

Δ^* -Irresolute Maps In Topological Spaces

5.1 Introduction

Irresolute functions are one of the most important concepts in topological spaces. Crossley and Hildebrand (1972) introduced the notion of irresoluteness. In this chapter, a new class of irresolute maps namely, Δ^* -irresolute and contra Δ^* -irresolute maps are introduced. Also the association of these maps with other existing irresolute maps and their properties are studied.

5.2 Δ^* -Irresolute Maps

Definition 5.2.1 A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called Δ^* -irresolute if $f^{-1}(V)$ is Δ^* -open set in (X, τ) for every Δ^* -open set V in (Y, σ) .

Proposition 5.2.2 A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is Δ^* -irresolute if and only if for every Δ^* -closed set V in (Y, σ) , $f^{-1}(V)$ is a Δ^* -closed set in (X, τ) .

Proof: (Necessary) : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a Δ^* -irresolute map. Let V be any Δ^* -closed set in (Y, σ) . Then $(Y - V)$ is Δ^* -open in (Y, σ) . Since f is Δ^* -irresolute, $f^{-1}(Y - V) = X - f^{-1}(V)$ is Δ^* -open in (X, τ) and hence $f^{-1}(V)$ is Δ^* -closed in (X, τ) .

(Sufficiency) : Assume that $f^{-1}(V)$ is Δ^* -closed in (X, τ) for each Δ^* -closed set V in (Y, σ) . Let U be any Δ^* -open set in (Y, σ) . Then $(Y - U)$ is a Δ^* -closed set in (Y, σ) . By hypothesis $f^{-1}(Y - U)$ is a Δ^* -closed set in (X, τ) . That is $f^{-1}(Y - U) = X - f^{-1}(U)$ is a Δ^* -closed set in (X, τ) which implies that $f^{-1}(U)$ is a Δ^* -open set in (X, τ) . Hence f is a irresolute map.

Proposition 5.2.3 If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is Δ^* -irresolute then for every subset A of (X, τ) , $f[\Delta^* \text{cl}(A)] = \delta \text{cl}[f(A)]$.

Proof : Let $A \subseteq X$. Then $\delta \text{cl}[f(A)]$ is δ -closed in (Y, σ) . $\delta \text{cl}[f(A)]$ is also Δ^* -closed in (Y, σ) . Since f is Δ^* -irresolute, $f^{-1}[\delta \text{cl}\{f(A)\}]$ is Δ^* -closed in (X, τ) . Since $f(A) \subseteq \delta \text{cl}[f(A)]$ we have $A \subseteq f^{-1}[\delta \text{cl}\{f(A)\}]$. By the definition of Δ^* -closure, $\Delta^* \text{cl}(A) = \Delta^* \text{cl}\{f^{-1}[\delta \text{cl}\{f(A)\}]\} = f^{-1}[\delta \text{cl}\{f(A)\}]$. Hence $f[\Delta^* \text{cl}(A)] = \delta \text{cl}[f(A)]$.

Proposition 5.2.4 If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is Δ^* -irresolute then for any subset $B \subseteq Y$, $\Delta^* \text{cl}[f^{-1}(B)] = f^{-1}[\delta \text{cl}(B)]$.

Proof : Let $B \subseteq Y$. Then $\delta \text{cl}(B)$ is δ -closed in (Y, σ) . Since every δ -closed set is Δ^* -closed set, $\delta \text{cl}(B)$ is a Δ^* -closed in (Y, σ) . By hypothesis, $f^{-1}[\delta \text{cl}(B)]$ is Δ^* -closed in (X, τ) . Since $B \subseteq \delta \text{cl}(B)$, $f^{-1}(B) \subseteq f^{-1}[\delta \text{cl}(B)]$. By the definition of Δ^* -closure, $\Delta^* \text{cl}[f^{-1}(B)] = f^{-1}[\delta \text{cl}(B)]$.

Proposition 5.2.5 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be δ -open surjective and Δ^* -irresolute map. If (X, τ) is a $\Delta^* T_\delta$ -space then (Y, σ) is a $\Delta^* T_\delta$ -space.

Proof : Let V be any Δ^* -open in (Y, σ) . Since f is Δ^* -irresolute map, $f^{-1}(V)$ is Δ^* -open in (X, τ) . Since (X, τ) is a $\Delta^* T_\delta$ -space, $f^{-1}(V)$ is δ -open in (X, τ) . Since f is δ -open surjective map, $f[f^{-1}(V)] = V$ is δ -open in (Y, σ) . Hence (Y, σ) is a $\Delta^* T_\delta$ -space.

Proposition 5.2.6 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be δg^* -open surjective and Δ^* -irresolute map. If (X, τ) is a $\Delta^* T_{\delta g^*}$ -space then (Y, σ) is a $\Delta^* T_{\delta g^*}$ -space.

Proof : Let V be any Δ^* -open set in (Y, σ) . Since f is Δ^* -irresolute map $f^{-1}(V)$ is Δ^* -open in (X, τ) . Since (X, τ) is a $\Delta^* T_{\delta g^*}$ -space, $f^{-1}(V)$ is δg^* -open in (X, τ) . Since f is δg^* -open surjective $f[f^{-1}(V)] = V$ is δg^* -open in (Y, σ) . Hence (Y, σ) is a $\Delta^* T_{\delta g^*}$ -space.

Proposition 5.2.7 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be two Δ^* -irresolute maps. Then their composition $(g \circ f) : (X, \tau) \rightarrow (Z, \eta)$ is a Δ^* -irresolute map.

Proof : Follows from the definition of Δ^* -irresolute map.

Proposition 5.2.8 A perfectly Δ^* -continuous function is a Δ^* -irresolute map but not conversely.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a perfectly Δ^* -continuous function. Let V be a Δ^* -open set in (Y, σ) . Since f is perfectly Δ^* -continuous, $f^{-1}(V)$ is clopen. Since a clopen set is a Δ^* -open set (by Proposition 2.5.5), $f^{-1}(V)$ is Δ^* -open in (X, τ) . Hence f is a Δ^* -irresolute map.

Counter example 5.2.9 Let $X = \{a, b, c, d\} = Y$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is Δ^* -irresolute but not perfectly Δ^* -continuous since for the closed set $\{b\}$ in (Y, σ) , $f^{-1}\{b\} = \{b\}$ is not Δ^* -closed in (X, τ) .

Proposition 5.2.10 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a Δ^* -irresolute map and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be Δ^* -continuous map. Then their composition mapping $(g \circ f) : (X, \tau) \rightarrow (Z, \eta)$ is a Δ^* -continuous map.

Proof : Let V be any closed set in (Z, η) . Since g is Δ^* -continuous, $f^{-1}(V)$ is Δ^* -closed in (Y, σ) . Since f is Δ^* -irresolute $f^{-1}[g^{-1}(V)] = [g \circ f]^{-1}(V)$ is Δ^* -closed in (X, τ) . Hence $(g \circ f)$ is Δ^* -continuous.

Proposition 5.2.11 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a Δ^* -irresolute map and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be a contra Δ^* -continuous map. Then their composition mapping $(g \circ f) : (X, \tau) \rightarrow (Z, \eta)$ is a contra Δ^* -continuous function.

Proof : Let U be any open set in (Z, η) . Since g is a contra Δ^* -continuous map, $g^{-1}(U)$ is Δ^* -closed in (Y, σ) . Since f is Δ^* -irresolute, $f^{-1}[g^{-1}(U)]$ is Δ^* -closed in (X, τ) . That is $f^{-1}[g^{-1}(U)] = (g \circ f)^{-1}(U)$ is Δ^* -closed in (X, τ) . Hence $(g \circ f)$ is contra Δ^* -continuous.

Remark 5.2.12 The following examples show that Δ^* -irresolute map and irresolute map are independent .

Counter Example 5.2.13 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map such that $f(a) = a, f(b) = c, f(c) = c$. Then f is Δ^* -irresolute but not irresolute since for the semi open set $\{a\}$ in (Y, σ) , $f^{-1}\{a\} = \{a\}$ is not semi open in (X, τ) .

Counter Example 5.2.14 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is irresolute but not Δ^* -irresolute since for the Δ^* -open set $\{b\}$ in (Y, σ) , $f^{-1}\{b\} = \{b\}$ is not a Δ^* -open set in (X, τ) .

Remark 5.2.15 The following examples show that Δ^* -irresolute map and α -irresolute map are independent.

Counter Example 5.2.16 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map such that $f(a) = a, f(b) = c, f(c) = c$. Then f is Δ^* -irresolute but not α -irresolute since for the α -open set $\{a, b\}$ in (Y, σ) , $f^{-1}\{a, b\} = \{a\}$ is not α -open in (X, τ) .

Counter Example 5.2.17 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is α -irresolute but not a Δ^* -irresolute map since for the Δ^* -open set $\{a, b\}$ in (Y, σ) , $f^{-1}\{a, b\} = \{a, b\}$ is not Δ^* -open in (X, τ) .

Remark 5.2.18 The following examples show that Δ^* -irresolute map is independent with δg^* -irresolute map.

Counter Example 5.2.19 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}, \{a, c\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is δg^* -irresolute but not Δ^* -irresolute since for the Δ^* -open set $\{c\}$ in (Y, σ) , $f^{-1}\{c\} = \{c\}$ is not Δ^* -open in (X, τ) .

Counter Example 5.2.20 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is Δ^* -irresolute but not δg^* -irresolute map since for the δg^* -open set $\{a, c\}$ in (Y, σ) , $f^{-1}\{a, c\} = \{a, c\}$ is not δg^* -open in (X, τ) .

Remark 5.2.21 The following examples show that Δ^* -irresolute map is independent with Δ^* -continuous map.

Counter Example 5.2.22 Let $X = \{a, b, c, d\} = Y$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is Δ^* -irresolute but not Δ^* -continuous since for the closed set $\{b\}$ in (Y, σ) , $f^{-1}\{b\} = \{b\}$ is not Δ^* -closed in (X, τ) .

Counter Example 5.2.23 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}, \{a, c\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is Δ^* -continuous but not Δ^* -irresolute since for the Δ^* -open set $\{c\}$ in (Y, σ) , $f^{-1}\{c\} = \{c\}$ is not Δ^* -open in (X, τ) .

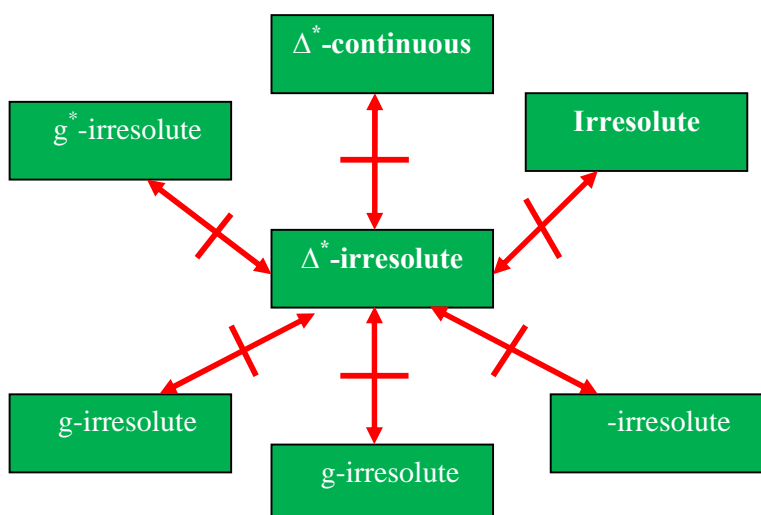
Remark 5.2.24 The following examples show that Δ^* -irresolute map is independent with δg -irresolute map and αg -irresolute map.

Counter Example 5.2.25 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}, \{a, c\}\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is

g -irresolute and g -irresolute but not Δ^* -irresolute since for the Δ^* -open set $\{c\}$ in (Y, τ) , $f^{-1}\{c\} = \{c\}$ is not Δ^* -open in (X, τ) .

Counter Example 5.2.26 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $\tau' = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \tau')$ be the identity map. Then f is Δ^* -irresolute but not g^* -irresolute map and not g -irresolute since for the g^* -open set $\{a, c\}$ in (Y, τ') , $f^{-1}\{a, c\} = \{a, c\}$ is not g^* -open in (X, τ) and for the g -open set $\{b\}$ in (Y, τ') , $f^{-1}\{b\} = \{b\}$ is not g -open in (X, τ) .

Remark 5.2.27 The above results are depicted by the following diagram.



5.3 Contra Δ^* -irresolute maps

Definition 5.3.1 A map $f : (X, \tau) \rightarrow (Y, \tau')$ is said to be **contra Δ^* -irresolute** if $f^{-1}(V)$ is Δ^* -open in (X, τ) for every Δ^* -closed set V in (Y, τ') .

Proposition 5.3.2 Let $f : (X, \tau) \rightarrow (Y, \tau')$ be a map. Then the following statements are equivalent.

a) f is a contra $*$ -irresolute map.

b) The inverse image of every $*$ -open set in (Y, τ) is $*$ -closed in (X, τ) .

Proof : Follows from the definition 5.3.1

Proposition 5.3.3 Every Strongly Totally $*$ -continuous map is contra $*$ -irresolute but not conversely.

Proof : Follows from the definition of strongly totally $*$ -continuous map (Definition 4.4.6) and contra $*$ -irresolute map (Definition 5.3.1).

Counter Example 5.3.4 Let $X = \{a, b, c\} = Y$ with $\tau = \{\phi, X, \{a, b\}\}$ and $\tau = \{\phi, Y, \{a\}\}$.

Let $f : (X, \tau) \rightarrow (Y, \tau)$ be a map such that $f(a) = c$, $f(b) = b$, $f(c) = a$. Then f is contra $*$ -irresolute but not a strongly totally $*$ -continuous map since for the subset $\{a\}$ in (Y, τ) , $f^{-1}[\{a\}] = \{c\}$ is $*$ -closed but not $*$ -open in (X, τ) .

Remark 5.3.5 The following examples show that the contra $*$ -irresolute and $*$ -irresolute maps are independent.

Counter Example 5.3.6 Let $X = \{a, b, c\} = Y$ with $\tau = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $\tau = \{\phi, Y, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \tau)$ be the identity map. Then f is a $*$ -irresolute map but not a contra $*$ -irresolute map. Since for the $*$ -closed set $\{b, c\}$ in (Y, τ) , $f^{-1}[\{b, c\}] = \{b, c\}$ is not $*$ -open in (X, τ) .

Counter Example 5.3.7 Let $X = \{a, b, c\} = Y$ with $\tau = \{\phi, X, \{a, b\}\}$ and $\tau = \{\phi, Y, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \tau)$ be a map such that $f(a) = c$, $f(b) = b$, $f(c) = a$. Then f is contra $*$ -irresolute but not a $*$ -irresolute map since for the $*$ -open set $\{a\}$ in (Y, τ) , $f^{-1}[\{a\}] = \{c\}$ is not $*$ -open in (X, τ) .

Remark 5.3.8 The Contra $*$ -irresolute and Quasi $*$ -continuous maps are independent as seen from the following examples.

Counter Example 5.3.9 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}\}$ and $\tau' = \{\emptyset, Y, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \tau')$ be a map such that $f(a) = a, f(b) = c, f(c) = c$. Then f is quasi τ' -continuous but not contra τ' -irresolute since for the τ' -closed set $\{c\}$ in (Y, τ') , $f^{-1}[\{c\}] = \{b, c\}$ is not τ -open in (X, τ) .

Counter Example 5.3.10 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\tau' = \{\emptyset, Y, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \tau')$ be a map such that $f(a) = c, f(b) = b, f(c) = a$. Then f is contra τ' -irresolute but not a quasi τ' -continuous map since for the τ' -open set $\{a\}$ in (Y, τ') , $f^{-1}[\{a\}] = \{c\}$ is not open in (X, τ) .

Remark 5.3.11 The contra τ' -irresolute and τ' -continuous maps are independent as seen from the following examples.

Counter Example 5.3.12 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\tau' = \{\emptyset, Y, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \tau')$ be the identity map. Then f is τ' -continuous but not contra τ' -irresolute since for the τ' -closed set $\{b, c\}$ in (Y, τ') , $f^{-1}[\{b, c\}] = \{b, c\}$ is not τ -open in (X, τ) .

Counter Example 5.3.13 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\tau' = \{\emptyset, Y, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \tau')$ be a map such that $f(a) = c, f(b) = b, f(c) = a$. Then f is contra τ' -irresolute but not a τ' -continuous map since for the closed set $\{b, c\}$ in (Y, τ') , $f^{-1}[\{b, c\}] = \{a, b\}$ is not τ -closed in (X, τ) .

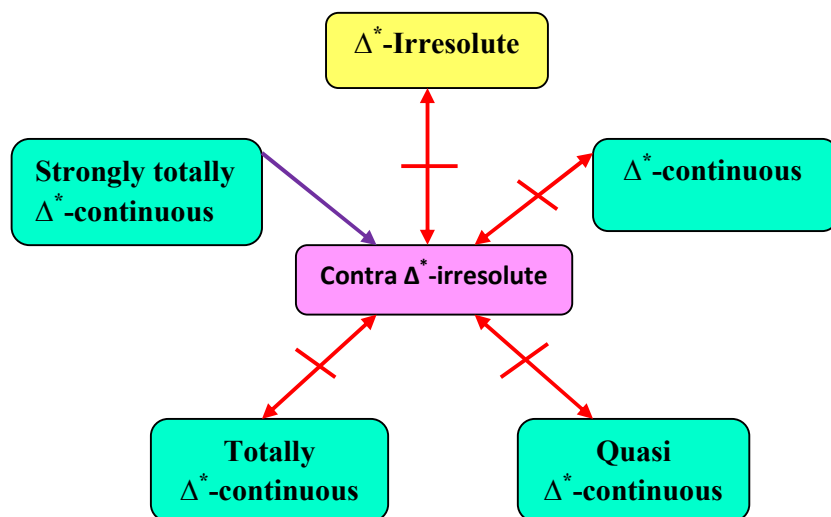
Remark 5.3.14 The contra τ' -irresolute and totally τ' -continuous are independent as seen from the following examples.

Counter Example 5.3.15 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ and $\tau' = \{\emptyset, Y, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \tau')$ be a map such that $f(a) = b, f(b) = c, f(c) = a$.

Then f is totally Δ^* -continuous but not contra Δ^* -irresolute since for the Δ^* -closed set $\{a, c\}$ in (Y, τ) , $f^{-1}[\{a, c\}] = \{b, c\}$ is not Δ^* -open in (X, τ) .

Counter Example 5.3.16 Let $X = \{a, b, c\} = Y$ with $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\tau' = \{\emptyset, Y, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \tau')$ be a map such that $f(a) = c, f(b) = b, f(c) = a$. Then f is contra Δ^* -irresolute but not totally Δ^* -continuous since for the open set $\{a\}$ in (Y, τ') , $f^{-1}[\{a\}] = \{c\}$ is Δ^* -closed but not Δ^* -open in (X, τ) .

Remark 5.3.17 The above results are depicted by the following diagram.



Composition of contra Δ^* -irresolute maps

Proposition 5.3.18 Let $f : (X, \tau) \rightarrow (Y, \tau')$ and $g : (Y, \tau') \rightarrow (Z, \tau'')$ be any two contra Δ^* -irresolute maps. Then their composition mapping $(g \circ f) : (X, \tau) \rightarrow (Z, \tau'')$ need not be a contra Δ^* -irresolute map as seen from the following example.

Counter Example 5.3.19 Let $X = \{a, b, c\} = Y = Z$ with $\tau = \{\emptyset, X, \{a, b\}\}$, $\tau' = \{\emptyset, Y, \{a\}\}$ and $\tau'' = \{\emptyset, Z, \{a\}\}$. Let $f : (X, \tau) \rightarrow (Y, \tau')$ be a map such that $f(a) = c,$

$f(b) = b, f(c) = a$. Then f is Contra * -irresolute. Let $g : (Y, \tau) \rightarrow (Z, \sigma)$ be a map such that $g(a) = b, g(b) = a, g(c) = a$. Then g is also a contra * -irresolute map. But the composition mapping $(g \circ f) : (X, \tau) \rightarrow (Z, \sigma)$ is not a contra * -irresolute map since for the * -closed set $\{b, c\}$ in (Z, σ) , $(g \circ f)^{-1}[\{b, c\}] = \{c\}$ is not * -open in (X, τ) . Hence $(g \circ f)$ is not a contra * -irresolute map.

Proposition 5.3.20 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \rho)$ be any two contra * -irresolute maps. Then their composition mapping $(g \circ f) : (X, \tau) \rightarrow (Z, \rho)$ is a * -irresolute map.

Proof : Let U be any * -closed set in (Z, ρ) . Since g is Contra * -irresolute, $g^{-1}(U)$ is * -open in (Y, σ) . Then by the assumption, $f^{-1}[g^{-1}(U)]$ is * -closed in (X, τ) . Hence $f^{-1}[g^{-1}(U)] = (g \circ f)^{-1}(U)$ is * -closed set in (X, τ) . Thus $(g \circ f) : (X, \tau) \rightarrow (Z, \rho)$ is a * -irresolute map.

Theorem 5.3.21 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \rho)$ be any two maps. Then the following two statements hold.

- a) If f is * -irresolute and g is contra * -irresolute maps then the composition mapping $(g \circ f) : (X, \tau) \rightarrow (Z, \rho)$ is a contra * -irresolute map.
- b) If f is contra * -irresolute and g is * -irresolute then $(g \circ f) : (X, \tau) \rightarrow (Z, \rho)$ is a contra * -irresolute map.
- c) If f is perfectly * -continuous and g is a contra * -irresolute map then $(g \circ f) : (X, \tau) \rightarrow (Z, \rho)$ is a Perfectly * -continuous map.

Proof : a) Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be * -irresolute and $g : (Y, \sigma) \rightarrow (Z, \rho)$ be a contra * -irresolute map. Let U be any * -closed set in (Z, ρ) . Since g is contra * -irresolute, $f^{-1}(U)$ is * -open in (Y, σ) . Since f is * -irresolute, $f^{-1}[f^{-1}(U)] = (g \circ f)^{-1}(U)$ is * -open in (X, τ) . Hence the composition mapping $(g \circ f)$ is a contra * -irresolute map.

b) Let f be contra \ast -irresolute and g be a \ast -irresolute map. Let V be any \ast -open set in (Z, τ) . Since g is \ast -irresolute, $f^{-1}(V)$ is \ast -open in (Y, τ) . Also f is Contra \ast -irresolute. Therefore $f^{-1}[g^{-1}(V)] = (g \circ f)^{-1}(V)$ is \ast -closed in (X, τ) . Hence the composition mapping $(g \circ f)$ is Contra \ast -irresolute map.

c) Let f be perfectly \ast -continuous and g be a contra \ast -irresolute map. Let V be any \ast -closed in (Z, τ) . Since g is contra \ast -irresolute, $f^{-1}(V)$ is \ast -open in (Y, τ) . Also f is perfectly \ast -continuous. Therefore $f^{-1}[g^{-1}(V)] = (g \circ f)^{-1}(V)$ is clopen in (X, τ) . Hence $(g \circ f)$ is perfectly \ast -continuous.