



# Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)

Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC

Coimbatore - 641 043, Tamil Nadu, India

## Continuous Internal Assessment Test I – August 2025

### Semester I

Class : I UG

Time : 2 Hours

Branch : Mathematics

Max.Marks : 60

### 23BMAC01 – Calculus

#### Course Outcomes:

CO1: assimilate the notions of limit of a sequence and convergence of a series of real numbers.

CO2: calculate the limit and examine the continuity of a function at a point.

CO3: understand the consequences of various mean value theorems for differentiable functions.

CO4: sketch curves in Cartesian and polar coordinate systems.

CO5: apply derivative tests in optimization problems appearing in social sciences, physical sciences, life sciences and a host of other disciplines.

#### Part A

6 x 1 = 6

#### Choose the Correct Answer

1. The Cantor set  $K$  is \_\_\_\_\_ CO1K1  
a. Countable      b. Uncountable      c. finite      d. bounded
2. A nonincreasing sequence which is not bounded below diverges to \_\_\_\_\_ CO1K2  
a. infinity      b. 0      c. minus infinity      d. 1
3. The value of  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$  is CO2K2  
a. 0      b.  $\pi$       c.  $\pi/2$       d. 1
4. A function  $f$  defined on a finite interval is \_\_\_\_\_ CO2K1  
a. continuous      b. closed      c. bounded      d. finite
5. Limit of  $f(x)$  exists only if both the left hand and right hand limits exists and are \_\_\_\_\_ CO2K1  
a. equal      b. not equal      c. equal to 1      d. equal to 0
6. The differentiable coefficient is also called the \_\_\_\_\_ CO3K1  
a. interval      b. function      c. limit      d. derivative

#### Part B

3 x 6 = 18

#### Answer ALL questions

- 7.a. Prove that, The set  $[0,1] = \{x | 0 \leq x \leq 1\}$  is uncountable. CO1K4  
(or)
- 7.b. Prove that a nondecreasing sequence which is bounded above is convergent. CO1K4

8.a. Integrate  $\frac{1}{\sqrt{(2x^2-x+2)}}$  CO1K3

(or)

8.b. Examine whether the limit of the function  $\frac{x^2-1}{x-1}$  as  $x$  tends to 1 exists. CO2K3

9.a. State and prove Intermediate value theorem. CO2K2

(or)

9.b. Show that the function  $f(x) = x \tan^{-1} x$  when  $x \neq 0$  and  $f(0) = 0$  is continuous but not differentiable at  $x=0$ . CO3K3

### Part C

3 x 12 = 36

#### Answer ALL questions

10.a. (i) Show that the sequence  $\left\{ \left( 1 + \frac{1}{n} \right)^n \right\}_{n=1}^{\infty}$  is convergent. CO1K4

(ii) Integrate  $(x+1)\sqrt{(x^2-x+1)}$

(or)

10.b.(i) Prove that  $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$  CO1K4

(ii) Integrate  $\frac{1}{\sin^3 x \cos^5 x}$

11.a. Prove that  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$  CO2K4

(or)

11.b. Prove that if  $f$  and  $g$  be two functions such that  $\lim_{x \rightarrow a} f(x) = A$  and  $\lim_{x \rightarrow a} g(x) = B$  where  $A$  and  $B$  are finite, then CO2K4

$\lim_{x \rightarrow a} (kf)(x) = kA$ , where  $k$  is a constant

(i)  $\lim_{x \rightarrow a} (kf)(x) = kA$ , where  $k$  is a constant (ii)  $\lim_{x \rightarrow a} (f \pm g)(x) = A \pm B$

(iii)  $\lim_{x \rightarrow a} (fg)(x) = AB$  (iv)  $\lim_{x \rightarrow a} (f/g)(x) = A/B$

12.a. (i) List the different types of discontinuities. Explain it. CO2K3

(ii) Examine the continuity of the following function at the origin  $f(x) = \frac{\sin^2 ax}{x^2}$

for  $x \neq 0, f(0) = 1$ .

(or)

12.b. (i) Prove that if a function is differentiable at a point, then it is continuous at that point.

(ii) Show that the function  $f(x) = x \sin(1/x)$ ,  $x \neq 0, f(0) = 0$  is continuous but not differentiable at  $x=0$ . CO3K4

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