Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination – May 2025
II Semester

Class : I P.G. **Time: 3 Hours**
Major : Physics **Max. Marks: 100**

23MPHC10 Mathematical Physics II

Course Outcomes:
CO1: Solve first order, second order homogeneous and non-homogeneous equations
CO2: Deliver mathematical modelling for Physics problems involving partial differential equations
CO3: Solve differential equations using Laplace transforms
CO4: Arrive at a solution for partial differential equation employing Fourier transform
CO5: Apply special functions in solving integral functions

Part A **10 x 1 = 10**
Choose the Correct Answer

1. The general solution of the differential equation dy/dx=xy is: CO1K2
   a. $y= x^2+C$ b. $y^2= x^2+C$ c. $y = e^x+C$ d. $y = \ln x+C$

2. The orthogonal trajectories of the family of curves y=kx are CO1K2
   a. $y^2=x^2+C$ b. $x^2+y^2=C$ c. $y=cx$ d. $y=cx^2$

3. A partial differential equation (PDE) contains: CO2K1
   a. Only one independent variable b. More than one independent variable
   c. No derivatives d. Only algebraic expressions

4. The Green's function for the one-dimensional Helmholtz equation CO2K1
   $d^2G/dx^2 + k^2 G = \delta(x-x')$ is
   a. $G(x,x')=(1/2k)e^{ik|x-x'|}$ b. $G(x,x')=(1/k)e^{ik|x-x'|}$
   c. $G(x,x')=(1/k^2)e^{ik|x-x'|}$ d. $G(x,x')=(1/2k^2)e^{ik|x-x'|}$

5. The Laplace transform of e^{-at} is given by: CO3K1
   a. 1/s b. 1/a c. 1/(s+a) d.1/(s-a)

6. The Laplace transform of the Dirac delta function $\delta(t-a)$is CO3K1
   a. e^{-a} b. e^{-s} c. e^{as} d. e^{-as}

7. The Fourier transform of a Gaussian function is: CO4K1
   a. Another Gaussian function b. A sinusoidal function
   c. A polynomial function d. A step function

8. The Fourier sine transform of $f(x)=e^{-ax}$ is CO4K1
   a. $a/(a^2-k^2)$ b. $a/(a^2+k^2)$ c. $k/(a^2+k^2)$ d. $k/(a^2+k^2)$

9. The recurrence relation for Bessel functions Jn(x) is CO5K1
   a. $J_{n-1}(x)+J_{n+1}(x)=(2n/x)J_n(x)$ b. $J_{n-1}(x)-J_{n+1}(x)=(2n/x)J_n(x)$
   c. $J_{n-1}(x)+J_{n+1}(x)=(2/x)J_n(x)$ d. $J_{n-1}(x)-J_{n+1}(x)=(2/x)J_n(x)$

10. The Legendre polynomial $P_n(x)$ satisfies the differential equation: CO5K1
    a. $(1-x^2)y''-2xy'+n(n+1)y=0$ b. $y''+xy'+ny=0$
    c. $y''+xy=0$ d. $y''-xy'+n^2y=0$

Part B **5 x 6 = 30**

Answer ALL questions

Each answer should not exceed 400 words or two pages

11.a. Solve $(x^2-yx^2)dy+(y^2+xy^2)dx=0$ CO1K3

(or)

11.b. Find the orthogonal trajectories of the family of curves $y=kx^2$, where k is a constant. CO1K3

12.a. Derive the expression for the Laplacian operator ∇^2in cylindrical polar coordinates. CO2K2

(or)

12.b. Derive the expression for the divergence of a vector field in cylindrical polar coordinates. Explain its physical interpretation. CO2K2

13.a. Find the Laplace transforms of (i) t sin at (ii) t cos at CO3K2

(or)

13.b. Define Dirac-delta function and show that $\int_{-\infty}^{+\infty} f(x)\delta(x-a)dx = f(a)$ CO3K2

14.a. Find the Fourier transform of $e^{-|t|}$. CO4K2

(or)

14.b. Find the finite Fourier sine transform of the function $f(x)=x^2$; $0<x<4$. CO4K2

15.a. Find the value of (i) $\Gamma\frac{1}{9}\Gamma\frac{2}{9}\Gamma\frac{3}{9}$ .... $\Gamma\frac{8}{9}$ (ii) $\Gamma 0.1\,\Gamma 0.2\Gamma 0.3$ ....$\Gamma 0.9$ CO5K3

(or)

15.b. Show that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}dx = \frac{\beta(m,n)}{a^n(1+a)^m}$ CO5K3

Part C **5 x 12 = 60**

Answer ALL questions

Each answer should not exceed 800 words or four pages

16.a. Solve the second-order inhomogeneous differential equation:$d^2y/dx^2+4y=\sin(2x)$. CO1K3

(or)

16.b. Using the Frobenius method, find the series solution of the differential equation: $x^2(d^2y/dx^2)+x(dy/dx)+(x^2-1)y=0$. CO1K3

17.a. Solve the one-dimensional diffusion equation using the separation of variables method: $\partial u/\partial t=D\partial^2 u/\partial x^2$ CO2K2

(or)

17.b. Solve the Helmholtz equation in spherical polar coordinates. CO2K2

18.a. Elaborate the properties of Laplace transform. CO3K3

(or)

18.b. Obtain the Laplace transform of the function CO3K2

$$F(t)=\begin{cases}\sin\omega t; 0<t<\frac{\pi}{\omega}\\ 0; \quad \frac{\pi}{\omega}<t<\frac{2\pi}{\omega}\end{cases}$$

19.a. Find the Fourier series expansion of the function f(x) given by f(x)=x, $-\pi<x<\pi$. CO4K3

(or)

19.b. Compute the Fourier transform of $f(x)=e^{-|x|}$ and discuss its properties. CO4K3

20.a. Derive the Rodrigues formula for Legendre polynomials and verify it for $P_2(x)$. CO5K3

(or)

20.b. Solve the differential equation for Laguerre polynomials: $xy''+(1-x)y'+ny=0$. CO5K3

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