CHAPTER IV

MULTIFACETED CONTINUOUS LOT BY LOT ACCEPTANCE SAMPLING PLAN

This chapter gives a newly proposed multifaceted continuous lot by lot acceptance sampling plan governed by clearance number for unit by unit inspection along with clearance number, sampling fraction and sampling rate for lot by lot inspection. Operating procedure of the plan and the derivation of performance measures by using Markov-chain approach are given. The strength of the proposed plan for varying parameters is analyzed. Procedure for construction of tables is given. Numerical examples are provided to illustrate the selection of the multifaceted plan.

Modern production processes, particularly those occurring in the electronics industry, are of high quality when the fraction of nonconforming products is in the range of parts per million. For control of such superior quality processes, the procedures generally assume cent percent inspection of product. This control strategy may not be effective and subject to test errors.

To overcome this drawback, Pesotchinsky (1987) proposed a strategy of combining the best features of continuous sampling plan, CSP-1 of Dodge (1943) with the lot by lot inspection plan. Due to its complexity, Govindaraju and Bebbington (2000) proposed a simplified scheme called combined continuous lot by lot acceptance sampling plan and provided performance measures, using the Markov chain approach of Stephens (1995).

Combining the best features of continuous sampling plan and lot by lot inspection is desirable for manufacturing processes with the superior quality. This made the investigator to propose a new multifaceted continuous lot by lot acceptance sampling plan, consisting of three phases of inspection in the order of unit by unit inspection, lot by lot inspection and skipping lot inspection.

Procedure is reverted to screening inspection by units when a non conforming unit/lot is found in any phase. The quality indices and the performance measure are obtained to the proposed plan for the specified parameters

The operating procedure of the multifaceted continuous lot by lot acceptance sampling plan consists of the following steps. When units are offered for inspection in the order of production from a high quality process

Step 1 : Start with the screening inspection of units.

Step 2 : If i units in succession are found to be clear of defects. Start forming lots of desired size and move to lot by lot inspection

Step 3 : Inspect lots one by one applying any lot by lot sampling plan

Step 4 : If l consecutive lots are accepted on the lot by lot sampling plan, move to skipping inspection of lots at rate f

Step 5 : If a lot is rejected either on the inspection of lot by lot or on the skipping inspection of lots then revert to screening inspection as in step 1, otherwise continue with skipping inspection at rate f.

The proposed multifaceted continuous lot by lot acceptance sampling plan is indexed by the parameters i, l and f and the parameters of the reference lot by lot sampling plan (N, n, c).

Derivation of performance measures

Various performance measures of multifaceted continuous lot by lot acceptance sampling plan are derived using the Markov chain approach due to Roberts (1965).

Let $[X_n], n = 1,2, \dots$ denote the discrete parameter Markov chain with finite state space $S_k,\ k = 1,2, \dots \dots, (i + l + 5)$.

The states of the multifaceted continuous lot by lot acceptance sampling plan are defined as

$S_k \quad = A_{k-1}, k = 1,2, \dots (i+1)$

= 100 percent inspection of units is being conducted, including the latest unit is inspected and the last (k-1) consecutive units were found to be conforming.

$S_k \quad = B_k, k = 1,2, \dots \dots, l$

= Lot by lot inspection is in effect and the last k consecutive lots submitted were inspected and accepted.

$S_{i+l+2} = R$

= Lot by lot sampling plan is in effect and the last lot submitted was inspected and rejected.

$S_{i+l+3} = SA$

= Skipping inspection is in effect and the last lot submitted was inspected and found to be accepted.

$S_{i+l+4} = SR$

= Skipping inspection is in effect and the last lot submitted was inspected and found to be rejected.

$S_{i+l+5} = SN$

= Skipping inspection is in effect and the last lot submitted was not inspected.

These set of $(i + l + 5)$ states defined above completely describes the mutually exclusive states of inspection for multifaceted continuous lot by lot acceptance sampling plan.

This procedure applied to an inspection process which is in statistical control and may be viewed as a discrete parameter Markov chain which is finite, irreducible and aperiodic. The vectors of the limiting probabilities π may be determined using the steady state equations. The transitional probability matrix is presented in Table 4.1

The steady state probabilities π_j satisfy the following conditions

$$\pi_j = \sum_{k=1}^{i+l+5} \pi_k \, p_{kj}$$

$$\pi_j > 0 \text{ for } j = 1,2, \dots \dots, (i + l + 5)$$

$$such\ that \sum_{k=1}^{i+l+5} \pi_k = 1$$

These conditions result in the following equations

$$\pi_0 = p(\pi_0 + \pi_1 + \cdots + \pi_{(i-1)} + \pi_R + \pi_{SR}$$

$$\pi_{A1} = q(\pi_{A0} + \pi_R + \pi_{SR})$$

$$\pi_{Aj} = q^{j-1} \pi_{A1} \text{ for } j = 2,3, \dots, i$$

$$\pi_{B1} = P\pi_{Ai} = Pq^{(i-1)} \pi_{A1}$$

$$\pi_{Bm} = P^m q^{i-1} \pi_{A1} \text{ for } m = 1,2, \dots, l$$

$$\pi_R = (1 - P)(\pi_{Ai} + \pi_{Bi} + \pi_{B(l-1)}$$

$$\pi_{SA} = fP\,(\pi_{Bl} + \pi_{SA} + \pi_{SN})$$

$$\pi_{SR} = f(1-P)\,(\pi_{Bl} + \pi_{SA} + \pi_{SN})$$

$$\pi_{SN} = (1-f)\,(\pi_{Bl} + \pi_{SA} + \pi_{SN})$$

Table 4.1 Transition Probability Matrix of Multifaceted continuous lot by lot plan

	A_0	A_1	. . .	A_{i-1}	A_i	B_1	B_2	. . .	B_{l-1}	B_l	R	SA	SR	SN
A_0	p	q		0	0	0	0		0	0	0	0	0	0
A_1	p	0		0	0	0	0		0	0	0	0	0	0
.														
.														
.														
A_{i-1}	p	0		0	q	0	0		0	0	0	0	0	0
A_i	0	0		0	0	P	0		0	0	(1-P)	0	0	0
B_1	0	0		0	0	0	P		0	0	(1-P)	0	0	0
B_2	0	0		0	0	0	0		0	0	(1-P)	0	0	0
.														
.														
.														
B_{l-1}	0	0		0	0	0	0		0	P	(1-P)	0	0	0
B_l	0	0		0	0	0	0		0	0	0	fP	f(1-P)	1-f
R	p	q		0	0	0	0		0	0	0	0	0	0
SA	0	0		0	0	0	0		0	0	0	fP	f(1-P)	1-f
SR	p	q		0	0	0	0		0	0	0	0	0	0
SN	0	0		0	0	0	0		0	0	0	fP	f(1-P)	1-f

On simplifying, the steady state probabilities are

$$\pi_0 \quad = p\big(1-q^i\big)(1-P)f/D$$

$$\pi_j \quad = pq^i(1-P)f/D \text{ where } j = 1,2,\dots,i$$

$$\pi_{Bm} = P^m q^i p(1-P)f/D \text{ where } m = 1,2,\dots,l$$

$$\pi_R \ = pq^i(1-P^l)(1-P)f/D$$

$$\pi_{SA} \ = pq^iP^{l+1}f/D$$

$$\pi_{SR} \ = pq^iP^l(1-P)f/D$$

$$\pi_{SN} \ = pq^iP^l(1-f)/D$$

where $D \ = \left(1-q^i\right)(1-P_a)f + pq^i(f+(1-f)P_a^l)$

The performance measures of the plan from the derived steady state probabilities are

(i) The average number of units inspected under the screening inspection is

$$U = (1-q^i)/pq^i \tag{4.1}$$

(ii) The average number of lots passed under the lot by lot inspection is

$$V_1 = (1-\pi_{100(i)})/\pi_i$$

$$= (f+(1-f)P^l)/(1-P)f \tag{4.2}$$

$where \quad \pi_{100(i)} = \sum_{i=0}^{i} \pi_i$

(iii) The average number of lots inspected under the skipping inspection is

$$V_2 = (1-\pi_{100(l)}-\pi_R)/\pi_{Bl}$$

$$= (\left(1-q^i\right)(1-P)f + pq^iP^l)/(P^lpq^if(1-P) \tag{4.3}$$

$where \quad \pi_{100(l)} = \sum_{l=1}^{l} \pi_l$

(iv) The OC function of the combined continuous lot by lot plan is

$$P_a = 1 - \pi_{100(i)} - \pi_R - \pi_{SR}$$

$$= pq^i(fP + (1-f)P^l)/D \tag{4.4}$$

(v) If f is the fraction of the lot sampled then average outgoing quality is

$$AOQ = pP_a = ((N-n)/N)p^2q^i(fP + (1-f)P^l)/D \tag{4.5}$$

AOQL is the maximum of AOQ function.

(vi) The average fraction inspected (AFI) in the sampling inspection is

$$AFI = 1 - \pi_{SN} = f((1-P)(1-q^i) + pq^i)/D \tag{4.6}$$

Application of the plan

For the purpose of illustration we suppose that the reference plan to be used is the single sampling plan. Tables 4.1 to 4.3 give the quality indices for multifaceted continuous lot by lot acceptance sampling plan. From which one can choose the required AOQL, given in parts per million closer to the value where the process is to be maintained.

If one needs the AOQL value as 0.001, there are many choices of the plan to be selected. One may observe that there are many useful combinations of i, l, n and c for the specified AOQL such as $(i, n, f, c, l) =$ (1000,100,1/3,2,5) with AQL=0.000862 and LQL=0.0003095; (1000,200,1/3,3,5) with AQL=0.0001116 and LQL=0.0003119; (500,200,1/10,2,5) with AQL =0.000892 and LQL=0.0003949.

From these different choices of the parameters one may choose the required multifaceted continuous lot by lot acceptance sampling plan. Tables give the Acceptable Quality Level (AQL) with 95% probability of acceptance and Limiting Quality Level (LQL) with 10% probability of acceptance in parts per million.

These constraints imply that the process is at the AQL, the lot by lot and fractional sampling will operate 95% of the time and remaining 5% of the time 100% inspection of units will operate and when the process is at the LQL, the lot by lot and fractional sampling will function 10% of the time and remaining 90% of the time 100% inspection will function.

Numerical values in these table reveal the following features

(i) Increase in n, decreases AOQL, AQL and LQL irrespective of n/N

(ii) Increase in f, decreases AOQL, AQL and LQL irrespective of n/N

(iii) As n/N increases the AOQL decreases for any f

(iv) AQL and LQL are independent of n/N

Construction of tables 4.1 to 4.3

The OC function of the multifaceted continuous lot by lot acceptance sampling plan derived in (4.4) is

$$P_a = pq^i(fP + (1-f)P^l)/D$$

where P is the probability of acceptance of the reference sampling plan for lot by lot inspection. Under the conditions of Poisson model with single sampling reference plan

$$P = \sum_{d=0}^{c} e^{-np}\,(np)^d/d!$$

For the assumed values of i, n, c, f the values of

(i) AQL with $P_a(p) = 0.95$ and

(ii) LQL with $P_a(p) = 0.10$

(iii) AOQL, the maximum of AOQ using (4.6) are evaluated using the iterative procedure varying p between 0 and 1 in steps of 0.000001.

Table 4.1 Quality indices for the multifaceted continuous lot by lot acceptance sampling plan for $c = 1$ and $l = 5$

i	n	100			200			300		
		n/N			n/N			n/N		
	f	0.05	0.2	0.1	0.05	0.2	0.1	0.05	0.2	0.1
500	1/3	246 472.9 2439	246 448.0 2439	246 398.3 2439	125 250.8 1412	125 237.6 1412	125 211.2 1412	83 170.8 996	83 161.8 996	83 143.8 996
	1/5	314 590.6 2890	314 559.5 2890	314 497.4 2890	160 317.2 1708	160 300.5 1708	160 267.1 1708	107 217.1 1214	107 205.7 1214	107 182.8 1214
	1/8	402 735.7 3392	402 696.9 3392	402 619.5 3392	206 400.9 2044	206 379.9 2044	206 337.6 2044	138 276.1 1466	138 261.6 1466	138 232.5 1466
	1/10	438 794.2 3581	438 752.4 3581	438 668.8 3581	225 435.3 2172	225 412.3 2172	225 366.5 2172	151 300.4 1562	151 284.6 1562	151 252.9 1562
1000	1/3	171 318.8 1553	171 302.0 1553	171 268.5 1553	87 172.1 937	87 163.1 937	87 144.9 937	58 118.1 673	58 111.9 673	58 99.4 673
	1/5	218 394.9 1826	218 374.1 1826	218 332.5 1826	112 216.6 1131	112 205.2 1131	112 182.4 1131	75 149.6 823	75 141.7 823	75 125.9 823
	1/8	278 487.5 2127	278 461.9 2127	278 410.6 2127	143 272.4 1254	143 258.1 1354	143 229.4 1354	97 189.6 998	97 179.6 998	97 159.7 998
	1/10	302 524.6 2240	302 496.9 2240	302 441.8 2240	157 295.1 1439	157 279.6 1439	157 248.5 1439	106 206.1 1066	106 195.2 1066	106 173.5 1066

AQL, AOQL and LQL values in ppm are presented as first, second and third values in each and every cell of the table

Table 4.2 Quality indices for the multifaceted continuous lot by lot acceptance sampling plan for $c = 2$ and $l = 5$

i	n	100			200			300		
		n/N			n/N			n/N		
	f	0.05	0.2	0.1	0.05	0.2	0.1	0.05	0.2	0.1
500	1/3	1154 1437.3 4753	1154 1361.7 4753	1154 1210.4 4753	606 792.0 2941	606 750.3 2941	606 666.9 2941	411 547.5 2139	411 518.7 2139	411 461.1 2139
	1/5	1345 1656.3 5299	1345 1569.2 5299	1345 1394.8 5299	714 925.9 3344	714 877.1 3344	714 779.7 3344	485 643.9 2455	485 610.1 2455	485 542.3 2455
	1/8	1570 1906.8 5886	1570 1806.4 5886	1570 1605.7 5886	841 1083.1 3785	841 1026.1 3785	841 912.1 3785	575 758.6 2803	575 718.6 2803	575 638.8 2803
	1/10	1658 2002.9 6102	1658 1897.5 6102	1658 1686.7 6102	892 1144.6 3949	892 1084.3 3949	892 963.8 3949	610 803.8 2933	610 761.5 2933	610 676.9 2933
1000	1/3	862 1028.4 3095	862 974.3 3095	862 866.0 3095	465 587.4 2020	465 556.4 2020	465 494.6 2020	318 412.4 1514	318 390.7 1514	318 347.3 1514
	1/5	996 1170.5 3406	996 1108.9 3406	996 985.7 3406	543 680.0 2272	543 644.2 2272	543 572.6 2272	374 405.2 1724	374 455.9 1724	374 405.2 1724
	1/8	1150 1329.8 3737	1150 1259.8 3737	1150 1119.8 3737	636 786.9 2546	636 745.6 2546	636 662.7 2546	440 561.8 1955	440 532.3 1955	440 473.1 1955
	1/10	1210 1390.2 3858	1210 1317.0 3858	1210 1170.7 3858	673 828.3 2648	673 784.8 2648	673 697.6 2648	466 593.4 2042	466 562.2 2042	466 499.7 2042

AQL, AOQL and LQL values in ppm are presented as first, second and third values in each and every cell of the table

Table 4.3 Quality indices for the multifaceted continuous lot by lot acceptance sampling plan for $c = 3$ and $l = 5$

i	n	100			200			300		
		n/N			n/N			n/N		
	f	0.05	0.2	0.1	0.05	0.2	0.1	0.05	0.2	0.1
500	1/3	2599 2742.7 6998	2599 2603.1 6998	2599 2313.9 6998	1423 1568.9 4514	1423 1486.3 4514	1423 1321.2 4514	980 1100.6 3358	980 1042.7 3358	980 926.8 3358
	1/5	2896 3039.8 7574	2896 2879.8 7574	2896 2559.8 7574	1603 1760.7 4971	1603 1667.9 4971	1603 1482.7 4971	1109 1243.2 3733	1109 1177.8 3733	1109 1046.9 3733
	1/8	3228 3361.0 8184	3228 3184.1 8184	3228 2830.3 8184	1810 1977.2 5464	1810 1873.1 5464	1810 1664.9 5464	1258 1406.4 4140	1258 1332.4 4140	1258 1184.3 4140
	1/10	3353 3481.5 8406	3353 3298.2 8406	3353 2931.7 8406	1889 2059.8 5645	1889 1951.4 5645	1889 1734.5 5645	1316 1469.2 4293	1316 1391.9 4293	1316 1237.2 4293
1000	1/3	1949 1984.6 4579	1949 1880.2 4579	1949 1671.2 4579	1116 1190.4 3119	1116 1127.7 3119	1116 1002.4 3119	782 854.5 2401	782 809.6 2401	782 719.6 2401
	1/5	2146 2168.4 4897	2146 2054.3 4897	2146 1826.0 4897	1246 1321.2 3392	1246 1251.7 3392	1246 1112.6 3392	879 956.1 2639	879 905.8 2639	879 805.2 2639
	1/8	2361 2366.8 5231	2361 2242.2 5231	2361 1993.1 5231	1393 1466.1 3683	1393 1388.9 3683	1393 1234.6 3683	990 1070.3 2896	990 1013.9 2896	990 901.3 2896
	1/10	2443 2440.2 5354	2443 2311.8 5354	2443 2054.9 5354	1449 1520.6 3789	1449 1440.6 3789	1449 1280.5 3789	1032 1113.7 2990	1032 1055.1 2990	1032 937.9 2990

AQL, AOQL and LQL values in ppm are presented as first, second and third values in each and every cell of the table