CHAPTER - IV

FUZZY SOFT RELATIONS

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Definition : 4.1 [21]

Let (F, A) and (G, B) be two fuzzy soft sets in the fuzzy soft class (U, E). Their **intersection** is defined as the fuzzy soft set (H, C) where $C = A \cap B$ and $H(\alpha) = F(\alpha) * G(\alpha)\ \forall\ \alpha \in C$, where $*$ is the operation t - norm between the fuzzy sets $F(\alpha)$ and $G(\alpha)$.

Example : 4.2

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and $E = \{e_1$(expertise in english), e_2(expertise in mathematics), e_3(expertise in chemistry), e_4(expertise in computer science)$\}$ be the set of parameters and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$ and $B = \{e_1, e_5\} \subseteq E$ Then

$(F, A) = \{F(e_1) = \{S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8\},$

$F(e_2) = \{S_1/0.3, S_2/0.9, S_3/0.4, S_4/0.3\},$

$F(e_3) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8\},$

$F(e_4) = \{S_1/0.1, S_2/0.7, S_3/0.3, S_4/0.2\}\}$

$(G, B) = \{G(e_1) = \{S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8\},$

$G(e_5) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7\}\}$

$(F \cap G)(e_1) = \{S_1/0.72, S_2/0.04, S_3/0.30, S_4/0.64\}$

Definition : 4.3 [21]

Let (F, A) and (G, B) be two fuzzy soft sets in the fuzzy soft class (U, E). Their **union** is defined as the fuzzy soft set $(H, C) = (F, A) \cup (G, B)$

$$\text{where } C = A \cup B \text{ and } H(\alpha) = \begin{cases} F(\alpha) & \text{if } \alpha \in A - B \\ G(\alpha) & \text{if } \alpha \in B - A \\ F(\alpha) \lozenge G(\alpha) & \text{if } \alpha \in B \cap A \end{cases}$$

where $\lozenge$ is the operation t – conorm between the fuzzy sets $F(\alpha)$ and $G(\alpha)$.

Example : 4.4

From Example 4.2

$(F \cup G)(e_1) = \{S_1/0.98, S_2/0.36, S_3/0.80, S_4/0.96\}$,

$(F \cup G)(e_2) = F(e_2)$,

$(F \cup G)(e_3) = F(e_3)$,

$(F \cup G)(e_4) = F(e_4)$,

$(F \cup G)(e_5) = G(e_5)$.

Definition : 4.5 [21]

Let (F, A) and (G, B) be two fuzzy soft sets in the fuzzy soft class (U, E). Their **Cartesian product** is defined as $(F, A) \times (G, B) = (H, C)$ Where $C = A \times B$ and $H(\alpha, \beta) = F(\alpha) * G(\beta)\ \forall\ (\alpha, \beta) \in C$.

Here $\times$ is the operation t - norm between the fuzzy sets $F(\alpha)$ and $G(\beta)$.

Example : 4.6

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and $E = \{e_1$(expertise in english), e_2(expertise in mathematics), e_3(expertise in chemistry), e_4(expertise in computer science)$\}$ be the set of parameters and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$ and $B = \{e_1, e_5\} \subseteq E$ Then

$(F, A) = \{F(e_1) = \{S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8\}$,

$F(e_2) = \{S_1/0.3, S_2/0.9, S_3/0.4, S_4/0.3\}$,

$F(e_3) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8\}$,

$F(e_4) = \{S_1/0.1, S_2/0.7, S_3/0.3, S_4/0.2\}\}$

$(G, B) = \{G(e_1) = \{S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8\}$,

$G(e_5) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7\}\}$

Then $(F, A) \times (G, B) = (H, C)$ where $C = A \times B$ and

$(H, C) = \{H(e_1, e_1) = \{S_1/0.72, S_2/0.04, S_3/0.30, S_4/0.64\}$,

$H(e_1, e_5) = \{S_1/0.54, S_2/0.06, S_3/0.54, S_4/0.56\}$,

$H(e_2, e_1) = \{S_1/0.24, S_2/0.18, S_3/0.20, S_4/0.24\}$,

$H(e_2, e_5) = \{S_1/0.18, S_2/0.27, S_3/0.36, S_4/0.21\}$,

$H(e_3, e_1) = \{S_1/0.48, S_2/0.06, S_3/0.45, S_4/0.64\}$,

$H(e_3, e_5) = \{S_1/0.36, S_2/0.09, S_3/0.81, S_4/0.56\}$,

$H(e_4, e_1) = \{S_1/0.08, S_2/0.14, S_3/0.15, S_4/0.16\}$,

$H(e_4, e_5) = \{S_1/0.06, S_2/0.21, S_3/0.27, S_4/0.14\}\}$

Definition : 4.7 [21]

Let (F, A) and (G, B) be two fuzzy soft sets in the fuzzy soft class (U, E). Then **fuzzy soft relation** (R, C) from (F, A) to (G, B) is a fuzzy soft subset of $(F, A) \times (G, B)$ where $C \subseteq A \times B$ and $(R, C) \subseteq (F, A) \times (G, B)$. If (R, C) is a fuzzy soft subset of (F, A) to (F, A), then it is called a fuzzy soft relation on (F, A).

Example : 4.8

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and

A = { expertise in physics (p), expertise in chemistry (c), expertise in biology (b)},
B = { expertise in physics (p), expertise in history (h)}.

(F, A) = {F(p) = {S_1/0.9, S_2/0.2, S_3/0.6, S_4/ 0.8},

F(c) = {S_1/0.3, S_2/0.9, S_4/0.4, S_5/0.3},

F(b) = {S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8}}

(G, B) = {G(p) = {S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8},

G(h) = {S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7}}

We take C = {(p, p), (c, p), (b, p)} $\subseteq$ A $\times$ B

Let (R, C) = {R (p, p) = {S_1/0.72, S_2/0.04, S_3/0.30, S_4/0.64},

R (c, p) = {S_1/0.24, S_2/0.18, S_3/0.20, S_4/0.24},

R (b, p) = (S_1/0.48, S_2/0.06, S_4/0.45, / 0.64}}

Clearly (R, C) $\subseteq$ (F, A) $\times$ (G, B) and hence a fuzzy soft relation from (F, A) to (G, B).

Definition : 4.9

A fuzzy soft relation R on (F, A) is said to be **fuzzy soft symmetric relation** if R (p, q) = R (q, p), $\forall$ p, q $\in$ A.

Definition : 4.10

Let R be fuzzy soft relation from (F, A) to (G, B) then **inverse relation** R^{-1} is defined as R^{-1} (p, q) = R (q, p), $\forall$ p, q $\in$ D $\subseteq$ B $\times$ A.

Example : 4.11

Define a fuzzy soft inverse relation R^{-1} from (G, B) to (F, A). From Example 4.8 (p, p), (p, c), (p, b) $\in$ D $\subseteq$ B $\times$ A. Then

$(R^{-1}, D) = \{ R^{-1}(p, p) = \{S_1/0.72, S_2/0.04, S_3/0.30, S_4/0.64\},$

$R^{-1}(p, c) = \{S_1/0.24, S_2/0.18, S_3/0.20, S_4/0.24\},$

$R^{-1}(p, b) = \{S_1/0.48, S_2/0.06, S_3/0.45, S_4/0.64\}\}.$

Theorem : 4.12

If R is fuzzy soft relation from (F, A) to (G, B) then R^{-1} is a fuzzy soft relation from (G, B) to (F, A).

Proof:

$R^{-1}(p, q) = R(q, p) \subseteq G(q) \cap F(p), \forall (p, q) \in C \subseteq B \times A.$

Hence R^{-1} is fuzzy soft relation from (G, B) to (F, A).

Theorem : 4.13

If R and S be two fuzzy soft relations from (F, A) to (G, B) then

1) $(R^{-1})^{-1} = R$
2) $R \subseteq S \Rightarrow R^{-1} \subseteq S^{-1}$

Proof:

1) $(R^{-1})^{-1}(p, q) = R^{-1}(q, p) = R(p, q)$

   It follows that $(R^{-1})^{-1} = R$

2) $R(p, q) \subseteq S(p, q) \Rightarrow R^{-1}(q, p) \subseteq S^{-1}(q, p)$

   $\Rightarrow R^{-1} \subseteq S^{-1}$

Definition : 4.14

Let R and S be two fuzzy soft relations from (F, A) to (G, B) and

(G, B) to (H, C) respectively. Then the **composition** $\circ$ of R and S is defined by $(R \circ S)(p, q) = R(p, r) \cap S(r, q)$.

Example : 4.15

Let U = {S_1, S_2, S_3, S_4} be the set of students under consideration and A = { expertise in physics (p), expertise in chemistry (c), expertise in biology (b)}, B = { expertise in physics (p), expertise in history (h)} and C = { expertise in chemistry (c), expertise in education (e)}. Then

(F, A) = {F(p) = {S_1/0.9, S_2/0.2, S_3/0.6, S_4/ 0.8},

F(c) = {S_1/0.3, S_2/0.9, S_4/0.4, S_5/0.3},

F(b) = {S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8}}

(G, B) = {G(p) = {S_1/0.8, S_2/0.2, S_3/0.5, S_4/0.8},

G(h) = {S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.7}}

(H, C) = {H(c) = {S_1/0.3, S_2/0.5, S_3/0.2, S_4/0.1},

H(e) = {S_1/0.4, S_2/0.2, S_3/0.7, S_4/0.1}}

Thus we have

(R, E) = {R (p, p) = {S_1/0.72, S_2/0.04, S_3/0.30, S_4/0.64},

R (c, p) = {S_1/0.24, S_2/0.18, S_3/0.20, S_4/0.24},

R (b, p) = {S_1/0.48, S_2/0.06, S_3/0.45, S_4/0.64}} where $E \subseteq A \times B$ and

(S, F) = {S (p, c) = {S_1/0.24, S_2/0.10, S_3/0.10, S_4/0.08}} where $F \subseteq B \times C$

$\therefore$ $(R \circ S)(p, c) = R(p, p) \cap S(p, c)$

= {S_1/0.1728, S_2/0.004, S_3/0.03, S_4/0.0512}

Theorem : 4.16

If R and S be two fuzzy soft relations from (F, A) to (G, B) and (G, B) to (H, C) respectively, satisfying t - norm condition. Then $R \circ S$ is fuzzy soft relation from (F, A) to (H, C).

Proof:

By definition

$R(p, q) = \{x / F(p) * G(q) : x \in U\}, \forall (p, q) \in A \times B$

$S(q, r) = \{x / G(q) * H(r) : x \in U\}, \forall (q, r) \in B \times C$

Therefore

$(R \circ S)(p, r) = \{x / (F(p) * G(q)) * (G(q) * H(r)) : x \in U\}, \forall (p, q, r) \in A \times B \times C$

Now

$(F(p) * G(q)) * (G(q) * H(r))$

$= (F(p) * G(q)) * H(r) \subseteq F(p) * 1 * H(r)$

$= F(p) * H(r)$

Hence $R(p, q) \cap S(q, r) \subseteq F(p) \cap H(r)$

Thus $R \circ S$ is fuzzy soft relation from (F, A) to (H, C).

Theorem : 4.17

The composition operation of soft mappings is associative.

Proof:

Let R, S, T are three fuzzy soft relation from (F, A) to (G, B), (G, B) to (H, C) and (H, C) to (L, D) .Then

$$((R \circ S) \circ T)(a, d) = (R \circ S)(a,c) \cap T(c, d)$$

$$= (R(a, b) \cap S(b, c)) \cap T(c, d)$$

$$= R(a, b) \cap ((S(b, c) \cap T(c, d))$$

$$= R(a, b) \cap (S \circ T)(b, d)$$

$$= (R \circ (S \circ T))(a, d)$$

Hence $(R \circ S) \circ T = R \circ (S \circ T)$

Definition : 4.18

A fuzzy soft relation R on (F, A) is said to be **fuzzy soft reflexive relation** if $R(p, q) \subseteq R(p, p)$ and $R(q, p) \subseteq R(p, p)$, $\forall\ p, q \in A$

Definition : 4.19

A fuzzy soft relation R on (F, A) is said to be **fuzzy soft irreflexive relation** if $R(p, q) \not\subset R(p, p)$ and $R(q, p) \not\subset R(p, p)$, $\forall\ p, q \in A$.

Example : 4.20

Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration and A = { expertise in physics (p), expertise in chemistry (c), expertise in biology (b)}. Then

$(F, A) = \{F(p) = \{S_1/0.9, S_2/0.2, S_3/0.6, S_4/0.8\},$

$F(c) = \{S_1/0.3, S_2/0.9, S_3/0.4, S_4/0.3\},$

$F(b) = \{S_1/0.6, S_2/0.3, S_3/0.9, S_4/0.8\}\}$

Define a fuzzy soft relation (R, C) from (F, A) to (F, A) as (p, p), (c, p), (p, c) $\in C \subseteq A \times A$. Then

$(R, C) = \{R(p, p) = \{S_1/0.81, S_2/0.04, S_3/0.36, S_4/0.64\},$

$R(p, c) = \{S_1/0.27, S_2/0.18, S_3/0.24, S_4/0.24\},$

$R(c, p) = \{S_1/0.26, S_2/0.11, S_3/0.20, S_4/0.14\}\}$

$\therefore$ $R(p, c) \not\subset R(p, p)$ and $R(c, p) \not\subset R(p, p)$ and hence R is irreflexive relation.

Definition : 4.21

A fuzzy soft relation R on (F, A) is said to be **fuzzy soft transitive relation** if $R \circ R \subseteq R$.

Definition : 4.22

If a fuzzy soft relation R on (F, A) is simultaneously reflexive, symmetric and transitive then it is known as a **fuzzy soft equivalence relation**.

Theorem : 4.23

For a fuzzy soft relation S on (F, A), $S = S^{-1}$ if and only if S is symmetric fuzzy soft relation on (F, A).

Proof:

Let $S = S^{-1}$

$S(p, q) = S^{-1}(q, p) = S(q, p)$

$\therefore$ S is symmetric.

Conversely, Let S a symmetric fuzzy soft relation.

$S^{-1}(p, q) = S(q, p) = S(p, q)$

$\therefore S = S^{-1}$

Theorem : 4.24

If S be a fuzzy soft equivalence relation on (F, A) then $S \circ S$ is also a fuzzy soft equivalence relation.

Proof:

Since S is a fuzzy soft equivalence relation therefore S is reflexive, symmetric and transitive.

Now $S \circ S\ (p, q) \subseteq S\ (p, p)$

$\therefore$ $S \circ S$ is reflexive.

Again

$S \circ S\ (p, q) = S\ (p, r) \cap S\ (r, q)$

$= S\ (r, q) \cap S\ (p, r)$

$= S\ (q, r) \cap S\ (r, p)$, since S is symmetric

$= S \circ S\ (q, p)$

$\therefore$ $S \circ S$ is symmetric.

$S \circ S\ (p, q) = S\ (p, r) \cap S(r, q)$

$\supseteq S \circ S\ (p, r) \cap S \circ S\ (r, q)$, since S is symmetric.

$= (S \circ S \circ S \circ S)\ (p, q)$

$\therefore$ $S \circ S \circ S \circ S \subseteq S \circ S$

$\therefore$ $S \circ S$ is transitive.

Hence $S \circ S$ is equivalence relation.

Theorem : 4.25

If S be a fuzzy soft equivalence relation on (F, A) then S^{-1} is also a fuzzy soft equivalence relation.

Proof:

S^{-1} (p, q) = S (q, p) $\subseteq$ S(p, p) = S^{-1} (p, p), since S is reflexive.

S^{-1} (q, p) = S (p, q) $\subseteq$ S (p, p) = S^{-1} (p, p)

S^{-1} is reflexive

S^{-1} (p, q) = S (q, p) = S (p, q) = S^{-1} (q, p), since S is symmetric

S^{-1} is symmetric.

S^{-1} (p, q) = S (q, p) $\supseteq$ $S \circ S$ (q, p)

= S (q, r) $\cap$ S (r, p)

= S (r, q) $\cap$ S (p, r)

= S^{-1} (q, r) $\cap$ S^{-1} (r, p)

= ($S^{-1} \circ S^{-1}$) (p, q)

$S^{-1} \circ S^{-1} \subseteq S^{-1}$

S^{-1} is transitive

Thus S^{-1} is a fuzzy soft equivalence relation.

Definition : 4.26

Let R and S be two fuzzy soft sets from (F, A) to (G, B) .Then

($R \cap S$) (p, q) = **min** {R (p, q), S (p, q)}

$(R \cup S)(p, q) = \mathbf{max}\{R(p, q), S(p, q)\}.\ \forall\ (p, q) \in A \times B$

Theorem : 4.27

If R and S, T are fuzzy soft relations from (F, A) to (G, B) and (G, B) to (H, C), then

1) $R \circ (S \cap T) = (R \circ S) \cap (R \circ T)$
2) $R \circ (S \cup T) = (R \circ S) \cup (R \circ T)$

Proof :

Suppose $p \in A$, $q \in B$, $r \in C$

1) $R(p, q) \circ S(q, r) \cap T(q, r)$

$= R(p, q) \cap \min\{S(q, r), T(q, r)\}$

$= \min\{(R(p, q) \cap S(q, r)), ((R(p, q) \cap T(q, r))\}$

$= \min\{(R \circ S)(p, r), (R \circ T)(p, r)\}$

$= (R \circ S)(p, r) \cap (R \circ T)(p, r)$

Hence $R \circ (S \cap T) = (R \circ S) \cap (R \circ T)$.

2) $R(p, q) \circ S(q, r) \cup T(q, r)$

$= R(p, q) \cap \max\{S(q, r), T(q, r)\}$

$= \max\{(R(p, q) \cap S(q, r)), (R(p, q) \cap T(q, r))\}$

$= \max\{(R \circ S)(p, r), (R \circ T)(p, r)\}$

$= (R \circ S)(p, r) \cup (R \circ T)(p, r)$

Hence $R \circ (S \cup T) = (R \circ S) \cup (R \circ T)$.

Theorem : 4.28

If R and S are fuzzy soft relations from (F, A) to (G, B) then

1) $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

2) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$

Proof :

Suppose $p \in A$, $q \in B$

1) $(R \cap S)^{-1} (p, q) = (R \cap S) (p, q)$

$= \min \{R(q, p), S(q, p)\}$

$= \min \{R^{-1} (p,q), S^{-1}(p, q)\}$

$= (R^{-1} \cap S^{-1}) (p, q)$

Hence $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$

2) $(R \cup S)^{-1} (p, q) = (R \cup S) (q, p)$

$= \max \{R (q, p), S (q, p)\}$

$= \max \{R^{-1}(p, q), S^{-1}(p, q)\}$

$= (R^{-1} \cup S^{-1}) (p, q)$

Hence $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$