements are equivalent. (i) (Y,ζ) is JTC-space. (ii) (Y,ζ) is gδTJ-space.
- In a $_{gs}T_{\delta g^*}$-space, gs-closed sets coincide with J-closed sets.
- Let (Y,ζ) be a topological space. Then the following conditions are equivalent.
- (Y,ζ) is a JTC-space.
- Every singleton set is either open or η*-closed in (Y,ζ).
- When a space is JTδg* and $_{\delta g^*}T_\delta$,then it is a $T_{3/4}$ -space.
- If (Y,ζ) is both a T_d-space and a JTδg*-space(resp. a T_b-space and a JTδg*-space or a T_c-space and a JTδg*-space), then the following closed sets are equivalent:
    - A gs-closed set
    - A J-closed set
    - A δg*-closed set.

- If a space (Y,ζ) is both a T_d-space and a JTδg*-space(resp. a T_b-space and a JTδg*-space or a T_c-space and a JTδg*-space),then (Y,ζ) is a ${}_{gs}T_{\delta g^*}$ -space.
- If (Y,ζ) is a JTδg and a T_b-space (resp. a $T_{1/2}$-space), then it becomes a JTC-space.
- If (Y,ζ) is JTδg and T_d-space, then it becomes JTg-space.
- If (Y,ζ) is a JTδg-space and a T_δ-space, then it becomes a gδTJ-space.
- If (Y,ζ) is both a T_d-space and a JTδg-space(resp. a T_b-space and a JTδg-space or a T_c and JTδg-space) then the following closed sets are equivalent:
    - A gs-closed set
    - A J-closed set
    - A δg-closed set.
- A topological space (Y,ζ) is a gδTJ-space and a JTC-space, then (Y,ζ) becomes a $T_{1/2}$ - space.
- A topological space (Y,ζ) is a JTδ-space and a gδTJ-space, then (Y,ζ) becomes a $T_{3/4}$ - space.

CHAPTER V:

In this **Chapter**, the concept of J-continuous functions is established and the dependency of J-continuous functions with other existing continuous functions are analysed.

The following definitions are introduced in this chapter:

- A function f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is said to be **J-continuous** if the inverse image of every closed set in (Z,σ) is J-closed in (Y,ζ).
- A function f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is said to be **quasi J -continuous** if the inverse image of every J-closed set in (Z,σ) is closed in (Y,ζ).
- A function f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is said to be **totally J -continuous** if the inverse image of every closed set in (Z,σ) is J-clopen in (Y,ζ).
- A function f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is said to be **strongly J -continuous** if the inverse image of every subset in (Z,σ) is J-clopen in (Y,ζ).
- A function f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is said to be **Quasi totally J-continuous** if the inverse image of every J-open set in (Z,σ) is a clopen set in (Y,ζ).

- A function f : (Y,ζ) → (Z,σ) is said to be **contra J-continuous** if the inverse image of every closed set in (Z,σ) is a J-open set in (Y,ζ).

This Chapter provides some important properties of J-continuous functions:

- A function f : (Y,ζ)→ (Z,σ) is J-continuous if and only if the inverse image of every open set in (Z,σ) is J-open in (Y,ζ).
- If f: (Y,ζ) → (Z,σ) is a J-continuous function and (Y,ζ) is a JTC-space (resp. JTδ-space,JTδg*-space,JTδg-space,JTg-space) then f is continuous (resp.super continuous, δg*- continuous, δg-continuous, g-continuous).
- If f : (Y,ζ)→ (Z,σ) is a gδ-continuous function and (Y,ζ) is a gδTJ-space. Then f is J-continuous.
- For every subset D of (Y,ζ),
  - f(JCl(D)) ⊆ Cl(f(D)) if f : (Y,ζ) → (Z,σ) is a J-continuous function.
  - f(JCl(D)) ⊆ Cl(f(D)) if f : (Y,ζ)→ (Z,σ) is a super continuous function.
  - f(JCl(D)) ⊆ Cl(f(D)) if f : (Y,ζ)→ (Z,σ) is a totally continuous function.
- Let f: (Y,ζ) → (Z,σ) be a function. If for each point x∈Y and each open set V in (Z,σ) containing f(x), there exists a J-open set U in (Y,ζ) containing x such that f(U)⊆V, then for each subset D of (Y,ζ), f(JCl(D)) ⊆ Cl(f(D)).

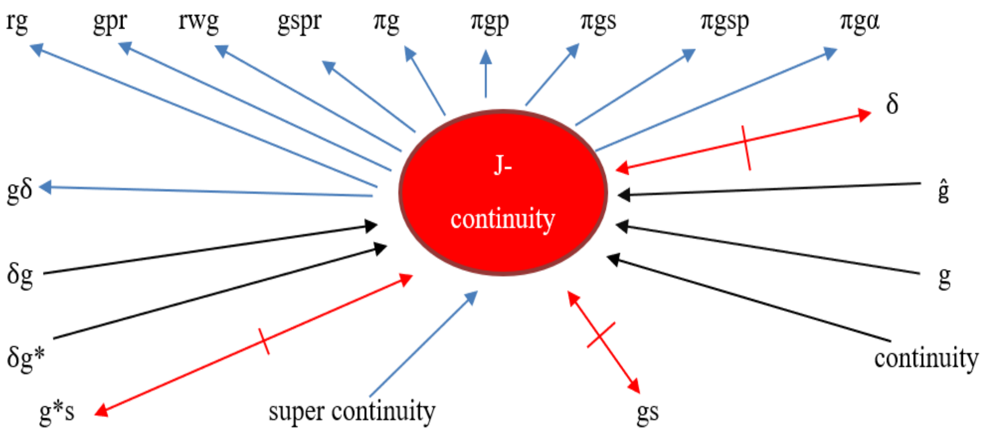
In addition, the following important properties under composition of functions are established.

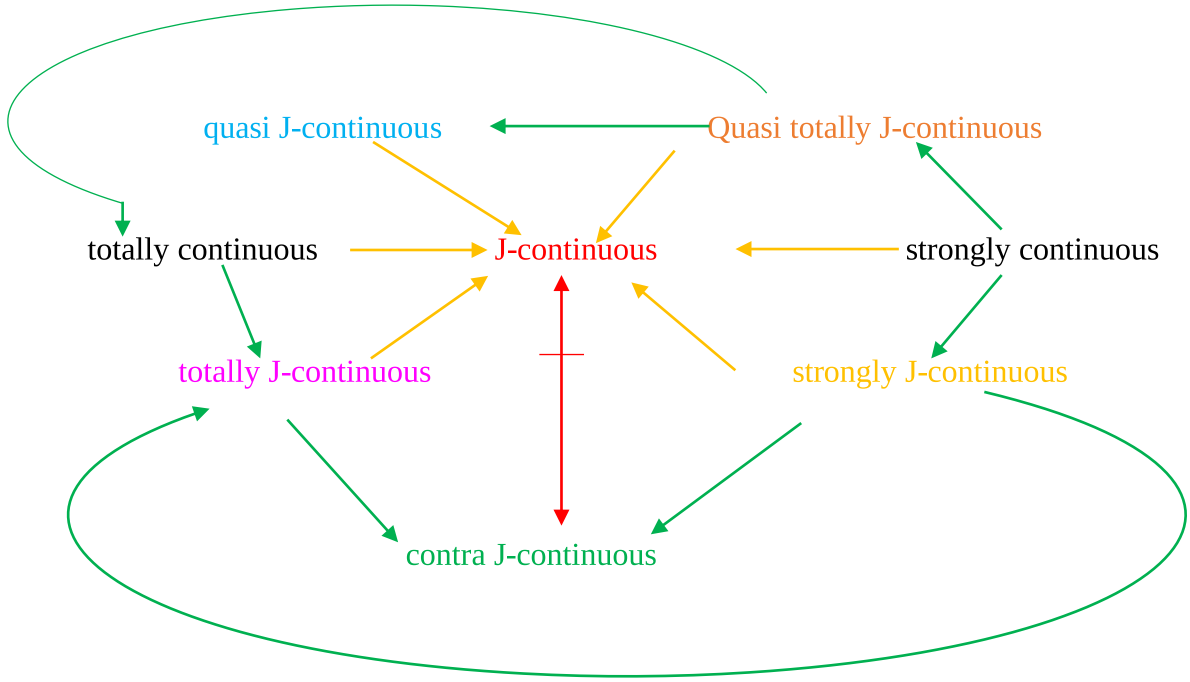
- The composition of two J-continuous functions need not be J-continuous. **(Counter Example obtained).**
- If f : (Y,ζ) → (Z,σ) is a J-continuous function and g : (Z,σ) →(P,μ) is a J-continuous function, then g ∘ f : (Y,ζ)→ (P, μ) is a J-continuous function when (Z,σ) is a JTC-space (resp.T_δ-space).
- If f : (Y,ζ)→ (Z,σ) is a J-continuous function and g : (Z,σ) →(P,μ) is a super continuous function (resp.continuous function), then g ∘ f : (Y,ζ)→ (P, μ) is a J-continuous function.
- If f : (Y,ζ)→ (Z,σ) is a super continuous function and g : (Z,σ) →(P,μ) is a super continuous function, then g ∘ f : (Y,ζ)→ (P, μ) is a J-continuous function.

Further characteristics of J-continuous functions,quasi J-continuous, totally J-continuous, Quasi totally J-continuous, strongly J-continuous and contra J-continuous are studied in this chapter:

- ❖ A totally continuous function f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is a quasi J-continuous function if (Z,σ) is a JTC-space.
- ❖ A function f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is totally J-continuous if and only if the inverse image of every open set in (Z,σ) is J-clopen in (Y,ζ).
- ❖ If (Y,ζ) is a discrete topological space,the following results are equivalent.
- f is J-continuous
- f is totally J-continuous
- f is totally continuous
- f is continuous.
- ❖ A totally J-continuous function f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is a strongly J-continuous function,if (Z,σ) is a discrete topological space.
- ❖ A function f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is Quasi totally J-continuous if and only if the inverse image of every J-closed set in (Z,σ) is clopen in (Y,ζ).
- ❖ If f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is a function. Then the following functions are equivalent under some restrictions as (Y,ζ) is a discrete topological space.
- f is Quasi totally J-continuous
- f is quasi J-continuous.
- ❖ If f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is a J-continuous function and g : $(Z,\sigma)\rightarrow(P,\mu)$ is a strongly continuous function, then $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a J-continuous function.
- ❖ If f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is a continuous function and g : $(Z,\sigma)\rightarrow(P,\mu)$ is a Quasi totally J-continuous function, then $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a Quasi totally J-continuous function.
- ❖ The composition of two contra J-continuous functions need not be a contra J-continuous function. **(Counter Example obtained).**
- ❖ If f : $(Y,\zeta)\rightarrow(Z,\sigma)$ is a contra J-continuous function and g : $(Z,\sigma)\rightarrow(P,\mu)$ is a continuous function, then $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a contra J-continuous function.

The following figure depicts the relations of J-continuous functions with various existing continuous functions.





CHAPTER VI:

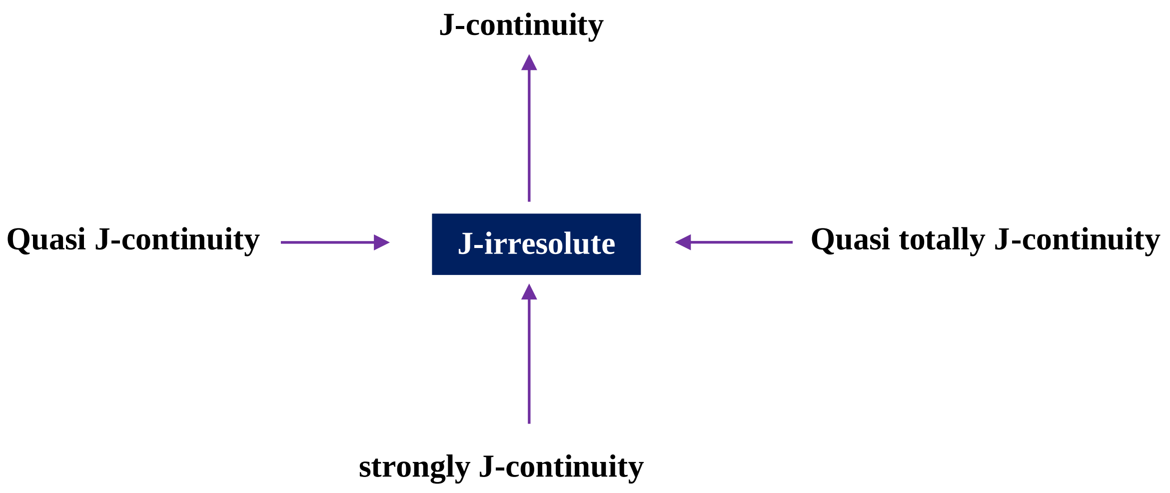
In **Chapter VI**, a new class of irresolute functions called J-irresolute and contra J-irresolute functions are introduced.

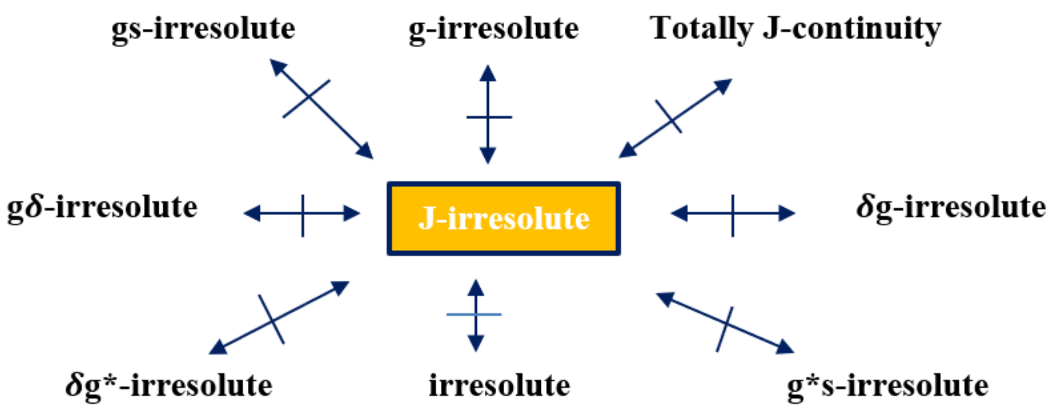
The following definitions are introduced in this chapter:

- A function f : (Y,ζ)⟶ (Z,σ) is said to be
- **J-irresolute function** if the inverse image of every J-open in (Z,σ) is J-open in (Y,ζ).

- **contra J-irresolute function** if the inverse image of every J-closed set in (Z,σ) is a J-open set in (Y,ζ).

The following figure depicts the relations of J-irresolute functions with various types of J-continuous functions and existing irresolute functions.





Fascinating Properties of J-irresolute and contra J-irresolute functions:

- A function f : (Y,ζ)$\rightarrow$ (Z,σ) in which (Z,σ) is a JTC-space. Then J-irresolute functions and J-continuous functions are equivalent.
- If f : (Y,ζ)$\rightarrow$ (Z,σ) and g : (Z,σ) $\rightarrow$(P,μ) are J-irresolute functions, then g $\circ$ f : (Y,ζ)$\rightarrow$ (P, μ) is a J-irresolute function.
- If f: (Y,ζ)$\rightarrow$ (Z,σ) is J-irresolute and g : (Z,σ) $\rightarrow$(P,μ) is a J-continuous (resp.contra J-continuous, totally J-continuous and strongly J-continuous) function then g $\circ$ f : (Y,ζ)$\rightarrow$ (P, μ) is a J-continuous (resp.contra J-continuous, totally J-continuous and strongly J-continuous) function.

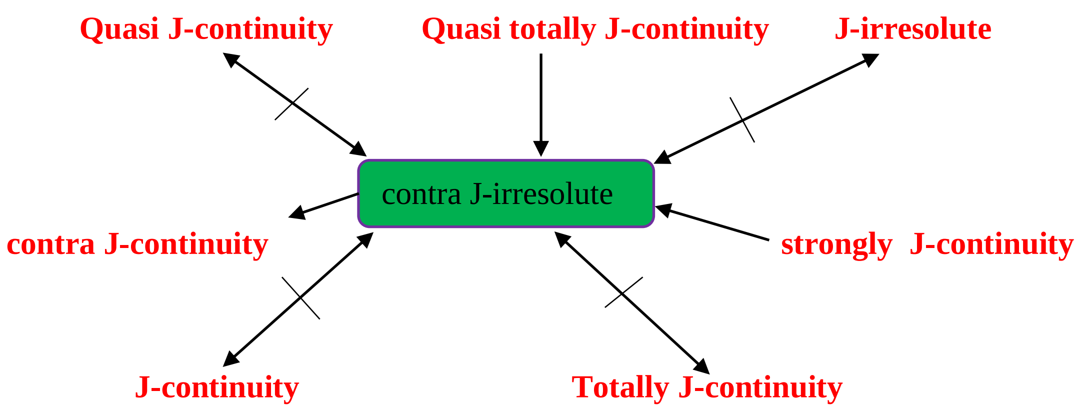
- ❖ A function $f : (Y,\zeta) \rightarrow (Z,\sigma)$ is a J-irresolute function if and only if $f^{-1}(U)$ is J-closed in (Y,ζ) for every J-closed set U in (Z,σ).
- ❖ If a function $f : (Y,\zeta) \rightarrow (Z,\sigma)$ is δ-open (resp. closed,δg-open, δg*-open and g-closed) surjective, J-irresolute function and (Y,ζ) is a JTδ -space (resp. JTC-space) then (Z,σ) is a JTδ -space (resp. JTC-space, JTδg-space, JTδg*-space and JTg-space).
- ❖ Let $f : (Y,\zeta) \rightarrow (Z,\sigma)$ be a J-irresolute function. Then for each subset D of (Y,ζ),

(i) $f(JCl(D)) \subseteq Cl(f(D))$.

(ii) $JCl(f^{-1}(D)) \subseteq f^{-1}(Cl(D))$.

- ❖ A function $f : (Y,\zeta) \rightarrow (Z,\sigma)$ is a contra J-irresolute function iff the inverse image of each J-open set in (Z,σ) is J-closed in (Y,ζ).
- ❖ The composition $g \circ f : (Y,\zeta) \rightarrow (P, \mu)$ of two contra J-irresolute functions $f: (Y,\zeta) \rightarrow (Z,\sigma)$ and $g : (Z,\sigma) \rightarrow (P,\mu)$ is a J-irresolute function.
- ❖ If $f: (Y,\zeta) \rightarrow (Z,\sigma)$ is a J-irresolute function and $g : (Z,\sigma) \rightarrow (P,\mu)$ is a contra J-irresolute function, then their composition $g \circ f$ is a contra J-irresolute function.
- ❖ If $f: (Y,\zeta) \rightarrow (Z,\sigma)$ is a contra J-irresolute function and $g : (Z,\sigma) \rightarrow (P,\mu)$ is a J-irresolute function, then their composition $g \circ f$ is a contra J-irresolute function.
- ❖ If $f: (Y,\zeta) \rightarrow (Z,\sigma)$ is a Quasi totally J-continuous function and $g : (Z,\sigma) \rightarrow (P,\mu)$ is a contra J-irresolute function, then their composition $g \circ f$ is a Quasi totally J-continuous function.

The following figure depicts the relations of contra J-irresolute functions with various types of J-continuous functions and J-irresolute functions.



CHAPTER VII:

In this **Chapter**, J-open functions, J-quotient maps and J-Homeomorphisms are introduced. Three types of quotient maps namely J-quotient maps,Strongly J-quotient maps and [J]-quotient maps are introduced and their properties are established.

The following definitions are introduced in this chapter:

- A function f : $(Y,\zeta)\rightarrow (Z,\sigma)$ is said to be
  - **J-closed function** if the image of every closed set in (Y,ζ) is J-closed in (Z,σ).
  - **J-open function** if the image of every open set in (Y,ζ) is J-open in (Z,σ).
  - **strongly J-open function** if the image of every J-open set in (Y,ζ) is J-open in (Z,σ).
- A surjective function f: $(Y,\zeta) \rightarrow (Z,\sigma)$ is said to be a
  - **J-quotient map** if f is a J-continuous function and $f^{-1}(U)$ is open in (Y,ζ) implies U is J-open in (Z,σ).
  - **strongly J-quotient map** provided a subset U of (Z,σ) is open $\Leftrightarrow$ $f^{-1}(U)$ is J-open in (Y,ζ).
  - **[J]-quotient map** if f is a J-irresolute function and $f^{-1}(U)$ is J-open in (Y,ζ) implies U is open in (Z,σ).
- A bijective function f: $(Y,\zeta) \rightarrow (Z,\sigma)$ is said to be
  - **J-Homeomorphism** if f is both a J-continuous function and a J-open function.
  - **J$\mathcal{C}$-Homeomorphism** if both the functions f and f^{-1} are J-irresolute.

The features of J-closed function under the composition of functions are shown in this chapter:

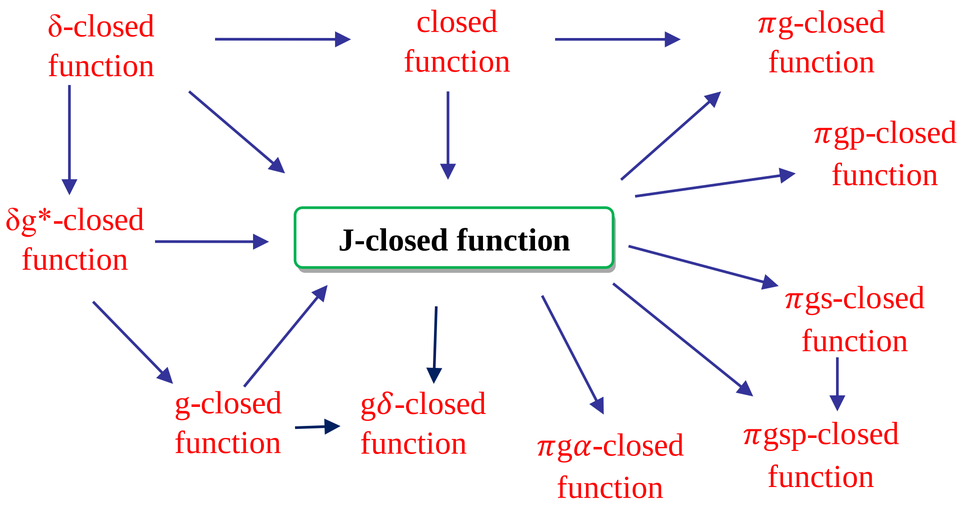
- The composition of two J-closed functions need not be J-closed. **(Counter Example obtained).**
- If f : $(Y,\zeta)\rightarrow (Z,\sigma)$ is a closed (resp.δ-closed) function and g : $(Z,\sigma) \rightarrow (P,\mu)$ is a J-closed function, then $g \circ f : (Y,\zeta)\rightarrow (P, \mu)$ is a J-closed function.
- The composition of two J-closed function is a J-closed function when (Z,σ) is a JTC-space (resp.JTδ-space).

- A function $f:(Y,\zeta)\rightarrow(Z,\sigma)$ is a J-closed function if $g:(Z,\sigma)\rightarrow(P,\mu)$ is a J-irresolute injective function and $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a J-closed function.
- A function $f:(Y,\zeta)\rightarrow(Z,\sigma)$ is a J-continuous function if $g:(Z,\sigma)\rightarrow(P,\mu)$ is a J-closed injective function and $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a J-irresolute function.
- A function $g:(Z,\sigma)\rightarrow(P,\mu)$ is a J-closed function if $f:(Y,\zeta)\rightarrow(Z,\sigma)$ is a surjective continuous function and $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a J-closed function.
- A function $g:(Z,\sigma)\rightarrow(P,\mu)$ is a J-closed function when (Y,ζ) is a JTC-space(resp. a $T_{1/2}$-space) if $f:(Y,\zeta)\rightarrow(Z,\sigma)$ is a surjective J-continuous(resp. g-continuous) function and $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a J-closed function.
- A function $f:(Y,\zeta)\rightarrow(Z,\sigma)$ is a closed function if $g:(Z,\sigma)\rightarrow(P,\mu)$ is a quasi J-continuous function, injective and $g\circ f:(Y,\zeta)\rightarrow(P,\mu)$ is a J-closed function.

The attractive properties of J-closed function are presented in this chapter:

- A function $f:(Y,\zeta)\rightarrow(Z,\sigma)$ is a J-closed function and V is a closed subset of (Y,ζ),then $f|_V:(V,\zeta|_V)\rightarrow(Z,\sigma)$ is a J-closed function.
- A function $f:(Y,\zeta)\rightarrow(Z,\sigma)$ is J-closed if and only if for each subset G of (Z,σ) and for each open set U of (Y,ζ) containing $f^{-1}(G)$, there exists a J-open set B of (Z,σ) such that $G\subseteq B$ and $f^{-1}(B)\subseteq U$.
- A bijection function $f:(Y,\zeta)\rightarrow(Z,\sigma)$ is a J-closed function if and only if f(U) is J-open in (Z,σ) for every open set U in (Y,ζ).

A J-closed function is stronger as well as weaker than other closed functions are shown in this chapter.



The exciting properties of J-open function are presented in this chapter:

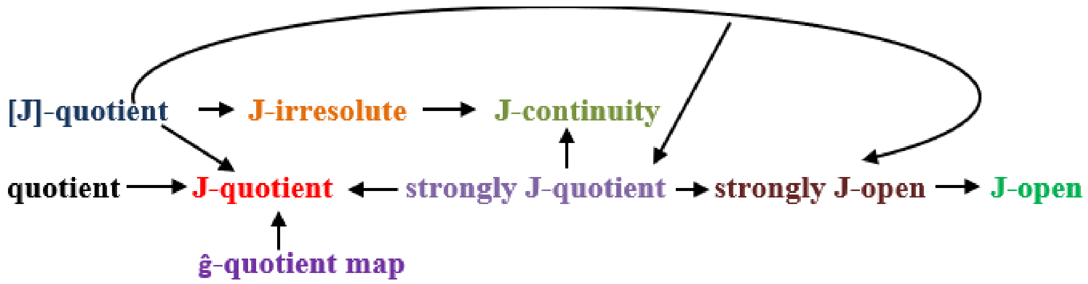
- If f : $(Y,\zeta) \to (Z,\sigma)$ is any function, g : $(Z,\sigma) \to (P,\mu)$ is an injective function and also a J-irresolute function, their composite function $g \circ f : (Y,\zeta) \to (P,\mu)$ is a J-open function then f is a J-open function in (Z,σ).
- For a bijective function f : $(Y,\zeta) \to (Z,\sigma)$,the following statements are equivalent.
  - $f^{-1} : (Z,\sigma) \to (Y,\zeta)$ is a J-continuous function.
  - f is a J-open function.
  - f is a J-closed function.

The exciting properties of J-quotient map, strongly J-quotient map and [J]-quotient map are presented in this chapter:

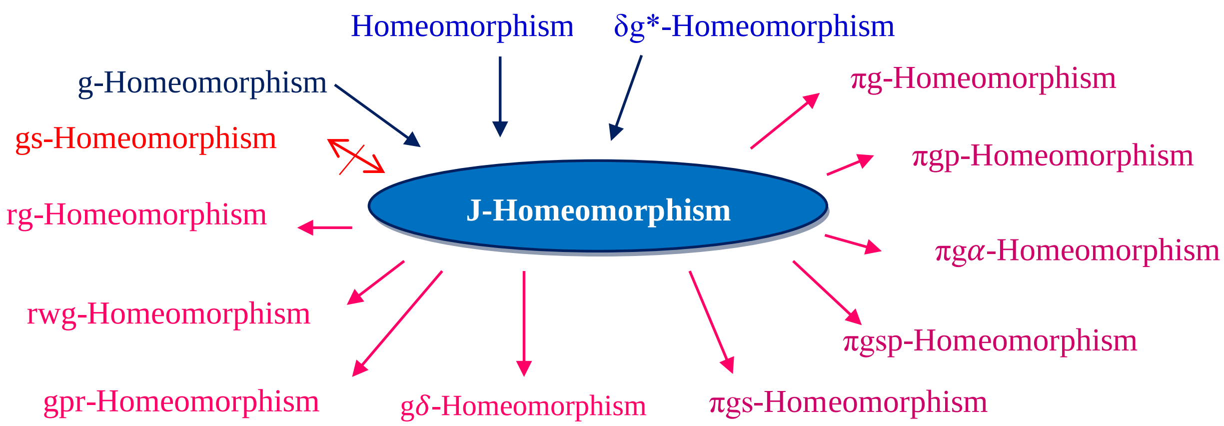
- If f : $(Y,\zeta) \to (Z,\sigma)$ is any function, g : $(Z,\sigma) \to (P,\mu)$ is an injective function and also a J-irresolute function, their composite function $g \circ f : (Y,\zeta) \to (P,\mu)$ is a strongly J-open function then f is a strongly J-open function in (Z,σ).
- If a surjective function f :$(Y,\zeta) \to (Z,\sigma)$ is a J-continuous function and J-open (resp.J-closed) function, then f is a J-quotient map.
- If an open surjective function f: $(Y,\zeta) \to (Z,\sigma)$ is a J-irresolute function and g : $(Z,\sigma) \to (P,\mu)$ is a J-quotient map, then $g \circ f : (Y,\zeta) \to (P, \mu)$ is a J-quotient map.
- If a map h: $(Y,\zeta) \to (Z,\sigma)$ is a J-quotient map and g : $(Y,\zeta) \to (P, \mu)$ is a continuous function that is constant on each set $h^{-1}(y)$,for $y \in Z$, then g induces a J-continuous function f : $(Z,\sigma) \to (P, \mu)$ such that $f \circ h = g$.
- If a surjective function f: $(Y,\zeta) \to (Z,\sigma)$ is strongly J-open, J-Irresolute and g : $(Z,\sigma) \to (P,\mu)$ is a J-quotient map (resp. strongly J-quotient map, [J]-quotient map), then $g \circ f : (Y,\zeta) \to (P, \mu)$ is a J-quotient map (resp.strongly J-quotient map, [J]-quotient map).
- If a function f: $(Y,\zeta) \to (Z,\sigma)$ is strongly J-quotient , J-irresolute and g : $(Z,\sigma) \to (P,\mu)$ is a [J]-quotient map, then $g \circ f : (Y,\zeta) \to (P, \mu)$ is a [J]-quotient map.
- If f : $(Y,\zeta) \to (Z,\sigma)$ and g : $(Z,\sigma) \to (P,\mu)$ are [J]-quotient maps, then $g \circ f : (Y,\zeta) \to (P, \mu)$ is a [J]-quotient map.

- If a function f: (Y,ζ)→ (Z,σ) is a J-quotient map where (Y,ζ) and (Z,σ) are JTC-spaces. Then g : (Z,σ) →(P,μ) is quasi J-continuous ⇔ the composite map g ∘ f : (Y,ζ)→ (P, μ) is quasi J-continuous.
- An injective [J]-quotient map f :(Y,ζ) →(Z,σ) is a J-open (resp.J-closed) function.
- If f :(Y,ζ) →(Z,σ) is any function from a JTC-space to another JTC-space, then the following conditions are equivalent.
  - f is a [J]-quotient map
  - f is a strongly J-quotient map
  - f is a J-quotient map.

A J-quotient map is stronger as well as weaker than newly defined quotient map is shown here.



A pictorial representation of J-Homeomorphism is stronger and weaker than existing homeomorphism is shown below.



An interesting results of J-Homeomorphism and J$\mathcal{C}$-Homeomorphism are analysed in this chapter:

- If a function $f : (Y,\zeta) \to (Z,\sigma)$ is a bijective function and a J-continuous function, then $f : (Y,\zeta) \to (Z,\sigma)$ is a J-open function (resp.J-closed function) if and only if f is a J-Homeomorphism.
- If a J-Homeomorphism $f : (Y,\zeta) \to (Z,\sigma)$ is a Homeomorphism when both spaces (Y,ζ) and (Z,σ) are JTC-spaces (resp. JTδ-spaces).
- The composition of two J-Homeomorphisms is not a J-Homeomorphism, since composition of two J-continuous functions is not a J-continuous function.
- If a J-Homeomorphism $f : (Y,\zeta) \to (Z,\sigma)$ is a Homeomorphism when both spaces (Y,ζ) and (Z,σ) is a JTC-spaces(resp. JTδ -spaces).
- If a J-Homeomorphism $f : (Y,\zeta)\to(Z,\sigma)$ is a g-Homeomorphism (resp.δg*-Homeomorphism) when both spaces (Y,ζ) and (Z,σ) are JTg-spaces(resp. JTδg*-spaces).
- If $f : (Y,\zeta) \to (Z,\sigma)$ and $g : (Z,\sigma) \to (P,\mu)$ are J-Homeomorphisms then $g \circ f : (Y,\zeta) \to (P, \mu)$ is a J-Homeomorphism when (Z,σ) is a JTC-space (resp. JTδ-space).
- The composition of two J$\mathcal{C}$-Homeomorphisms is a J$\mathcal{C}$-Homeomorphism.
- If a J-Homeomorphism $f : (Y,\zeta) \to (Z,\sigma)$ is a J$\mathcal{C}$-Homeomorphism when both spaces (Y,ζ) and (Z,σ) are JTC-spaces (resp. JTδ-spaces).
- The set $J\mathcal{C}\hbar(Y, \zeta)$ is a group under the composition of functions.
- A function $f : (Y,\zeta) \to (Z,\sigma)$ is a J$\mathcal{C}$-Homeomorphism, then the function f induces an isomorphism from the group $J\mathcal{C}\hbar(Y,\zeta)$ onto the group $J\mathcal{C}\hbar(Z,\sigma)$.
- The J$\mathcal{C}$-Homeomorphism is an equivalence relation in the collection of all topological spaces.

CHAPTER VIII:

In this **Chapter**, soft regular*-open sets, soft regular*-closed sets, soft η*-open sets, soft η*-closed sets are introduced. Using soft η*-open sets, soft J-closed sets, soft J*-closed sets and soft J**-closed sets are initiated and related properties are discussed.

The following definitions are introduced in this chapter:

- A soft subset (F,E) of a soft topological space (Y,ζ,E) is said to be a **soft regular*-open (soft r*-open) set** if $(F,E) = Int(Cl^*(F,E))$.

- Let (Y,ζ,E) be a soft topological space. Let (A,E) be a soft subset. A point x in Y is called a **soft r* - interior point** of (A,E) if (A,E) contains a soft r* - open set containing x.
- Let (Y,ζ,E) be a soft topological space over Y then the **soft r*-interior** of the soft set (F,E) over Y is denoted by **Sr*Int(F,E)** and is defined as the union of all soft r*-open sets contained in (F,E).
- The complement of a soft regular* - open set is called a **soft regular* - closed set (soft r* - closed).**
- The **soft r*** -c**losure** of (A,E) is defined as the intersection of all soft r* -closed sets of (Y,ζ,E) containing (A,E). It is denoted by **Sr*Cl(A,E)**.
- Let (A,E) be a subset of soft topological space.
- An element x ∈ Y is called **soft r*-adherent point** of (A,E) if every soft r*-open set in (Y,ζ,E) containing x intersect (A,E).
- An element x ∈ Y is called **soft r* -limit point** of (A,E) if every soft regular* -open set in (Y,ζ,E) containing x intersect (A,E) in a point different from x.
- The set of all soft regular* - limit point of (A,E) is called the **soft r* - Derived set** of (A,E). It is denoted by SD_{r*}(A,E).
- A subset (D,E) of a soft topological space (Y,ζ,E) is called
- **soft η*-open set** if it is a union of soft regular*-open sets (soft r*-open sets).
- **soft η*-Interior of (D,E)** is the union of all of soft η*-open sets of Y contained in (D,E).It is denoted by Sη*int(D,E).
- The complement of a soft η*-open set is called a **soft η*-closed set**.We denote soft η*-closed sets in (Y,ζ,E) by Sη*C(Y,ζ,E).
- The intersection of all soft η*-closed sets of Y containing (D,E) is called as the **soft η*-closure** of (D,E) and denoted by Sη*Cl(D,E).
- Let (D,E) be a subset of the topological space (Y,ζ,E),then
- an element (y,E)∈(Y,E) is called **soft η*-adherent point** of (D,E) if every soft η*-open set in Y containing (y,E) intersects (D,E).
- a point (y,E)∈Y is called **a soft η*-cluster point** of D if for every soft η*-open set (V,E) containing (y,E) intersects (D,E) in a point different from (y,E).
- the set of all soft η*-cluster points of (D,E) is denoted by **Sη*-D(D,E)**.
- A soft set (D,E) of a soft topological space (Y,ζ,E) is said to be

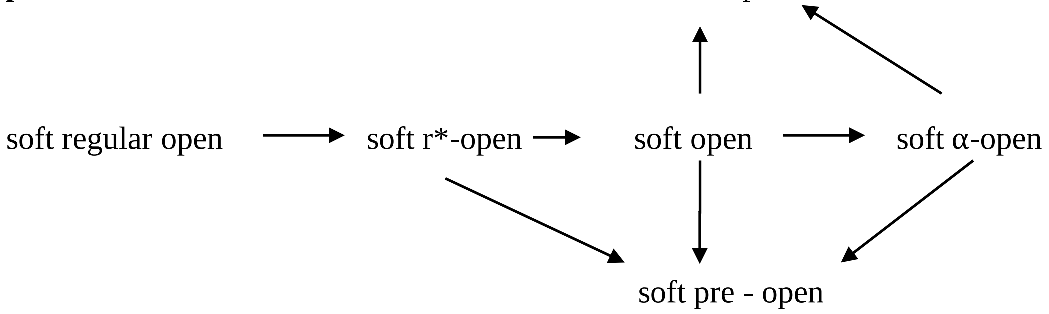
- **soft J-closed** set if Cl(D,E) $\tilde{\subseteq}$(M,E) whenever (D,E) $\tilde{\subseteq}$(M,E) and (M,E) is soft η*-open in Y.
- **soft J*-closed** set if Sη*Cl(D,E) $\tilde{\subseteq}$ (M,E) whenever (D,E) $\tilde{\subseteq}$ (M,E) and (M,E) is soft open in Y.
- **soft J**-closed** set if Sη*Cl (D,E) $\tilde{\subseteq}$ (M,E) whenever (D,E) $\tilde{\subseteq}$ (M,E) and (M,E) is soft η*-open in Y.

The exciting properties of soft r*-open sets are presented in this chapter:

- ❖ The union of two soft r*-open sets need not be soft r*-open.
- ❖ Intersection of any two soft r*-open sets is soft r*-open.
- ❖ SR*O(Y,ζ,E) forms a soft topology on Y if and only if it is closed under arbitrary union.
- ❖ In any soft topological space (Y,ζ,E), SRO(Y,ζ,E) $\tilde{\subseteq}$SR*O(Y,ζ,E)$\tilde{\subseteq}$ ζ.
- ❖ In any soft topological space (Y,ζ,E), if (A,E) and (B,E) are subsets of soft topological space, then the following holds.

- Sr*Int(ϕ, E) = (ϕ, E)
- Sr*Int(Y, E) = (Y,E)
- Sr*Int(A, E) $\tilde{\subseteq}$ (A, E)
- (A,E)$\tilde{\subseteq}$ (B,E)⇒ Sr*Int(A,E) $\tilde{\subseteq}$ Sr*Int(B,E)
- Int(Sr*Int(A,E)) $\tilde{\subseteq}$ Int(A,E)
- srint(A,E) $\tilde{\subseteq}$ Sr*Int(A,E) $\tilde{\subseteq}$ Int(A,E) $\tilde{\subseteq}$ (A,E)
- Sr*Int((A,E)∪ (B,E)) $\tilde{\supseteq}$ Sr*Int(A,E) ∪ Sr*Int(B,E)
- Sr*Int((A, E) ∩(B,E)) = Sr*Int(A,E) ∩S r*Int(B,E)
- (A,E) is soft regular* - open iff (A,E) = Sr*Int(A,E).

soft r*-open sets is stronger as well as weaker than other soft sets are shown in this chapter.

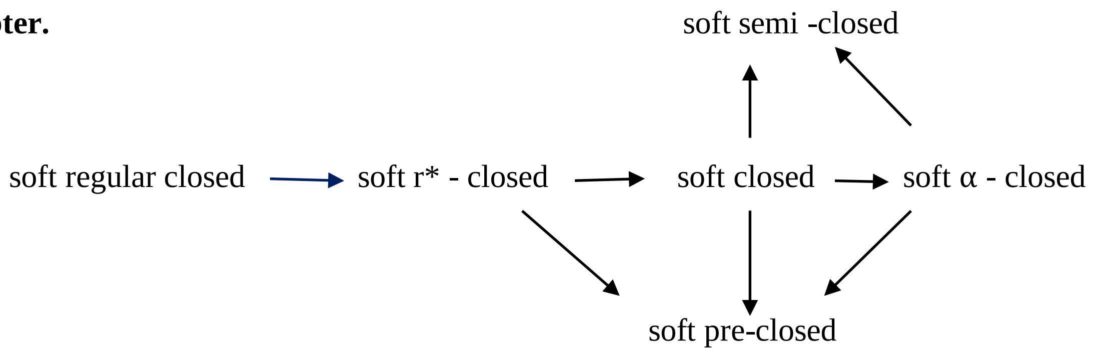


Properties of soft r*-closed sets:

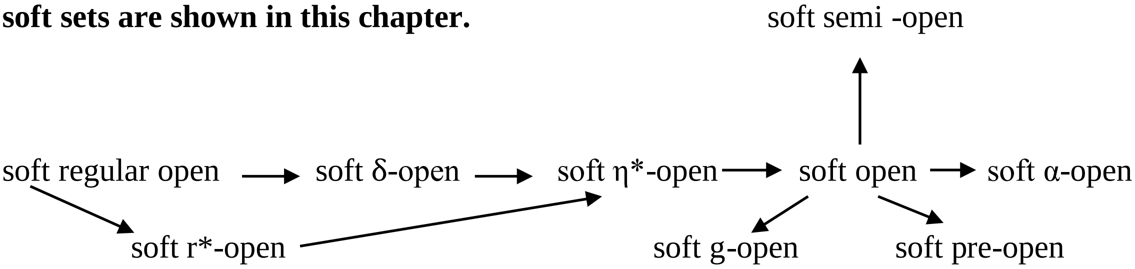
- ❖ If a soft set is soft r* - closed, then (A,E) = Cl(Int*(A,E)).
- ❖ Union of any two soft r* - closed sets is soft r* - closed.
- ❖ In any soft topological space (Y,ζ,E), the following results hold:

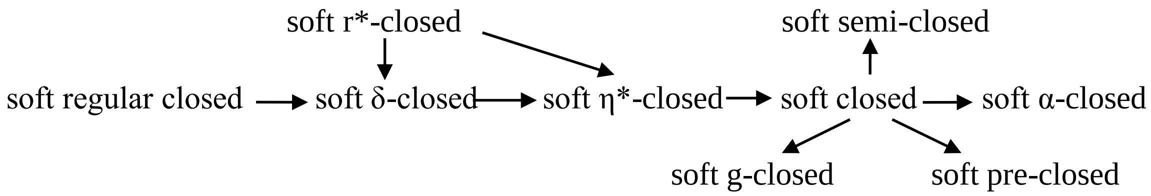
- Sr*Cl(ϕ, E) = (ϕ,E)
- Sr*Cl(Y,E) = (Y,E)
- (A,E) $\tilde{\subseteq}$ Sr*Cl(A,E)
- (A,E) $\tilde{\subseteq}$ (B,E)⇒ Sr*Cl(A,E) $\tilde{\subseteq}$ Sr*Cl(B,E), if (A,E) and (B,E) are soft subsets of (Y,ζ,E).
- (A,E) $\tilde{\subseteq}$Cl(A,E) $\tilde{\subseteq}$ Sr*Cl(A,E) $\tilde{\subseteq}$srcl(A,E).
- Sr*Cl((A,E)∪ (B,E)) = Sr*Cl(A,E) ∪ Sr*Cl(B,E), if (A,E) and (B,E) are soft subsets of (Y,ζ,E).
- Sr*Cl((A,E) ∩(B,E) ⊆ Sr*Cl(A,E) ∩ Sr*Cl(B,E), if (A,E) and (B,E) are soft subsets of (Y,ζ,E).
- Cl(A,E) $\tilde{\subseteq}$Cl(Sr*Cl(A,E)).

soft r*-closed sets is stronger as well as weaker than other soft sets are shown in this chapter.



soft η*-open sets and soft η*-closed sets are stronger as well as weaker than other soft sets are shown in this chapter.



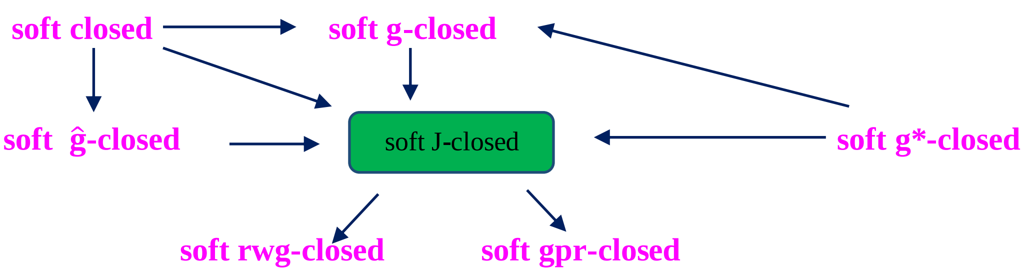


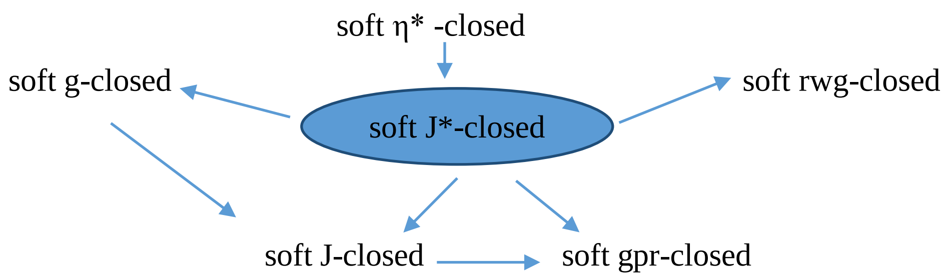
The exciting properties of soft η*-open sets and soft η*-closed sets are presented in this chapter:

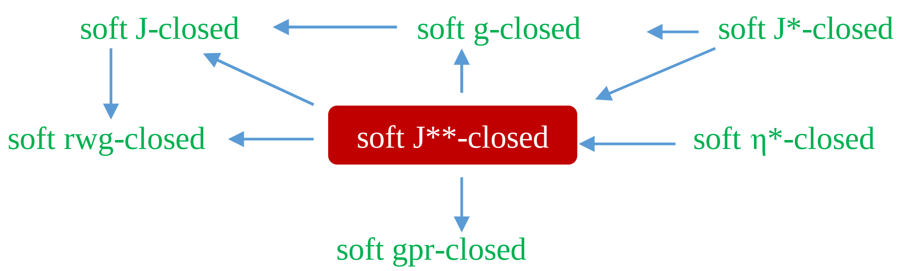
- In any soft topological space (Y,ζ,E),Y and ϕ are soft η* -open sets.
- Any arbitrary union of soft η* -open sets are soft η* -open sets.
- The finite intersection of soft η* -open sets are soft η* -open sets.
- (D,E) is soft η*-open iff (D,E) = Sη*int(D,E).
- In any soft topological space (Y,ζ,E), if (C,E) and (D,E) are subsets of Y then we get the following :
  - Sη*int(ϕ) = (ϕ,E)
  - Sη*int(Y,E) = (Y,E)
  - Sη*int (D,E) $\tilde{\subseteq}$ (D,E)
  - (C,E) $\tilde{\subseteq}$ (D,E)⇒Sη*int(C,E) $\tilde{\subseteq}$Sη*int(D,E)
  - Sη*int(Sη*int(D,E)) $\tilde{\subseteq}$Sη*int(D,E)
  - Sη*int($\bigcup_{i\in\varepsilon}$\{ D_i, E\}) = $\bigcup_{i\in\varepsilon}$ Sη*int(\{D_i,E\})
  - Sη*int((C,E)∩(D,E)) = Sη*int(C,E) ∩Sη*int(D,E).
- In any soft topological space (Y,ζ,E), if (C,E) and (D,E) are subsets of Y then we get the following :
  - Sη*Cl(∅,E) = (∅,E)
  - Sη*Cl(Y,E) = (Y,E)
  - (D,E) $\tilde{\subseteq}$ Sη*Cl(D,E)
  - (C,E) $\tilde{\subseteq}$(D,E) ⇒ Sη*Cl(C,E) $\tilde{\subseteq}$ Sη*Cl(D,E),if (C,E) and (D,E) are subsets of (Y,ζ,E)
  - Sη* Cl((C,E)∪(D,E)) = Sη* Cl (C,E) ∪ Sη* Cl (D,E)

- $S\eta^*Cl(\cap_{i\in\varepsilon}\ \{D_i, E\}) = \cap_{i\in\varepsilon}\ S\eta^*Cl\ (\{D_i, E\})$
- $Cl(D, E) \widetilde{\subseteq} Cl(S\eta^*Cl(D, E))$
- $S\eta^*\ Cl(\ S\eta^*Cl(C,E)) = S\eta^*Cl(C,E)$.

soft J-closed sets, soft J*-closed sets and soft J-closed sets are stronger as well as weaker than other soft sets are shown in this chapter.**







The exciting properties of soft J-closed sets, soft J*-closed sets and soft J-closed sets are presented in this chapter:**

- ❖ If (D,E) is soft J-closed in Y and (D,E) $\widetilde{\subseteq}$ (B,E) $\widetilde{\subseteq}$ Cl(D,E),then (B,E) is soft J-closed.
- ❖ If (D,E) and (H,E) are soft J-closed sets then their union also soft J-closed.
- ❖ A soft set (D,E) is soft J-closed iff Cl(D,E) \ (D,E) contains only null soft η*-closed set.
- ❖ A soft J-closed set (D,E) is soft closed iff Cl(D,E) \ (D,E) is soft η* -closed.

- ❖ Let (G,E) be a soft subset of soft topological space (Y,ζ,E).If (G,E) is soft η*-open and soft J-closed,then (G,E) is a soft closed set.
- ❖ If (D,E) is soft J*-closed in Y and (D,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$Sη*Cl(D,E),then (B,E) is soft J*-closed.
- ❖ If (D,E) and (H,E) are soft J*-closed sets then their union also soft J*-closed.
- ❖ A soft set (D,E) is soft J*-closed iff Sη*Cl(D,E) \ (D,E) contains only null soft closed set.
- ❖ A soft J*-closed set (D,E) is soft η* -closed iff Sη*Cl(D,E) \ (D,E) is soft closed.
- ❖ Let (G,E) be a soft subset of soft topological space (Y,ζ,E).If (G,E) is soft open and soft J*-closed,then (G,E) is a soft η*-closed set.
- ❖ If (D,E) is soft J**-closed in Y and (D,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$Sη*Cl(D,E),then (B,E) is soft J**-closed.
- ❖ If (D,E) and (H,E) are soft J**-closed sets then their union also soft J**-closed.
- ❖ A soft set (D,E) is soft J**-closed iff Sη*Cl(D,E) \ (D,E) contains only null soft η* -closed set.
- ❖ A soft J**-closed set(D,E) is soft η* -closed iff Sη*Cl(D,E) \ (D,E) is soft η*-closed.
- ❖ Let (G,E) be a soft subset of soft topological space (Y,ζ,E).If (G,E) is soft η*-open and soft J**-closed, then (G,E) is a soft η*-closed set.

- ♣ **During the course of my work, it is essential to produce many counter examples to substantiate the results. Hence, for the topological spaces containing three elements and four elements, various open sets and closed sets are derived and the collections are tabulated in Appendix I and Appendix II.**
- ♣ **These appendices are like dictionary of examples for those who work in a similar line.**