Avinashilingam Institute for Home Science and Higher Education for Women
Deemed to be University Estd. u/s 3 of UGC Act 1956, Category 'A' by MHRD (now MoE)
Re-accredited with 'A++' Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Continuous Internal Assessment Test I – February 2025
Semester-II

Class : I PG **Time : 2 Hours**
Branch :Mathematics **Max.Marks : 60**

23MMAC07 –Advanced Algebra II

Course Outcomes:

CO1:Reduce the geometric problem to an algebraic problem.
CO2: Calculate the group of all automorphisms of a given field.
CO3: Findeigen values and eigen vectors of linear transformations.
CO4: Testorthogonality of given vectors.
CO5: Construct finite fields corresponding to a prime number.

Part A 6 x 1 = 6

Choose the Correct Answer

1. When twoautomorphismsσ and τ of K are said to be equal then CO1K1

a.$\sigma(a) \neq \tau(a)$ b.$\sigma(a) = \tau(a)$ c.$\sigma(a) \neq 0$ d.$\sigma(a) = 0$

2.If k is a normal extension of a field F then k is a..... of some polynomial over F. CO1K1

a.Splitting field b.fixed field c.finite extension d.normal extension

3. If V is finite dimensional over F then T $\in$A(V) is ...... if and only if T maps V onto V. CO2K3

a. regular b. singular c. left invertible d. Both (a) and (c)

4.If dim V over a field F is greater than one then A(V) is CO2K2

a. commutative b.field c.group d. not commutative

5. If T, S $\in$A(V) and if S is regular then T and STS^{-1} have .... minimal polynomial. CO3K2

a. same b. different c .n d. 1

6. The subspace W of V is invariant under T$\in$A(V) if CO3K1

a.$WT\subset W$ b.WT= W c.WT= V d.$WT\subset V$

Part B

Answer ALL questions **3 x 6 = 18**

7. a. Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F. CO1K3

(or)

7. b.If p(x) $\in$ F[x] i solvable by radicals over F then prove that the galois group of p(x) is a solvable group. CO1K5

8. a. If V is finite dimensional over F then show that T $\in$A(V) is singular if and only if there exists v $\neq$ 0in V such that VT=0. CO2K4

(or)

8. b.If V is finite dimensional over F, then prove that T $\in$A(V) is regular if and only if T maps V onto V. CO2K4

9. a.If V is n-dimensional over F and if T $\in$A(V) has the matrix $m_1(T)$ in the basis $v_{1,.....}v_n$ and the matrix $m_2(T)$ in the basis $w_1,w_2.....w_n$of V over F, then there is an element C $\in F_n$ such that $m_2(T) = Cm_1(T)C^{-1}$. CO2K4

(or)

9. b. If V is n dimensional over F and if T $\in$A(V) has all its characteristic roots in F then prove that T satisfies a polynomial of degree n over F. CO3K4

Part C

Answer ALL questions **3 x 12 = 36**

10. a. If K is a fnite extension of F, then prov that G(K,F) is a finite group and its order o(G(K,F)) satisfies the condition o(G(K,F)) $\leq$ [K: F] CO1K3

(or)

10. b. If K is a splitting field of some polynomial over F, then show that K is a normal extension of F. CO1K4

11. a.Let G' be the commutator subgroup of G then prove that
   (i) G' is normal in G
   (ii) G / G' isabelian
   (iii) G' is the smallest subgroup of G such that G/ G' is abelian. CO2K3

(or)

11. b.(i) If V is finite dimensional over F, then T $\in$ A(V) is invertible iff the constant term of the minimal polynomial for T is not 0.
   (ii)If λ_1, λ_2, λ_3,...,λ_k are distinct characteristic roots ofT $\in$ A(V) and if $v_1,v_2,v_3,...,v_k$ are characteristic vectors of T belonging to λ_1, λ_2, λ_3,...,$\lambda_k$respectively, then $v_1,v_2,v_3,...,v_k$ are linearly independent over F. CO2K4

12. a. (i) Define Characteristic vector. (ii) Prove that if V isan n-dimensional vector spaces over F, then A(V) and F_n are isomorphic as algebras over F. CO2K3

(or)

12. b. If T $\in$ A(V) has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular. CO3K3

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