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Abstract

In chapter 1, definitions of various characteristics of queueing process, descriptions of the models under consideration, review of literature, methodology and some preliminary results are presented.In Chapter II,it is assumed thata cycle starts whenever the system empties and the server is deactivated and leaves the system immediately for vacation of random length.The server operates (m, N)-policy with at most J-multiplevacations.During busy period,the server provides singleFES to allarriving customers in the first phase and C-optional heterogeneous services in the second phase.As soon as the FES is completed, each customer may either opt for a certain (i^{th})service in the second phase (with probability r_i, $0 \le r_i \le 1$) or may leave the system(with probability $(1-\sum_{i=1}^{C} r_i)$).The server is subject to random breakdowns during busy period and the server's lifetime follows the exponential distribution in the first phase with parameter **a**. In the second phase, the server fails at an exponentialrate $\mathbf{a_i}$($1 \le i \le C$).Wheneverbreakdowns occur,it is assumed that the server is sent for repair immediately. The customer,just being served just before breakdown, waits for the server to return from the repair facility to complete the remaining service. As soon as the server is fixed,the service resumes for the waiting customer. The vacation period,buildup period,setup period,dormant period,busy period and breakdown period constitute a cycle for the model.The model analyzed in chapter III differs from the model of chapter II, in service facility and the customers' behaviour during breakdown period. The author scrutinizes the (m, N)-policy by considering three possible scenarios to restoring an interrupted service together in a single $M^X/G/1$ queueing system. They include, if the server fails,then the customer in service mayjoin the head of the queue and opt for a new service (with probability q_1) or may leave the system without completing the service (with probability q_2) or else stay in the service facility (with probability $q_3 = 1-(q_1 + q_2)$) to complete the remaining service.The repair times of the server follow distinct general distributions of finite moments. The hypothesis regarding (m, N)-policy with at most J-vacations is as in Chapter II.In Chapters II and III, it is assumed that the server takes multiple vacations only when the system becomes empty. In some situations,especially when the services are executed in two or more phases, maintenance of the system is required at the completion of each service.In Chapter IV, the (m, N)-policy is inspected for an $M^X/G/1$ queueing system with the provisions of various types of generally distributed long and short vacations. It is assumed that at the end of busy period, the server becomes free andtakes a single vacation (VI)(with probability p')or stays idle in the system (with complementary probability).During busy period, after completing a service to a customer,the server may choose (i^{th}) type of vacationVB_i(with probability p_i, ($1 \le i \le M$)) or continueto serve the next customer(with probability $(1-\sum_{i=1}^{M} p_i)$).The other assumptions regarding (m, N)-policy, busy period,breakdown period and repair facility are as in Chapter II with C =1

and J= 1.In Chapter V, the (m, N)-policy is studied for an $M^X/G/1$ queue, in which the unsatisfied customers, after completing the regular service, may demand a re-service with probability **f**or leave the system with probability **(1−f)**is admissible.It is also assumed that the server may take Bernoulli Scheduled Vacations in between services.Two different vacation distributions, $VB_1(t)$ and $VB_2(t)$ are considered. The server, after completing each service, either takes a vacation of type VB_1(with probability p_{b_1})orVB_2(with probabilityp_{b_2}), depending on whether the customer leaves the system or opts for feedback.Otherwise,the server continues to serve with probability $(1-p_{b_1})$ in case 1 or $(1-p_{b_2})$ in case 2.During idle period, a more general J-vacation policy controlled by the parameter **p** is considered.That is,when the system becomes empty, the server immediately takes a vacation.Upon returning from the vacation,the server is activated if there are at least**m** customers waiting in the queue.On the other hand, if the number of

customers waiting in the queue is less than **m**,the server either joins the system with probability (1 – p) or leaves for another vacation with probability **p**.The pattern continues until the number of vacations reaches **J**. At the end of the $\mathbf{J}^{\mathbf{th}}$vacation if the queue length is still less than **m**, the server joins the system and stays idle until the queue length reaches at least **m** to start the setup work. The process after setup period regarding (m,N)-policy is as in Chapter II.Most of the queueing models with service interruptions assume that interrupted server is sent for repairimmediately. On the contrary, this is possible only when the repair facility is available on a permanent basisin the service station.In chapter VI, the author analyses a repairable batch arrival queueing modelwith two phasesof heterogeneous service where the repair work is delayed.The lifetime of the server follows exponential distributionswith parameters a_i, i = 1, 2 in phase 1 and 2 respectively. The customer whose service has been interrupted will repeat the service from the beginning. After the first round of service,the customer who wants to repeat the service will immediately reiterate service with feedback probability **f**. Bernoulli Schedule Vacation policy controlled by the parameter **p** is considered during busy period and MAV policy is adopted during idle period.A setup time is also introduced at the beginning of every cycle.Chapters VII and VIII examine $M^X/G/1$ queueing system with immediate feedbacksand multi second optional service facilities. The server operates single service in the first phase and different kinds of heterogeneous services in the second phase. A customer is said to complete the first round service if he undergoes the first phase service and any one of the second phase services. After having completed the first round service, the customer is permitted to repeat services from the second multi-optional services which may be different from the one chosen earlier. Kalidass and Kasturi (2013) analyzed a reliable Poisson arrival M/G/1 queue with two phases of heterogeneous service and a finite number of immediate Bernoulli feedbacks before leaving the system. In Chapter VII of the present work, the author studies the $M^X/G/1$ queueing system where customers can feedback finitely many times, the server undergoes unpredictable breakdowns and takes optional vacations between services. When the server fails, the customer in service may resume or repeat service. These two cases are considered separately in sections 7.1 and 7.2. The corresponding infinite feedback cases are investigated in sections 8.1 and 8.2.

i) Major objectives :

- The steady state system size equations governing the mathematical models are derived using the supplementary variable technique. The probability generating functions of the system size at arbitrary epoch are obtained.
- The performance indices namely the probability that the server is in different states and the mean queue length when the system is in different states are also established.
- A procedure to find the optimal threshold values under a linear cost structure is developed for the models from Chapters II to V.
- The effects of various system parameters on the system performance are numerically analyzed and graphically depicted to interpret the data more comprehensively.
- The stochastic decomposition property has been verified.
- The systems studied are among the most general queuing systems and include many previous works as special cases.

ii) Hypothesis:

iii) Methodology :

- **Transient and Steady-State Solution :**

Let N(t) denote the number of customers in the system at time t and its probability distribution be denoted by $P_n(t)$ = Pr (N(t) = n / N(0) = •). For a complete description of the queueing process, the transient or time-dependent solutions are necessary. But it is often difficult to obtain such solutions. Further in many practical situations, we need to know the behaviour in steady-state, i.e., when the system reaches an equilibrium state, after being in operation for a pretty long time. The time-dependent solutions are called transient solutions. And the solutions obtained as $t \to \infty$ are called steady-state solutions. Throughout the present work, the queueing systems are analysed at steady state, assuming that the system size probabilities are independent of time t, as $t \to \infty$.

Markovian and Non-Markovian Queueing Models

- The exponential distribution is the only continuous distribution which satisfies the Markov property. The queueing models in which all the continuous time random variables involved, follow exponential distribution are said to be Markovian queueing models. In non-markovian queueing models atleast one random variable follows distribution other than exponential.

The first step in analyzing a queueing system is to set it up as a Markov process. There are a number of techniques or approaches that are used for this purpose. In most practical queueing systems, supplementary variables are usually needed to achieve this. The alternative to that is an embedded Markov chain method.

- **Supplementary Variable Technique (SVT) (Alfa and Srinivasa Rao, 2000)**

In queueing literature, there exists two kinds of supplementary variables, in general. They are elapsed time and the remaining time of random variables. In both the cases, the approaches of deriving the queueing characteristics are different. The main reason for adding supplementary variables to a stochastic process variable is to make the system Markovian. Cox (1955) considered elapsed service time as supplementary variable to study the M/G/1 queueing system. The use of the supplementary variable technique (SVT) in queueing was introduced by Kosten (1973). Later, the technique became popular for most stochastic models. The technique of remaining time as the supplementary variable is simple and elegant. The supplementary variable technique analysis for the queueing problems by considering the remaining time as supplementary variable involves, probability density function and partial differential equations.

iv) Findings:

Mathematically, the (m,N) policy vacation queueing models are the generalizations of N-policy and the classical 1-policy queueing models. The system size of every (m,N) policy model is decomposed into two random variables, one of which is the system size of the corresponding $M^X/G/1$ model without (m,N)policy and the other random variable gives the PGF of the conditional system size distribution ($\frac{I_{(m,N)}(z)}{I_{(m,N)}(1)}$) during the server idle period.Thus deriving the expression for $I_{(m,N)}(z)$ is the primary task for the (m,N) policy queueing models. Lee et al. (2003) have defined β_n, ψ_n and ϕ_n for the non-vacation, single and multiple vacation $M^X/G/1$ reliable single server queueing systems independently and obtained $I_{(m,N)}(z)$ in terms of these functions. In chapters II to V of the presentwork, the author has made an attempt to derive the expression for $I_{(m,N)}(z)$ by re-defining β_n, ψ_n and ϕ_n suitably for J-vacation policy, <p,J> vacation policy, Bernoulli schedule single vacation policy and has shown that the corresponding results of the single , multiple and non-vacation queueing models can be deduced as special cases of these variant vacation policies.

The present work also concentrates on the following features which are not considered in the literature of (m,N) policy models.

- The server operates two phases of services whereby, the first phase is a single essential phase and the second phase being an optional phase and contains multi- optional service facilities.
- The server is subject to breakdowns while serving the customers and the service interrupted customers thus may resume the service from where they were interrupted or may repeat their service from the beginning or quit the system without completing the service.
- The dissatisfied customers may demand for re-services after completing their regular service.
- The server can take optional vacation between any two consecutive services during busy period.

Chapters VI to VIII deal with the 1-policy queueing systems with distinct features:

Chapter VI considers two phase service channels wherein each customer undergoes two stages of services one after another and the server operates, Multiple Adapted Vacation policy (MAV) during idle period and optional Bernoulli Schedule Vacation policy during busy period. The dissatisfied customers are allowed to demand re-services after completing the regular service. The server is subject to breakdowns and whenever the system breaks down, the repair time is delayed and the service interrupted customers will repeat their service from the beginning as soon as the server is fixed. The MAV policy generalizes the other vacation policies. It is shown that the results including that of <p,J> vacation policy can be derived from the results of MAV models.Most of the existing queueing models allow the customers to take infinite number of feedbacks which are hardly possible in everyday life. In chapter VII, a repairable $M^X/G/1$ queueing model with finite number of immediate feedbacks (at most m) is considered. The system also contains multi-optional service facilities . A corresponding queueing model with infinite number of feedbacks is also analysed in chapter VIII to justify that as $m\to\infty$ the results of both the models coincide with each other.The cases of repeat or resumption of interrupted service when the server is fixed are considered as two different cases in chapters VII and VIII and it is shown that if the service time is exponential, then the two types of behaviours of the interrupted customers will lead to the same result. All the models analysed are unique and general in nature. The analytical treatment of all models are done by Supplementary Variable Technique. The PGFs of the queue length distribution are presented in a compact form. The partial generating functions of the system size when the system is in different states are calculated and the formulae for the system size probabilities and the expected number of customers waiting in the system when the system is in different states are also derived. Moreover, expected cycle length, expected idle time and expected busy period are also calculated.A cost model is introduced for the models of chapters II to V, and a procedure to find the optimal values of (m,N) that minimize the average cost is also developed. The results of various queueing models are derived as particular cases of the models presented in the work.The decomposition property of vacation queues is established for all the models and the PGFs of the departure point system size distribution are also derived .The numerical computation of performance measures for batch arrival models (especially for (m,N) policy) are not easy in general. The measures for all the batch arrival models are calculated using C++ programs.

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