On fuzzy γ^* generalized closed sets

Keerthana, R

(16PMA009)

Thesis Submitted to

Avinashilingam Institute for Home science and Higher Education for Women

Coimbatore-641 043

In Partial Fulfilment of the Requirements for the Degree of

Master of Science in Mathematics

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CONTENT

Introduction

L. A. Zadeh (1965) first introduced the concepts of fuzzy sets and fuzzy set operations in his classical paper. A fuzzy set in X is a mapping from X in to I where X is a non empty set and I is the unit interval [0,1]. The notion of fuzzy sets naturally plays a very significant role in the study of fuzzy topology.

Chang (1968) introduced the concept of fuzzy topological space which is a natural generalization of topological spaces.Subsequently several researchers have worked on various basic concepts from general topology using fuzzy sets and developed the theory of fuzzy topological spaces.

Closed sets are fundamental objects in topological spaces. Levine (1970) introduced the concept of generalized closed sets in general topology. Using Levine's idea many researchers have introduced and studied various types of generalized closed sets. K.K.Azad (1981)introduced the fuzzy semi-continuous mappings, fuzzy weaklycontinuous mappings,fuzzy open mappings and fuzzy semi-open mappings. The same author also introducedconcept of fuzzy almost continuous mappings. Thakur and Malviya (1995) introduced fuzzy generalized closed sets. Andrijevic (1996) introduced b-open sets in topological spaces. Benchalli and Jenifer (2010) introduced fuzzy b open sets in fuzzy topological spaces.

Fuzzy continuity is oneof the main topics in fuzzy topology. Various authors introduced various types of fuzzy continuity. One of them was fuzzy γcontinuity. I. M., Hanafy (1999) introduced the concept of fuzzy γ continuity and fuzzy generalized γ continuity in fuzzy topologicalspaces.

The definition of fuzzy point by Wong (1974) gained an importance and it is also added a new dimension to the study of fuzzy topological concepts. Pu and Liu (1980) introduced the concept of quasi coincidence in fuzzy topological spaces. M. N. Mukherjee and S. P. Sinha (1989) studied some weaker forms of fuzzy continuous and fuzzy open maps between fuzzy topological spaces.

In this research work, we have analyzed the following topics:

- Fuzzy $\gamma*$ generalized closed sets and fuzzy$\gamma*$ generalized open sets
- Fuzzy $\gamma*$ generalized continuous mappings and fuzzy almost $\gamma*$ generalized continuous mappings
- Fuzzy $\gamma*$ generalized irresolute mappings

Chapter 1 deals with the preliminary definitions that are needed for the present study.

In **Chapter 2,** fuzzy$\gamma*$ generalized closed sets and fuzzy$\gamma*$ generalized open sets are introduced. The relationship between these newly introduced fuzzy set and few of the already existing fuzzy closed sets are being discussed. Some characterizations of these newly introduced sets are also studied.

In **Chapter 3,** fuzzy $\gamma*$ generalized continuous mappings and fuzzy almost $\gamma*$ generalized continuous mappings are introduced which is followed by the relation of it with some of the already existing continuous mappings in fuzzy topological spaces. Also we have investigated some of their properties.

In **Chapter 4,** fuzzy $\gamma*$ generalized irresolute mappings are introduced. Also we have investigated some of their properties.

Review of literature

Research on the field of fuzzy topology was developed by many authors. Many researchers have contributed to the study of generalized forms of fuzzy closed sets and their basic properties. We present a review of literature in some of the important article published that are related to this topic.

Levine (1963) introduced the concepts of semi open sets and semi continuity in topological spaces. Chang (1968) introduced the notions of fuzzy topology and most basic concepts like fuzzy open set, fuzzy closed set, fuzzy neighborhood, fuzzy interior of a fuzzy set. Azad (1981) introduced the notions of fuzzy semi open and fuzzy semiclosed, fuzzy regular open and fuzzy regular closed sets.

Palaniappan (1993) introduced the concept of regular generalized closed sets in topological spaces. Fukutake, Nasef and El-Maghrabi (2003) introduced γ generalizedclosed sets in fuzzy topological spaces.Ganster and Steiner (2007) investigated many relationships between b generalized closed sets with generalized notions of closed sets. Luay.A. Al. Swidi and Amed. S. A. Oon (2011) studied some of the properties of fuzzy γ open sets and fuzzyγclosed sets.

Chang (1968) studied and introduced fuzzy continuity which was proved to be fundamental importance in fuzzy topology since then various notions in classical topology have been extended to fuzzy topological spaces by various authors. Mashour (1983) established α continuous and α open mappings in topological spaces. A. S. Bin Shahana (1991) introduced fuzzy pre open sets, and fuzzy pre continuity. M. K. Singhal and NitiRajvanshi (1992) introduced the notion of fuzzy α sets and also introduced the concepts of fuzzy α continuous mappings and fuzzy α open mappings. G. Balasubramaniam and P. Sundaram (1997) have introduced fuzzy generalized continuous mappings and $T_{1/2}$ spaces.

M. K. Singhal and A. R. Singhal (1968) introduced almost continuous mapping in topological spaces. Balachandran (1991) introduced a class of generalized continuous maps in topological spaces and studied their relationships with other irresolute maps. Min (2002) introduced γ open sets, and the notion of γ closure, γ interior and γ continuity in topological spaces and investigated some of their properties.

Here we present a brief survey of some of the articles published on fuzzy closed sets and fuzzy continuous functions.

1. FUZZY SETS

[Zadeh, L.A., 1965]

The author have introduced the concept of fuzzy set. The notions of inclusion, union, intersection, complement, relation and various properties of these notions of fuzzy sets are established.

2. FUZZY TOPOLOGICAL SPACES

[Chang, C. L., 1968]

Theauthor have introduced the basic concepts such as fuzzy open sets, fuzzy closed sets, fuzzy neighborhood, fuzzy interior, fuzzy continuity and fuzzy compactness.

3. GENERALIZED CLOSED SETS IN TOPOLOGY

[Levine., 1970]

In this paper the author have introduced generalized closed sets and generalized open sets in topological spaces. The author also studied and investigated standard properties of these sets.

4. ON FUZZY SEMI CONTINUITY, FUZZY ALMOST CONTINUITY AND FUZZY WEAKLY CONTINUITY

[Azad, K. K., 1981]

In this paper the author have introduced fuzzy semiopen set, fuzzy semiclosed set, fuzzy regular open set, fuzzy regular closed set, fuzzy semi continuous mapping, fuzzy semi open mapping, fuzzy semi closed mapping, fuzzy almost mapping and fuzzy weakly continuous mapping and discussed their character.

5.α-CONTINUOUS AND α-OPEN MAPPINGS

[Mashour., 1983]

The author established and studied α-continuous and α-open mapping in topological spaces.

6.REGULARLY OPEN SETS IN FUZZY TOPOLOGICAL SPACES

[Singal, M. K., NitiRajvanshi 1992 [b]]

This paper is devoted to the study of role of fuzzy regularly open sets and to prove some properties of fuzzy almost continuous mappings and defined fuzzy almost open mappings.

7. GENERALIZED CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

[Thakur, S. S., Malviya, R., 1995]

The authors have introduced and extended the concept of generalized closed sets in fuzzy topology. Many authors utilized fuzzy generalized closed sets for the generalization of various fuzzy topological concepts in fuzzy topology.

8. FUZZY γ OPEN SETS AND FUZZY γ CONTINUITY

[Hanafy, I. M., 1999]

The author has introduced the concept of fuzzy γ open sets which is weaker than each of the concept of fuzzy semi open set or fuzzy pre open set. The author also studied their properties and discussed relationships between these concepts and in fuzzy topological spaces.

9. FUZZY GENERALIZED ALPHA CLOSED SETS AND ITS APPLICATIONS

[BayazDaraby, Nimse., S. B., 2007]

In this article the authors have defined and studied fuzzy generalized α closed sets and fuzzy α continuous functions and their applications.

10. ON GENERALIZED b CLOSED SETS

[Ahmad Al Omari, Mohammed Salmi Md. Noorani., 2009]

In this article the authors have studied the basic concepts of generalized b closed sets and used this notion to consider new weak and stronger forms of continuities associated with these sets.

11. ON FUZZY b OPEN SETS IN FUZZY TOPOLOGICAL SPACES

[Benchalli, S. S., Jenifer Karnel, 2010 [a]]

In this paper the authors have introduced a new form of fuzzy set called fuzzy b open and fuzzy b closed set and studied some of their properties. Also the concept of fuzzy generalized open set and fuzzy generalized closed set is introduced and studied. The interrelationship of fuzzy generalized b open set with fuzzy b open set is investigated.

12. ON FUZZY b NEIGHBORHOODS AND FUZZY b MAPPINGS IN FUZZY TOPOLOGICAL SPACES

[Benchalli, S. S., Jenifer Karnel., 2010 [b]]

In this article the authors have introduced the concept of fuzzy b neighborhood and fuzzy b continuous mappings in fuzzy topological spaces. The interrelationship of fuzzy b continuous mappings with various fuzzy mappings are investigated.

13. FUZZY γ OPEN SETS AND FUZZY γ CLOSED SETS

[Luay, A. Al. Swidi., Amed, S. A. Oon., 2011]

In this paper the authors have introduced fuzzyγ open sets and investigated its properties in fuzzy topological spaces.

14. ON ALMOST γ CONTINUOUS FUNCTIONS

[Hariwan, Z. Ibrahim., 2012]

In this paper the author have introduced a new class of functions called almostγ continuous functions which is contained in the class of almost continuous functions and contains the class of γ continuous functions.

15. MORE ON γ GENERALIZED CLOSED SETS IN TOPOLOGY

[Maghrabi, A. I. EL., 2013]

In this article the author has introduced and studied a new class of sets called γ generalized regularly weakly closed set. This new class of sets lies between the class of regularly weakly closed sets and the class of γ generalized closed set.

16. ON FUZZY γ SEMI OPEN SETS AND FUZZY γ SEMI CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

[UshaParameshwari, R., Bageemathi, K., 2013]

The authors have introduced fuzzy γ semi open sets and fuzzy γ semi closed sets and established their properties in fuzzy topological spaces. In addition, the authors have also introduced fuzzy γ semi interior and fuzzy γ semi closure operators and fuzzy γ t-set.

17. FUZZY GENERALIZED γ CLOSED SETS IN FUZZY TOPOLOGICAL SPACES

[Dipankar De., 2014]

This paper introduced and studied the concepts of the fuzzy generalized γ closed sets and analyzed their basic properties in fuzzy topological spaces.

CHAPTER 1

Preliminaries

Definition 1.1: (1965)

Let X be a non-empty set. A fuzzy set A in X is characterized by its membership function μ_A: X→ [0, 1] and $\mu_A(x)$ is interpreted as the degree of member of element x in a fuzzy set A, for each x ∈ X. It is clear that A is determined by the set of tuples of A = {(x, $\mu_A(x)$) : x ∈ X}.

Definition 1.2: (1965)

A family τ of fuzzy sets is called fuzzy topologyfor X if it satisfies the following three axioms:

(a) $\bar{0}, \bar{1} \in \tau$

(b) ∀ A, B $\in \tau$ ⇒ A ∧ B $\in \tau$

(c) $\forall (A_j)_{j\in J} \in \tau \Rightarrow \bigvee_{j\in J} A_j \in \tau$

The pair (X, τ) is called a fuzzy topological space.The elements of τ are called fuzzy open sets in X and their respective complements are called fuzzy closed sets of (X, τ).

Definition 1.3: (1965)

Let A and B be two fuzzy sets A = {(x, $\mu_A(x)$) : x ∈ X} and B = {(x, $\mu_B(x)$) : x ∈ X}.Then, their union A∨B, intersection A ∧B and complement A^c are also fuzzy sets with membership functions defined as follows :

(a) $\mu_{A^c}(x) = 1-\mu_A(x)$,∀ x ∈ X,

(b) $\mu_{A\vee B}(x) = \max\{\mu_A(x), \mu_B(x)\}$, ∀ x ∈ X,

(c) $\mu_{A\wedge B}(x) = \min \{\mu_A(x), \mu_B(x)\}$, ∀x ∈ X.

Further,

(a) A ⊆B if and only if $\mu_A(x) \leq \mu_B(x)$, ∀ x ∈ X,

(b) A = B if and only if $\mu_A(x) = \mu_B(x)$,∀x ∈ X.

Definition 1.4: (2011)

Let A be a fuzzy set in a fuzzy topological space X. Then we define γ interior andγ closure as

i. $\gamma cl(A) = \wedge \{ B : B \geq A$, B is a fuzzy$\gamma$ closed set in X$\}$,

ii. $\gamma int(A) = \vee\{B : B \leq A$, B is a fuzzy$\gamma$ open set in X$\}$.

Definition 1.5: (1981)

Let (X, τ) be a fuzzy topological space. Then a fuzzy set A is said to be

i. fuzzy semi closed set if $int(cl(A)) \leq A$

ii. fuzzy regular closed set if $cl(int(A)) = A$

Definition 1.6: (1998)

Let (X, τ) be a fuzzy topological space. Then a fuzzy set A is said to be a

i. fuzzy pre closed set if $cl(int(A)) \leq A$

ii. fuzzy α closed set if $cl(int(cl(A))) \leq A$

Definition 1.7: (1999)

Let (X, τ) be a fuzzy topological space. Then a fuzzy set A is said to be a fuzzy γ closed set (fuzzy b closed set)if $cl(int(A)) \wedge int(cl(A)) \leq A$

Definition 1.8: (1995)

Let (X, τ) be a fuzzy topological space. Then a fuzzy set A is said to be a fuzzy generalized closed set if $cl(A) \leq U$ whenever $A \leq U$ and U is a fuzzy open in X

Definition 1.9: (1981)

Let (X, τ) be a fuzzy topological space. Then a fuzzy set A is said to be a

i. fuzzy semi open set if $A \leq cl(int(A))$

ii. fuzzy regular open set if $A= int(cl(A))$

Definition 1.10: (1998)

Let (X, τ) be a fuzzy topological space. Then a fuzzy set A is said to be a

i. fuzzy pre open set if $A \leq int(cl(A))$
ii. fuzzy α open set if $A \leq int(cl(int(A)))$

Definition 1.11: (1999)

Let (X, τ) be a fuzzy topological space. Then a fuzzy set A is said to be a fuzzy γ open set (fuzzy b open set)if $A \leq int(cl(A)) \vee cl(int(A))$.

Definition 1.12:(1980)

A fuzzy set A isquasi-coincident with a fuzzy set B, denoted by A_qB, if there exists $x \in X$ such that $A(x)+B(x) > 1$.

Definition 1.13: (1980)

If A and B are not quasi-coincident then we write $A_{\bar{q}}B$ and $A \leq B \Leftrightarrow A_{\bar{q}}(1 - B)$.

Definition 1.14: (2013)

A fuzzy set A in a fuzzy topological space (X, τ) is fuzzy nowhere dense if there exists no non-zero fuzzy open set B in (X, τ) such that $B < cl(A)$, that is $int(cl(A)) = \bar{0}$.

Definition 1.15: (2014)

The intersection of all fuzzy open subsets of a fuzzy topological space (X, τ) containing A is called the Kernel of A, this means $ker(A) = \wedge\{G \in \tau, A \leq G\}$.

Definition 1.16: (1974)

Afuzzy point$\tilde{p}$ in a set X is also a fuzzy set with membership function

$$\mu_{\tilde{p}}(x) = \begin{cases} r, & \text{for } x = y \\ 0, & \text{for } x \neq y \end{cases}$$

where $x \in X$ and $0 < r \leq 1$, y is called the support of $\tilde{p}$ and r the value of $\tilde{p}$. We denote this fuzzy point by $x_r$or $\tilde{p}$. A fuzzy point $x_r$is said to be belonged to a fuzzy subset $\tilde{A}$ in X, denoted by $x_r \in \tilde{A}$ if and only if $r \leq \mu_{\tilde{A}}(x)$.

Definition 1.17: (1968)

Let f be a mapping from a fuzzy topological space (X, τ_1) into a fuzzy topological space (Y,τ_2). Then f is said to be a fuzzy continuous mapping if for every U$\in \tau_2$, f^{-1} (U) $\in \tau_1$.

Definition 1.18: (1981)

Let f be a mapping from a fuzzy topological space (X, τ_1) into a fuzzy topological space (Y,τ_2). Then f is said to be afuzzy semi continuous mappingif f^{-1}(A) is a fuzzy semi closed set in X for every fuzzy closed set A of Y.

Definition 1.19: (1991)

Let f be a mapping from a fuzzy topological space (X, τ_1) into a fuzzy topological space (Y,τ_2). Then f is said to be a

i. fuzzy pre continuous mappingif f^{-1}(A) is a fuzzy pre closed set in X for every fuzzy closed set A of Y
ii. fuzzy α continuous mapping if f^{-1}(A) is a fuzzy α closed set in X for every fuzzy closed set A of Y.

Definition 1.20: (1999)

Let f be a mapping from a fuzzy topological space (X, τ_1) into a fuzzy topological space (Y,τ_2). Then f is said to be a fuzzy γ continuous mapping (fuzzy b continuous mapping)if f^{-1}(A) is a fuzzy γ closed set (fuzzy b closed set) in X for every fuzzy closed set A of Y.

Definition 1.21: (1997)

Let f be a mapping from a fuzzy topological space (X, τ_1) into a fuzzy topological space (Y,τ_2). Then f is said to be a fuzzy generalized continuous mappingif f^{-1}(A) is a fuzzy generalized closed set in X for every fuzzy closed set A of Y.

Definition 1.22: (2008)

A fuzzy set A is said to be fuzzy dense in another fuzzy set B in a fuzzy topological space X, if cl(A) = B.

CHAPTER 2

Section 2.1

Fuzzy γ^* generalized closed sets

In this section we have introduced a new class of fuzzy set called fuzzyγ^* generalized closed sets and discussed some of their properties.

Definition 2.1.1:

A fuzzy set A of a fuzzy topological space (X, τ) is said to be a fuzzy γ^* generalized closed set if cl(int(A))∧int(cl(A)) ≤ U, whenever A ≤ U and U is a fuzzy open set in X.

The family of all fuzzy γ^* closed sets of a fuzzy topological spaces of a fuzzy topological space (X, τ) is denoted by Fγ^*C(X).

Example 2.1.2:

Let X = {a, b} and τ = {$\bar{0}$,$\bar{1}$, G_1, G_2} be a fuzzy topology on X, where G_1 = $\langle x, (0.5_a, 0.6_b)\rangle$, G_2 = $\langle x, (0.4_a, 0.5_b)\rangle$. Then (X, τ) is a fuzzy topological space. Let A = $\langle x, (0.5_a, 0.5_b)\rangle$ be a fuzzy set in (X, τ). We have A ≤ G_1. Now, cl(int(A)) ∧ int(cl(A)) = G_2^c∧ G_2 = G_2 ≤ G_1, where G_1 is a fuzzy open set in X. This implies A is a fuzzy γ^* generalized closed set in X.

Theorem 2.1.3:

Every fuzzy closed set is a fuzzy γ^* generalized closed set in (X, τ) but not conversely in general.

Proof:

Let A be a fuzzy closed set in (X, τ), then cl(A) = A. Let A ≤ U and U be a fuzzy open set in (X, τ). Now cl(int(A)) ∧ int(cl(A)) ≤ cl(A) = A ≤ U, by hypothesis. Hence A is a fuzzy γ^* generalized closed set in (X, τ).

Example 2.1.4:

Let X = {a, b} and $\tau = \{\bar{0}, \bar{1}, G_1, G_2\}$ be a fuzzy topology on X, where $G_1 = \langle x, (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.5_b)\rangle$. Then (X, τ) is a fuzzy topological space. Let $A = \langle x, (0.5_a, 0.5_b)\rangle$ be a fuzzy set in (X, τ). We have $A \leq G_1$. Now, $cl(int(A)) \wedge int(cl(A)) = G_2^c \wedge G_2 = G_2 \leq G_1$ where G_1 is a fuzzy open set in X. This implies A is a fuzzy γ^* generalized closed set in X but not a fuzzy closed set in (X, τ), as $cl(A) = G_2^c \neq A$.

Theorem 2.1.5:

Every fuzzy semi closed set in (X, τ) is a fuzzy γ^* generalized closed set but not conversely in general.

Proof:

Let A be a fuzzy semi closed set in X, then $int(cl(A)) \leq A$. Let $A \leq U$ and U be a fuzzy open set in (X, τ). Now $cl(int(A)) \wedge int(cl(A)) \leq cl(int(A)) \wedge A \leq cl(A) \wedge A = A \leq U$. Hence A is a fuzzy γ^* generalized closed set in (X, τ).

Example 2.1.6:

Let X = {a, b} and $\tau = \{\bar{0}, \bar{1}, G_1, G_2\}$ be a fuzzy topology on X, where $G_1 = \langle x, (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.6_a, 0.5_b)\rangle$. Then (X, τ) is a fuzzy topological space. Let $A = \langle x, (0.5_a, 0.3_b)\rangle$ be a fuzzy set in (X, τ). Now $cl(int(A)) \wedge int(cl(A)) = \bar{0} \wedge G_1 = \bar{0} \leq U$, then A is a fuzzy γ^* generalized closed set but not a fuzzy semi closed set in (X, τ), as $int(cl(A)) = G_1 \nleq A$.

Theorem 2.1.7:

Every fuzzy pre closed set is a fuzzy γ^* generalized closed set in (X, τ) but not conversely in general.

Proof:

Let A be a fuzzy pre closed set in X, then $cl(int(A)) \leq A$. Let $A \leq U$ and U be a fuzzy open set in (X, τ). Now $cl(int(A)) \wedge int(cl(A)) \leq A \wedge int(cl(A)) \leq A \wedge cl(A) = A \leq U$. Hence A is a fuzzy γ^* generalized closed set in (X, τ).

Example 2.1.8:

Let X = {a, b} and $\tau = \{\bar{0}, \bar{1}, G_1, G_2\}$ be a fuzzy topology on X, where $G_1 = \langle x, (0.3_a, 0.3_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.6_b)\rangle$. Then (X, τ) is a fuzzy topological space. Let

$A = \langle x, (0.4_a, 0.4_b)\rangle$ be a fuzzy set in (X, τ). Now $cl(int(A)) \wedge int(cl(A)) = G_2^c \wedge G_1 = G_1 \leq G_2$, where $A \leq G_2$. Then A is a fuzzy γ^* generalized closed set in X, but not a fuzzy pre closed set in (X, τ), as $cl(int(A)) = G_2^c \nleq A$.

Theorem 2.1.9:

Every fuzzy regular closed set is a fuzzy γ^* generalized closed set in (X, τ) but not conversely in general.

Proof:

Let A be a fuzzy regular closed set in X, then $cl(int(A)) = A$. Let $A \leq U$ and U be a fuzzy open set in (X, τ). Now $cl(int(A)) \wedge int(cl(A)) = A \wedge int(cl(A)) \leq A \wedge cl(A) = A \leq U$. Hence A is a fuzzy γ^* generalized closed set in (X, τ).

Example 2.1.10:

Let $X = \{a, b\}$ and $\tau = \{\bar{0}, \bar{1}, G_1, G_2\}$ be a fuzzy topology on X, where $G_1 = \langle x, (0.3_a, 0.3_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.6_b)\rangle$. Then (X, τ) is a fuzzy topological space. Let $A = \langle x, (0.4_a, 0.4_b)\rangle$ be a fuzzy set in (X, τ). Now $cl(int(A)) \wedge int(cl(A)) = G_2^c \wedge G_1 = G_1 \leq G_2$, where $A \leq G_2$. Then A is a fuzzy γ^* generalized closed set in X, but not a fuzzy regular set as $cl(int(A)) = G_2^c \neq A$.

Theorem 2.1.11:

Every fuzzy α closed set is afuzzy γ^* generalized closed set in (X, τ) but not conversely in general.

Proof:

Let A be a fuzzy α closed set in X, then $cl(int(cl(A))) \leq A$. Let $A \leq U$ and U be fuzzy open set in (X, τ). Now $cl(int(A)) \wedge int(cl(A)) \leq cl(int(cl(A))) \wedge int(cl(A)) \leq A \wedge cl(A) = A \leq U$. Hence A is a fuzzy γ^* generalized closed set in (X, τ).

Example 2.1.12:

Let $X = \{a, b\}$ and $\tau = \{\bar{0}, \bar{1}, G_1, G_2\}$ be a fuzzy topology on X, where $G_1 = \langle x, (0.3_a, 0.3_b)\rangle$, $G_2 = \langle x, (0.5_a, 0.6_b)\rangle$. Then (X, τ) is a fuzzy topological space. Let $A = \langle x, (0.4_a, 0.4_b)\rangle$ be a fuzzy set in (X, τ). Now $cl(int(A)) \wedge int(cl(A)) = G_2^c \wedge G_1 = G_1 \leq G_2$, where $A \leq G_2$. Then A is a fuzzy γ^* generalized closed set in X, but not a fuzzy α closed set as $cl(int(cl(A))) = G_2^c \nleq A$.

Theorem 2.1.13:

Every fuzzy γ closed set is a fuzzy γ^* generalized closed set in (X, τ) but not conversely in general.

Proof:

Let A be a fuzzy γ closed set in X, then $cl(int(A)) \wedge int(cl(A)) \leq A$. Let $A \leq U$ and U be a fuzzy open set in (X, τ). Now $cl(int(A)) \wedge int(cl(A)) \leq A \leq U$. Hence A is a fuzzy γ^* generalized closed set in (X, τ).

Example 2.1.14:

Let X = {a, b} and $\tau = \{\overline{0}, \overline{1}, G_1, G_2\}$ be a fuzzy topology on X, where $G_1 = \langle x,(0.3_a, 0.3_b)\rangle$, $G_2 = \langle x,(0.5_a, 0.5_b)\rangle$. Then (X, τ) is a fuzzy topological space. Let $A = \langle x,(0.4_a, 0.4_b)\rangle$be a fuzzy set in (X, τ). Now $cl(int(A)) \wedge int(cl(A)) = G_2^c \wedge G_2 = G_2 \leq G_2$where $A \leq G_2$. Then A is a fuzzy γ^* generalized closed set but not a fuzzy γ closed set as $cl(int(A)) \wedge int(cl(A)) = G_2 \nleq A$.

Theorem 2.1.15:

Every fuzzy generalized closed set is a fuzzy γ^* generalized closed set in (X, τ) but not conversely in general.

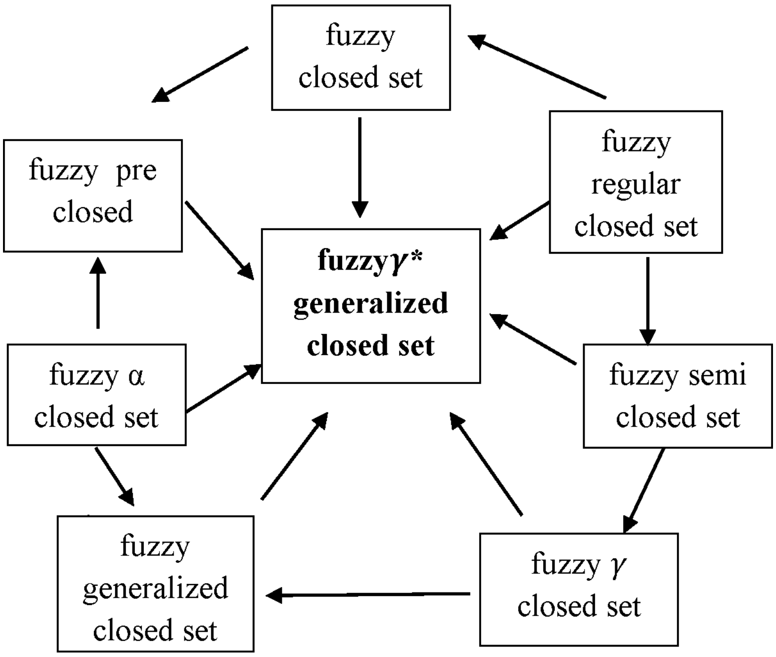
Proof:

Let A be a fuzzy generalized closed set in X. Let $A \leq U$ and U be a fuzzy open set in (X, τ). Now $cl(int(A)) \wedge int(cl(A)) \leq cl(A) \wedge cl(A) = cl(A) \leq U$, by hypothesis. Hence A is a fuzzy γ^* generalized closed set in (X, τ).

Example 2.1.16:

Let X = {a, b} and $\tau = \{\overline{0}, \overline{1}, G_1, G_2\}$ be a fuzzy topology on X, where $G_1 = \langle x,(0.5_a, 0.6_b)\rangle$, $G_2 = \langle x,(0.4_a, 0.5_b)\rangle$. Then (X, τ) is a fuzzy topological space. Let $A = \langle x,(0.4_a, 0.5_b)\rangle$be a fuzzy set in (X, τ). Now $cl(int(A)) \wedge int(cl(A)) = G_2^c \wedge G_2 = G_2 \leq G_1, G_2$ where $A \leq G_1, G_2$. Then A is a fuzzy γ^* generalized closed set but not a fuzzy generalized closed set as $cl(A) = G_2^c \nleq G_1, G_2$ but $A \leq G_1, G_2$.

In the following diagram we have provided relation between various types of fuzzy closedness.



Theorem 2.1.17:

Let (X, τ) be a fuzzy topological space. Then for every A ∈ Fγ*GC(X) and for every B ∈ FS(X), A ≤ B ≤ cl(int(A))⟹B ∈ Fγ*GC(X).

Proof:

Let B ≤ U and U be a fuzzy open set in X. Let A ≤ B, A ≤ U, by hypothesis. Since B ≤ cl(int(A)), cl(int(B)) ≤ cl(int(A)). Also int(cl(B)) ≤ int(cl(cl(int(A)))) ≤ int(cl(int(A))) ≤ int(cl(A)). Therefore cl(int(B)) ∧int(cl(B)) ≤ cl(int(A)) ∧ int(cl(A)) ≤ U, by hypothesis. Hence B ∈ Fγ*GC(X).

Theorem 2.1.18:

A fuzzy set A of a fuzzy topological space (X, τ) is a fuzzyγ* generalized closed set if and only if $A_{\bar{q}}F \Rightarrow (\text{int}(\text{cl}(A)) \wedge \text{cl}(\text{int}(A)))_{\bar{q}}F$ for every fuzzy closed set F of X.

Proof:

Necessity:

Let F be a fuzzy closed set and $A_{\bar{q}}F$, then A≤ F^c, where F^c is a fuzzy open set in X. Then int(cl(A)) ∧ cl(int(A)) ≤ F^c, by hypothesis. Hence by Definition 1.13 $(\text{int}(\text{cl}(A)) \wedge \text{cl}(\text{int}(A)))_{\bar{q}}$ F.

Sufficiency:

Let U be a fuzzy open set in X such that A ≤ U. Then U^c is a fuzzy closed set and A ≤ $(U^c)^c$. Therefore $A_{\bar{q}}U^c$.By hypothesis, $A_{\bar{q}}U^c \Rightarrow (\text{int}(\text{cl}(A)) \wedge \text{cl}(\text{int}(A)))_{\bar{q}}U^c$. Hence int(cl(A))∧ cl(int(A)) ≤ $(U^c)^c$ = U. Therefore int(cl(A)) ∧ cl(int(A)) ≤ U. Hence A is a fuzzy $\gamma*$ generalized closed set.

Theorem 2.1.19:

If A is both a fuzzy open set and a fuzzy$\gamma*$ generalized closed set then A is a fuzzy γ closed set in (X,τ).

Proof:

Let A be a fuzzy open set and a fuzzy $\gamma*$ generalized closed set in (X, τ). Then as A ≤ A, cl(int(A) ∧ int(cl(A)) ≤ A. Hence A is a fuzzy γ closed set in (X, τ).

Theorem 2.1.20:

For a fuzzy set A in (X, τ) the following are equivalent:

i. A is both a fuzzy open set and a fuzzy $\gamma*$ generalized closed set
ii. A is a fuzzy regular open set

Proof:

(i) ⟹ (ii) Let A be a fuzzy open set and a fuzzy$\gamma*$ generalized closed set in X. Then by Theorem 2.1.19, A is a fuzzy γ closed set. So cl(int(A)) ∧int(cl(A)) ≤ A. We have int(cl(A)) = int(cl(A)) ∧ cl(A) = int(cl(A)) ∧ cl(int(A)) ≤ A. Hence int(cl(A)) ≤ A⟶ (1). Since A is a fuzzy open set, it is a fuzzy pre open set. Hence A≤ int(cl(A)) ⟶ (2). Therefore from (1) and (2), A = int(cl(A)) and A is a fuzzy regular open set in X.

(ii)⟹(i) Let A be a fuzzy regular open set in X then A = int(cl(A)). Since every fuzzy regular open set is a fuzzy open set, A is a fuzzy open set in X and A ≤

A, Therefore cl(int(A)) ∧ int(cl(A)) = cl(int(A)) ∧ A = A ∧ cl(A) ≤ A. Hence A is a fuzzy γ* generalized closed set in X.

Theorem 2.1.21:

Let F ≤ A ≤ X where A is a fuzzy open set and a fuzzy γ* generalized closed set in X. Then F is a fuzzy γ* generalized closed set in A if and only if F is a fuzzy γ* generalized closed set in X.

Proof:

Necessity:

Let F be a fuzzy γ* generalized closed set in A. Let U be a fuzzy open set in X and F ≤ U. Then F ≤ A ∧ U and A ∧ U is a fuzzy open set in A. Hence $int_A(cl_A(F))$ ∧ $cl_A(int_A(F))$ ≤ A∧ U. By Theorem 2.1.19 and by hypothesis, A is a fuzzy γ closed set. Therefore int(cl(A)) ∧ cl(int(A)) ≤ A. Now int(cl(F)) ∧ cl(int(F)) ≤ [int(cl(F)) ∧ cl(int(F))]∧ [int(cl(A)) ∧ cl(int(A))] ≤ (int(cl(F)) ∧ cl(int(F))) ∧ A = $int_A(cl_A(F))$ ∧ $cl_A(int_A(F))$ ≤ A ∧ U ≤ U. That is int(cl(F)) ∧ cl(int(F)) ≤ U, whenever F ≤ U. Hence F is a fuzzy γ* generalized closed set in X.

Sufficiency:

Let V be a fuzzy open set in A such that F ≤ V. Since A is a fuzzy open set in X, V is a fuzzy open set in X. Therefore int(cl(F)) ∧ cl(int(F)) ≤ V as F is a fuzzy γ* generalized closed set in X. Thus, $int_A(cl_A(F))$ ∧ $cl_A(int_A(F))$ = int(cl(F)) ∧ cl(int(F)) ∧ A ≤ V ∧ A ≤ V. Hence F is a fuzzy γ* generalized closed set in A.

Theorem 2.1.22:

For a fuzzy γ* generalized closed set A in a fuzzy topological space (X, τ), the following conditions hold:

i. A is a fuzzy regular open set, then scl(A) is a fuzzy γ* generalized closed set
ii. A is a fuzzy regular closed set, then sint(A) is a fuzzy γ* generalized closed set

Proof:

(i) Let A be a fuzzy regular open set in (X, τ). Then int(cl(A)) = A. By the definition of semi closure we have scl(A) = A$\vee$ int(cl(A)) = A. Since A is a fuzzy γ^* generalized closed set in X, scl(A) is a fuzzy γ^* generalized closed set in X.

(ii) Let A be a fuzzy regular closed set in (X, τ). Then cl(int(A)) = A. By the definition of semi interior we have sint(A) = A $\wedge$ cl(int(A)) = A. Since A is a fuzzy γ^* generalized closed set in X, sint(A) is a fuzzy γ^* generalized closed set in X.

Theorem 2.1.23:

If every fuzzy set in (X, τ) is a fuzzy γ^* generalized closed set then FO(X) $\leq$ FγC(X).

Proof:

Suppose that every fuzzy set is a fuzzy γ^* generalized closed set in (X, τ). Let U $\in$FO(X) then as U $\leq$ U and by hypothesis, int(cl(U)) $\wedge$ cl(int(U)) $\leq$ U. Therefore U $\in$ FγC(X). Hence FO(X) $\leq$ FγC(X).

Theorem 2.1.24:

A fuzzy set A of X is afuzzy γ^* generalized closed set if int(cl(A)) $\wedge$ cl(int(A)) $\leq$ ker(A).

Proof:

Let A be any fuzzy set and let U be any fuzzy open set in X such that A $\leq$ U. By hypothesis int(cl(A)) $\wedge$cl(int(A)) $\leq$ ker(A). Since A $\leq$ U, ker(A) $\leq$ U. Therefore int(cl(A))$\wedge$cl(int(A)) $\leq$ U and hence A is a fuzzy γ^* generalized closed set in X.

Theorem 2.1.25:

If a fuzzy set A of a fuzzy topological space X is nowhere dense, then A is afuzzy γ^* generalized closed set in X.

Proof:

If A is a fuzzy nowhere dense subset, then by Definition 1.14, int(cl(A)) = $\bar{0}$. Let A $\leq$ U where U is a fuzzy open set in X. Then cl(int(A)) $\wedge$ int(cl(A)) = $\bar{0} \leq$ U and hence A is a fuzzy γ* generalized closed set in X.

Theorem 2.1.26:

Let A be afuzzy γ* generalized closed set in (X, τ) and $\mu_{\tilde{p}}(x)$ be a fuzzy point such that $\mu_{\tilde{p}}(x)_q$(cl(int(A)) $\wedge$ int(cl(A)). Then cl($\mu_{\tilde{p}}(x))_q$ A.

Proof:

Assume that A is afuzzy γ* generalized closed set in (X, τ) and $\mu_{\tilde{p}}(x)_q$(cl(int(A)) $\wedge$ int(cl(A)). Suppose that cl($\mu_{\tilde{p}}(x))_{\bar{q}}$A, then A $\leq$ (cl($\mu_{\tilde{p}}(x)))^c$ where (cl($\mu_{\tilde{p}}(x)))^c$ is a fuzzy open set in (X, τ). Then by hypothesis, cl(int(A)) $\wedge$ int(cl(A)) $\leq$ (cl($\mu_{\tilde{p}}(x)))^c$ = int($\mu_{\tilde{p}}(x))^c \leq (\mu_{\tilde{p}}(x))^c$. Therefore(cl(int(A)) $\wedge$ int(cl(A))$_{\bar{q}}$($\mu_{\tilde{p}}(x)$), which is a contradiction to the hypothesis. Hence cl($\mu_{\tilde{p}}(x))_q$ A.

Theorem 2.1.27:

If A is a fuzzy open set and a fuzzy γ* generalized closed set in (X, τ), then int(A) is a fuzzy regular open set in X.

Proof:

Since A is a fuzzy open set and a fuzzy γ* generalized closed set in (X, τ), then by Theorem 2.1.19, A is a fuzzy γ closed set, which implies int(cl(A)) $\wedge$cl(int(A)) $\leq$ A. Therefore int[int(cl(A)) $\wedge$ cl(int(A))] $\leq$ int(A) which implies int(cl(int(A)) $\leq$ int(A). Since A is a fuzzy regular open set, it is a fuzzy α open set. Hence int(A) $\leq$ int(cl(int(A)). Therefore int(A) = int(cl(int(A)). Thus int(A) is a fuzzy regular open set.

Section 2.2

Fuzzyγ* generalized open sets

In this section we have introduced a new type of fuzzy open set called fuzzyγ* generalized open set and studied some of its properties.

Definition 2.2.1:

The complement A^c of a fuzzy γ* generalized closed set A in a fuzzy topological space (X, τ) is called a fuzzyγ* generalized open set in X.

The family of all fuzzy γ* generalized open sets of a fuzzy topological space (X, τ) is denoted by Fγ*GO(X).

Example 2.2.2:

In Example 2.1.2, A = $\langle x,(0.5_a, 0.5_b)\rangle$ is a fuzzy γ* generalized open set in X.

Theorem 2.2.3:

Every fuzzy open set, fuzzy semi open set, fuzzy pre open set, fuzzy regular open set, fuzzy α open set, fuzzy γ open set, fuzzy generalized open set are fuzzy γ* generalized open setin (X, τ) but not conversely in general.

Proof:

Straight forward.

Example 2.2.4:

Obvious from Example 2.1.4, Example 2.1.6, Example 2.1.8, Example 2.1.10, Example 2.1.12, Example 2.1.14, Example 2.1.16 by taking complement of A in the respective examples.

Theorem 2.2.5:

Let (X, τ) be a fuzzy topological space. Then for every A $\in$ Fγ*GO(X) and for every B $\in$ FS(X), int(cl(A)) $\leq$ B $\leq$ A $\Rightarrow$ B $\in$Fγ*GO(X).

Proof:

Let A be a fuzzy γ* generalized open set of X. Let B $\leq$ U and U be a fuzzy open set in X. As A^c is afuzzy γ* generalized closed set and $A^c \leq B^c \leq$ cl(int(A^c)) from the hypothesis, B^c is a fuzzy γ* generalized closed set, by Theorem 2.1.17. This implies B is a fuzzy γ* generalized open set in X. Hence B $\in$ Fγ*GO(X).

Theorem 2.2.6:

If A is a fuzzy γ closed set and a fuzzy γ* generalized open set in (X, τ), then A is a fuzzy γ open set in (X, τ).

Proof:

Obvious from the Theorem 2.1.19 by taking complement.

Theorem 2.1.7:

A fuzzy set A of a fuzzy topological space (X, τ) is a fuzzy γ* generalized open set if and only if F $\leq$ cl(int(A)) $\vee$ int(cl(A)) whenever F is a fuzzy closed set and F $\leq$ A.

Proof:

Necessity:

Suppose A is a fuzzy γ* generalized open set in X. Let F be a fuzzy closed set, such that F $\leq$ A.Then F^c is a fuzzy open set and $A^c \leq F^c$, by hypothesis A^c is a fuzzy γ* generalized closed set. We have int(cl(A^c)) $\wedge$ cl(int(A^c)) $\leq F^c$. Therefore F $\leq$ cl(int(A)) $\vee$ int(cl(A)).

Sufficiency:

Let U be a fuzzy open set, such that $A^c \leq$ U. Now $U^c \leq$ A and U^c is a fuzzy closed set in X. Then by hypothesis, $U^c \leq$ cl(int(A)) $\vee$int(cl(A)). Therefore int(cl(A^c))

$\vee$ cl(int(A^c)) $\leq$ U and A^c is a fuzzy γ^* generalized closed set. Hence A is a fuzzy γ^* generalized open set in X.

Theorem 2.1.8:

A fuzzy set A of a fuzzy topological space (X, τ) is a fuzzy γ^* generalized open set, then F $\leq$cl(int(cl(A))) whenever F is a fuzzy closed set and F $\leq$ A.

Proof:

Suppose A is afuzzy γ^* generalized open setin X. Let F be a fuzzy closed set such that F $\leq$ A. Then F^c is a fuzzy open set and $A^c \leq F^c$. By hypothesis A^c is afuzzy γ^* generalized closed set, we have cl(int(A^c)) $\wedge$ int(cl(A^c)) $\leq F^c$. Now int(cl(int(A^c))) = cl(int(A^c)) $\wedge$ int(cl(int(A^c))) $\leq$ cl(int(A^c)) $\wedge$ int(cl(A^c)) $\leq$ F^c. Therefore F $\leq$ cl(int(cl(A))).

CHAPTER 3

Section 3.1

Fuzzy γ^* generalized continuous mappings

In this section we have introduced fuzzy γ^* generalized continuous mappings and investigated some of their properties.

Definition 3.1.1:

A mapping f: $(X,\tau_1) \to (Y, \tau_2)$ is called a fuzzy γ^* generalized continuous mapping if $f^{-1}(V)$ is a fuzzy γ^* generalized closed set in (X, τ_1) for every fuzzy closed set V of (Y, τ_2).

Example 3.1.2:

Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_2\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.5_b)\rangle$ and $G_2 = \langle y, (0.6_u, 0.6_v)\rangle$. Then (X,τ_1) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: $(X,\tau_1) \to (Y, \tau_2)$ by f(a) = u, f(b) = v. The fuzzy set $G_2^c = \langle y, (0.4_u, 0.4_v)\rangle$ is a fuzzy closed set in Y. Then $f^{-1}(G_2^c) = \langle x, (0.4_a, 0.4_b)\rangle$ is a fuzzy γ^* generalized closed setin (X, τ_1) as $f^{-1}(G_2^c) \leq G_1$ and $cl(int(f^{-1}(G_2^c))) \wedge int(cl(f^{-1}(G_2^c))) = \bar{0} \leq G_1$, where G_1 is a fuzzy open set in X.Therefore f is a fuzzy γ^* generalized continuous mapping.

Theorem 3.1.3:

Every fuzzy continuous mapping is a fuzzy γ^* generalized continuous mapping but not conversely in general.

Proof:

Let f: $(X,\tau_1) \to (Y, \tau_2)$ be a fuzzy continuous mapping. Let V be a fuzzy closed set in Y. Then $f^{-1}(V)$ is a fuzzy closed set in X. Since every fuzzy closed set is a fuzzy γ^* generalized closed set, $f^{-1}(V)$ is a fuzzy γ^* generalized closed set in X. Hence f is a fuzzy γ^* generalized continuous mapping.

Example 3.1.4:

Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_2\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.5_b)\rangle$ and $G_2 = \langle y, (0.6_u, 0.6_v)\rangle$. Then (X,τ_1) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: $(X,\tau_1) \to (Y, \tau_2)$ by f(a) = u, f(b) = v. Then f is a fuzzy γ^* generalized continuous mapping but not a fuzzy continuous mapping. Since the fuzzy set $G_2^c = \langle y, (0.4_u, 0.4_v)\rangle$ is a fuzzy closed set in Y, but $f^{-1}(G_2^c)$ is not a fuzzy closed set in X as $cl(f^{-1}(G_2^c)) = G_1^c \neq f^{-1}(G_2^c)$.

Theorem 3.1.5:

Every fuzzy semi continuous mapping is a fuzzy γ^* generalized continuous mapping but not conversely in general.

Proof:

Let f: $(X,\tau_1) \to (Y, \tau_2)$ be a fuzzy semi continuous mapping. Let V be a fuzzy closed set in Y. Then $f^{-1}(V)$ is a fuzzy semi closed set in X. Since every fuzzy closed set is afuzzy γ^* generalized closed set, $f^{-1}(V)$ is a fuzzy γ^* generalized closed set in X. Hence f is a fuzzy γ^* generalized continuous mapping.

Example 3.1.6:

Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_2\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.5_b)\rangle$ and $G_2 = \langle y, (0.6_u, 0.6_v)\rangle$. Then (X,τ_1) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: $(X,\tau_1) \to (Y, \tau_2)$ by f(a) = u, f(b) = v. Then $f^{-1}(G_2^c)$ is a fuzzy γ^* generalized continuous mapping but not a fuzzy semi continuous mapping. Since the fuzzy set $G_2^c = \langle y, (0.4_u, 0.4_v)\rangle$ is a fuzzy closed set in Y, but $f^{-1}(G_2^c)$ is not a fuzzy semi closed set in X as $int(cl(f^{-1}(G_2^c))) = G_1 \nleq f^{-1}(G_2^c)$.

Theorem 3.1.7:

Every fuzzy pre continuous mapping is a fuzzy γ^* generalized continuous mapping but not conversely in general.

Proof:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy pre continuous mapping. Let V be a fuzzy closed set in Y. Then $f^{-1}(V)$ is a fuzzy pre closed set in X. Since every fuzzy pre closed set is a fuzzy γ^* generalized closed set, $f^{-1}(V)$ is a fuzzy γ^* generalized closed set in X. Hence f is a fuzzy γ^* generalized continuous mapping.

Example 3.1.8:

Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1, G_2\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_3\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.5_b)\rangle$ and $G_2 = \langle x, (0.4_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.5_v)\rangle$. Then (X,τ_1) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ by f(a) = u, f(b) = v. Then f is a fuzzy γ^* generalized continuous mapping but not a fuzzy pre continuous mapping. Since the fuzzy set $G_3^c = \langle y, (0.4_u, 0.5_v)\rangle$is a fuzzy closed set in Y, but $f^{-1}(G_3^c)$ is not a fuzzy pre closed set as $cl(int(f^{-1}(G_3^c))) = G_1^c \nleq f^{-1}(G_3^c)$.

Theorem 3.1.9:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a mapping and $f^{-1}(A)$ be a fuzzy regular closed set in X for every fuzzy closed set A in Y. Then f is a fuzzy γ^* generalized continuous mapping but not conversely in general.

Proof:

Let A be a fuzzy closed set in Y and $f^{-1}(A)$ is a fuzzy regular closed set in X. Since every fuzzy regular closed set is afuzzy γ^* generalized closed set, $f^{-1}(A)$ is a fuzzy γ^*generalized closed set in X. Hence f is a fuzzy γ^* generalized continuous mapping.

Example 3.1.10:

Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1, G_2\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_3\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.5_b)\rangle$ and $G_2 = \langle x, (0.4_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.5_v)\rangle$. Then (X,τ_1) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ by f(a) = u, f(b) = v. Then f is a fuzzy γ^* generalized continuous mapping but not a mapping as defined in the Theorem 3.1.9. Since the fuzzy set $G_3^c = \langle y, (0.4_u, 0.5_v)\rangle$is a fuzzy closed set in Y, but $f^{-1}(G_3^c)$ is not a fuzzy regular closed set as $cl(int(f^{-1}(G_3^c))) = G_1^c \neq f^{-1}(G_3^c)$.

Theorem 3.1.11:

Every fuzzy α continuous mapping is a fuzzy γ^* generalized continuous mapping but not conversely in general.

Proof:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy α continuous mapping. Let V be a fuzzy closed set in Y. Then $f^{-1}(V)$ is a fuzzy α closed set in X. Since every fuzzy α closed set is a fuzzy γ^*generalized closed set, $f^{-1}(V)$ is a fuzzy γ^*generalized closed set in X. Hence f is a fuzzy γ^* generalized continuous mapping.

Example 3.1.12:

Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1, G_2\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_3\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.5_b)\rangle$ and$G_2 = \langle x, (0.6_a, 0.6_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.6_v)\rangle$. Then (X,τ_1) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ by f(a) = u, f(b) = v. Then f is a fuzzy γ^* generalized continuous mapping but not a fuzzy α continuous mapping. Since the fuzzy set $G_3^c = \langle y, (0.5_u, 0.4_v)\rangle$ is a fuzzy closed set in Y, but $f^{-1}(G_3^c)$ is not a fuzzy α closed set in X as $cl(int(cl(f^{-1}(G_3^c)))) = G_1^c \nleq f^{-1}(G_3^c)$.

Theorem 3.1.13:

Every fuzzy γ continuous mapping is a fuzzy γ^* generalized continuous mapping but not conversely in general.

Proof:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ continuous mapping. Let V be a fuzzy closed set in Y. Then $f^{-1}(V)$ is a fuzzy γ closed set in X. Since every fuzzy γ closed set is a fuzzy γ^*generalized closed set, $f^{-1}(V)$ is a fuzzy γ^*generalized closed set in X. Hence f is a fuzzy γ^* generalized continuous mapping.

Example 3.1.14:

Let X = {a, b} and Y = {u, v}. Then τ_1 = {$\bar{0}$, $\bar{1}$, G_1, G_2} and τ_2 = {$\bar{0}$, $\bar{1}$, G_3} are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.3_a, 0.3_b)\rangle$ and $G_2 = \langle x, (0.5_a, 0.5_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.6_v)\rangle$. Then (X,$\tau_1$) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: (X,τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. Then f is a fuzzy γ* generalized continuous mapping but not a fuzzy γ continuous mapping. Since the fuzzy set $G_3^c = \langle y, (0.4_u, 0.4_v)\rangle$ is fuzzy closed set in Y, but $f^{-1}(G_3^c)$ is not a fuzzy γ closed set in X as $cl(int(f^{-1}(G_3^c))) \wedge int(cl(f^{-1}(G_3^c))) = G_2 \nleq f^{-1}(G_3^c)$.

Theorem 3.1.15:

Every fuzzy generalized continuous mapping is a fuzzy γ* generalized continuous mapping but not conversely in general.

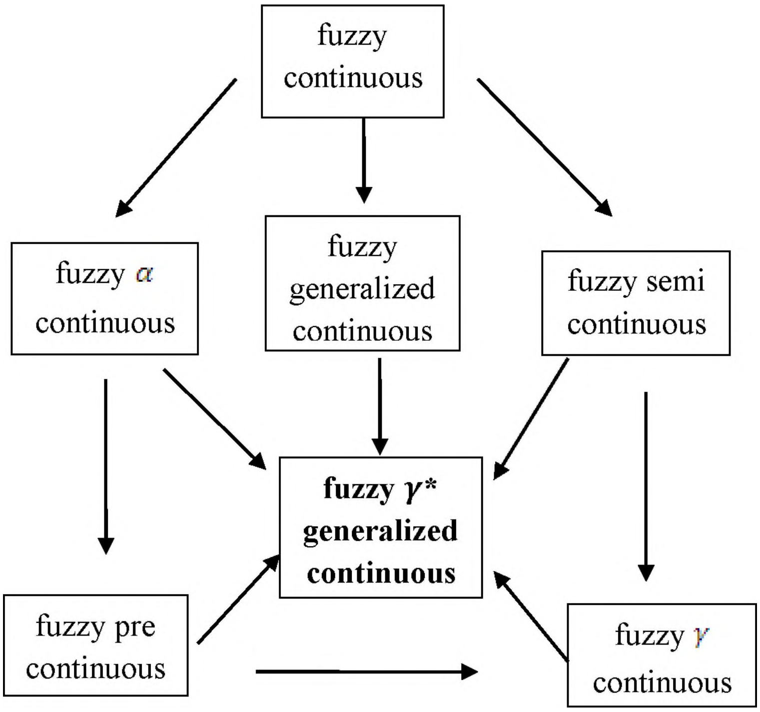
Proof:

Let f: (X, τ_1) → (Y, τ_2) be a fuzzy generalized continuous mapping. Let V be a fuzzy closed set in Y. Then $f^{-1}(V)$ is a fuzzy generalized closed set in X. Since every fuzzy generalized closed set is a fuzzy γ*generalized closed set, $f^{-1}(V)$ is a fuzzy γ*generalized closed set in X. Hence f is a fuzzy γ* generalized continuous mapping.

Example 3.1.16:

Let X = {a, b} and Y = {u, v}. Then τ_1 = {$\bar{0}$, $\bar{1}$, G_1, G_2} and τ_2 = {$\bar{0}$, $\bar{1}$, G_3}are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.6_a, 0.5_b)\rangle$ and $G_2 = \langle x, (0.4_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.5_v)\rangle$. Then (X,$\tau_1$) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: (X,τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. Then f is a fuzzy γ* generalized continuous mapping but not a fuzzy generalized continuous mapping. Since the fuzzy set $G_3^c = \langle y, (0.6_u, 0.5_v)\rangle$ is a fuzzy closed set in Y, but $f^{-1}(G_3^c)$ is not a fuzzy generalized closed set in X as $cl(f^{-1}(G_3^c)) = G_2^c \nleq G_1$.

The relation between various types of fuzzy continuity is given in the following diagram.



Theorem 3.1.17:

A mapping f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ^* generalized continuous mapping if and only if the inverse image of each fuzzy open set in Y is a fuzzy γ^* generalized open set in X.

Proof:

Necessity:

Let A be a fuzzy open set in Y. Then A^c is a fuzzy closed set in Y. Since f is a fuzzy γ^* generalized continuous mapping, $f^{-1}(A^c)$ is a fuzzy γ^* generalized closed set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a fuzzy γ^* generalized open set in X.

Sufficiency:

Let A be a fuzzy closed set in Y. Then A^c is a fuzzy open set in Y. By hypothesis $f^{-1}(A^c)$ is a fuzzy γ^* generalized open set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$,

$f^{-1}(A)$ is a fuzzy γ^* generalized closed set in X. Hence f is a fuzzy γ^* generalized continuous mapping.

Theorem 3.1.18:

If f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ^* generalized continuous mapping then for each fuzzy point $\mu_{\tilde{p}}(x)$ of X and each $A \in \tau_2$ such that $f(\mu_{\tilde{p}}(x)) \in A$, there exists a fuzzy γ^* generalized open set B of X such that $\mu_{\tilde{p}}(x) \in B$ and $f(B) \leq A$.

Proof:

Let $\mu_{\tilde{p}}(x)$ be a fuzzy point of X and $A \in \tau_2$ such that $f(\mu_{\tilde{p}}(x)) \in A$. Put $B = f^{-1}(A)$. Then by hypothesis, B is a fuzzy γ^* generalized open set S in X such that $\mu_{\tilde{p}}(x) \in B$ and $f(B) = f(f^{-1}(A)) \leq A$.

Theorem 3.1.19:

If f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ^* generalized continuous mapping then for each fuzzy point $\mu_{\tilde{p}}(x)$ of X and each $A \in \tau_2$ such that $f(\mu_{\tilde{p}}(x))\ _q\ A$, there exists a fuzzy γ^* generalized open set B of X such that $\mu_{\tilde{p}}(x))\ _q\ B$ and $f(B) \leq A$.

Proof:

Let $\mu_{\tilde{p}}(x)$ be a fuzzy point of X and $A \in \tau_2$ such that $f(\mu_{\tilde{p}}(x))\ _q\ A$. Put $B = f^{-1}(A)$. Then by hypothesis, B is a fuzzy γ^* generalized open set in X such that $\mu_{\tilde{p}}(x)_q$ B and $f(B) = f(f^{-1}(A)) \leq A$.

Definition 3.1.20:

If every fuzzy γ^* generalized closed set in (X,τ) is a fuzzy γ closed set in (X, τ), then the space can be called as a fuzzy γ^* $T_{1/2}$ space.

Example 3.1.21:

Let $X = \{a, b\}$ and $\tau = \{\bar{0},\bar{1}, G_1, G_2\}$ be a fuzzy topology on X, where $G_1 = \langle x, (0.6_a, 0.5_b)\rangle$, $G_2 = \langle x, (0.6_a, 0.6_b)\rangle$. Then $\tau = \{\bar{0},\bar{1}, G_1, G_2\}$ is a fuzzy topology on X and the space (X,τ) is a fuzzy $\gamma^*T_{1/2}$ space.

Definition 3.1.22:

A fuzzy topological space (X, τ) is a fuzzy $\gamma^*{}_cT_{1/2}$ space if every fuzzy γ^* generalized closed setis a fuzzy closed set in X.

Definition 3.1.23:

A fuzzy topological space (X, τ) is a fuzzy $\gamma^*{}_pT_{1/2}$ space if every fuzzy γ^* generalized closed set is a fuzzy pre closed set in X.

Theorem 3.1.24:

Let f: (X,τ_1) → (Y, τ_2) be a fuzzy γ^* generalized continuous mapping, then

(i) f is a fuzzy γ continuous mapping if X is a fuzzy $\gamma^*T_{1/2}$ space

(ii) f is a fuzzy continuous mapping if X is a fuzzy $\gamma^*{}_cT_{1/2}$ space

(iii) f is a fuzzy pre continuous mapping if X is a fuzzy $\gamma^*{}_pT_{1/2}$ space

Proof:

(i) Let V be a fuzzy closed set in Y. Then f^{-1}(V) is a fuzzy γ^* generalized closed set in X, by hypothesis. Since X is a fuzzy $\gamma^*T_{1/2}$ space, f^{-1}(V) is a fuzzy γ closed set in X. Hence f is a fuzzy γ continuous mapping.

(ii) Let V be a fuzzy closed set in Y. Then f^{-1}(V) is a fuzzy γ^* generalized closed set in X, by hypothesis. Since X is a fuzzy $\gamma^*{}_cT_{1/2}$ space, f^{-1}(V) is a fuzzy closed set in X. Hence f is a fuzzy continuous mapping.

(iii) Let V be a fuzzy closed set in Y. Then f^{-1}(V) is a fuzzy γ^* generalized closed set in X, by hypothesis. Since X is a fuzzy $\gamma^*{}_pT_{1/2}$ space, f^{-1}(V) is a fuzzy pre closed set in X. Hence f is a fuzzy pre continuous mapping.

Theorem 3.1.25:

Let f: (X,τ_1) → (Y, τ_2) be a fuzzy γ^* generalized continuous mapping and g:(Y,τ_2) → (Z, τ_3) be a fuzzy continuous mapping then g ∘ f : (X,τ_1) → (Z, τ_3) is a fuzzyγ^* generalized continuous mapping.

Proof:

Let V be a fuzzy closed set in Z. Then $g^{-1}(V)$ is a fuzzy closed set in Y, by hypothesis. Since f is a fuzzy γ^* generalized continuous mapping, $f^{-1}(g^{-1}(V))$ is a fuzzy γ^* generalized closed set in X. Hence $g \circ f$ is a fuzzy γ^* generalized continuous mapping.

Theorem 3.1.26:

The composition of two fuzzy γ^* generalized continuous mapping is a fuzzy γ^* generalized continuous mapping if Y is a fuzzy $\gamma^*{}_cT_{1/2}$ space.

Proof:

Let V be a fuzzy closed set in Z. Then $g^{-1}(V)$ is a fuzzy γ^* generalized closed set in Y, by hypothesis. Since Y is a fuzzy $\gamma^*{}_cT_{1/2}$ space, $g^{-1}(V)$ is a fuzzy closed set in Y. Therefore $f^{-1}(g^{-1}(V))$ is a fuzzy γ^* generalized closed set in X, by hypothesis. Hence $g \circ f$ is a fuzzy γ^* generalized continuous mapping. .

Theorem 3.1.27:

Let f: $(X,\tau_1) \to (Y, \tau_2)$ be a mapping. Then the following conditions are equivalent if X and Y are fuzzy $\gamma^*T_{1/2}$ spaces:

(i) f is a fuzzy γ^* generalized continuous mapping

(ii) $cl(int(f^{-1}(B))) \wedge int(cl(f^{-1}(B))) \leq f^{-1}(cl(B))$ for each fuzzy closed set B in Y

(iii) $f^{-1}(int(B)) \leq cl(int(f^{-1}(B))) \vee int(cl(f^{-1}(B)))$ for each fuzzy open set B in Y

(iv) $f(int(cl(A)) \wedge cl(int(A))) \leq cl(f(A))$ for each fuzzy set A of X.

Proof:

(i) ⇒ (ii) Let B be a fuzzy closed set in Y. Then $f^{-1}(B)$ is a fuzzy γ^* generalized closed set in X. Since X is a fuzzy $\gamma^*T_{1/2}$ space, $f^{-1}(B)$ is a fuzzy γ closed set in X. Therefore $cl(int(f^{-1}(B))) \wedge int(cl(f^{-1}(B))) \leq f^{-1}(B) = f^{-1}(cl(B))$.

(ii) ⇒(iii) can be easily proved by taking complement in (ii).

(iii) ⇒(iv) Let $A \in X$. Then $B = f(A)$ in Y and therefore $A \leq f^{-1}(f(A)) \leq f^{-1}(B)$. Here $int(f(A)) = int(B)$ is a fuzzy open set in Y. Then (iii) implies that

$f^{-1}(int(B)) \leq cl(int(f^{-1}(int(B)))) \vee int(cl(f^{-1}(int(B)))) \leq cl(int(f^{-1}(B))) \vee int(cl(f^{-1}(B)))$. Now $(cl(int(A^c)) \vee int(cl(A^c)))^c \leq (cl(int(f^{-1}(B^c) \vee int(cl(f^{-1}(B^c)))^c \leq (f^{-1}(int(B^c)))^c$. Therefore $int(cl(A)) \wedge cl(int(A)) \leq f^{-1}(cl(B))$. Now $f(int(cl(A)) \wedge cl(int(A))) \leq f(f^{-1}(cl(B))) \leq cl(f(A))$.

(iv) ⇒(i) Let B be any fuzzy closed set in Y, then $f^{-1}(B)$ is a fuzzy set in X. By hypothesis $f(int(cl(f^{-1}(B))) \wedge cl(int(f^{-1}(B)))) \leq cl(f(f^{-1}(B))) \leq cl(B) = B$. Now $(int(cl(f^{-1}(B))) \wedge cl(int(f^{-1}(B)))) \leq f^{-1}(f(int(cl(f^{-1}(B))) \wedge cl(int(f^{-1}(B)))) \leq f^{-1}(B)$. This implies $f^{-1}(B)$ is a fuzzy γ closed set and hence it is a fuzzy $\gamma*$ generalized closed set in X. Thus f is a fuzzy $\gamma*$ generalized continuous mapping.

Theorem 3.1.29:

A mapping f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ is a fuzzy $\gamma*$ generalized continuous mapping if $cl(int(cl(f^{-1}(A)))) \leq f^{-1}(cl(A))$ for every fuzzy set A in Y.

Proof:

Let A be a fuzzy closed set in Y. By hypothesis, $cl(int(cl(f^{-1}(A)))) \leq f^{-1}(cl(A)) = f^{-1}(A)$. Therefore $f^{-1}(A)$ is a fuzzy α closed set and hence it is a fuzzy $\gamma*$ generalized closed set. Thus f is a fuzzy $\gamma*$ generalized continuous mapping.

Theorem 3.1.30:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a mapping from a fuzzy topological space X into a fuzzy topological space Y. Then the following conditions are equivalent if X is a fuzzy $\gamma*T_{1/2}$ space:

i. f is a fuzzy $\gamma*$ generalized continuous mapping
ii. $cl(int(f^{-1}(A))) \wedge int(cl(f^{-1}(A))) \leq f^{-1}(cl(A))$ for every fuzzy set A in Y

Proof:

(i) ⇒(ii) Let A be a fuzzy set in Y. Then cl(A) is a fuzzy closed set in Y. By hypothesis, $f^{-1}(cl(A))$ is a fuzzy $\gamma*$ generalized closed set in X. Since X is a fuzzy $\gamma*T_{1/2}$ space, $f^{-1}(cl(A))$ is a fuzzy γ closed set in X. Therefore $cl(int(f^{-1}(cl(A)))) \wedge int(cl(f^{-1}(cl(A)))) \leq f^{-1}(cl(A))$. Now $cl(int(f^{-1}(A))) \wedge int(cl(f^{-1}(A))) \leq cl(int(f^{-1}(cl(A)))) \wedge int(cl(f^{-1}(cl(A)))) \leq f^{-1}(cl(A))$.

(ii) ⇒(i) Let A be a fuzzy closed set in Y. By hypothesis cl(int(f^{-1}(A))) ∧ int(cl(f^{-1}(A))) ≤ f^{-1}(cl(A)) = f^{-1}(A). This implies f^{-1}(A) is a fuzzy γ closed set in X and hence it is a fuzzy γ^* generalized closed set. Thus f is a fuzzy γ^* generalized continuous mapping.

Section 3.2

Fuzzy almost γ^* generalized continuous mappings

In this chapter we have introduced fuzzy almostγ^* generalized continuous mappings and investigated some of their properties.

Definition 3.2.1:

A mapping f: (X,τ_1) → (Y, τ_2) is called a fuzzy almost γ^* generalized continuous mapping if f^{-1}(V) is a fuzzy γ^* generalized closed set in (X, τ_1) for every fuzzy regular closed set V of (Y, τ_2).

Example 3.2.2:

Let X = {a, b} and Y = {u, v}. Then τ_1 = {$\bar{0}$, $\bar{1}$, G_1} and τ_2 = {$\bar{0}$, $\bar{1}$, G_2} are fuzzy topologies on X and Y respectively, where G_1 = ⟨x, (0.6_a, 0.6_b)⟩ and G_2 = ⟨y, (0.5_u, 0.5_v)⟩. Then (X,τ_1) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: (X,τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. The fuzzy set G_2^c = ⟨y, (0.5_u, 0.5_v)⟩ is a fuzzy regular closed set in Y. Then f^{-1}(G_2^c) = ⟨x, (0.5_a, 0.5_b)⟩ is a fuzzy γ^* generalized closed set in (X, τ_1) as f^{-1}(G_2^c) ≤ G_1 and cl(int(f^{-1}(G_2^c))) ∧ int(cl(f^{-1}(G_2^c))) = $\bar{0}$ ≤ G_1, where G_1 is a fuzzy open set in X. Therefore f is a fuzzy almost γ^* generalized continuous mapping.

Theorem 3.2.3:

Every fuzzy continuous mapping is a fuzzy almost γ^* generalized continuous mapping but not conversely in general.

Proof:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy continuous mapping. Let V be a fuzzy regular closed set in Y. Since every fuzzy regular closed is a fuzzy closed set in Y, V is a fuzzy closed set in Y. Then $f^{-1}(V)$ is a fuzzy closed set in X. Since every fuzzy closed set is afuzzy γ^* generalized closed set, $f^{-1}(V)$ is a fuzzy γ^* generalized closed set in X. Hence f is a fuzzy almost γ^* generalized continuous mapping.

Example 3.2.4:

Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_2\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.6_a, 0.6_b)\rangle$ and $G_2 = \langle y, (0.5_u, 0.5_v)\rangle$. Then (X,τ_1) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ by f(a) = u, f(b) = v. The fuzzy set $G_2^c = \langle y, (0.5_u, 0.5_v)\rangle$ is a fuzzy regular closed set in Y. Then f is a fuzzy almost γ^* generalized continuous mapping but not a fuzzy continuous mapping. Since the fuzzy set G_2^c = is a fuzzy closed set in Y, but $f^{-1}(G_2^c)$ is not a fuzzy closed in X as $cl(f^{-1}(G_2^c)) = G_1 \neq f^{-1}(G_2^c)$.

Theorem 3.2.5:

Every fuzzy semi continuous mapping is a fuzzy almost γ^* generalized continuous mapping but not conversely in general.

Proof:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy semi continuous mapping. Let V be a fuzzy regular closed set in Y. Since every fuzzy regular closed set is a fuzzy semi closed set in Y, V is a fuzzy semi closed set in Y. Then $f^{-1}(V)$ is a fuzzy semi closed set in X. Since every fuzzy semi closed set is afuzzy γ^* generalized closed set, $f^{-1}(V)$ is a fuzzy γ^* generalized closed set in X. Hence f is a fuzzy almost γ^* generalized continuous mapping.

Example 3.2.6:

Let X = {a, b} and Y = {u, v}. Then τ_1 = {$\bar{0}$, $\bar{1}$, G_1,G_2 } and τ_2 = {$\bar{0}$, $\bar{1}$, G_3} are fuzzy topologies on X and Y respectively, where G_1 = ⟨x, $(0.6_a, 0.5_b)$⟩ and G_2 = ⟨x, $(0.4_a, 0.5_b)$⟩ and G_3 = ⟨y, $(0.5_u, 0.5_v)$⟩. Then (X,τ_1) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: (X,τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. Then f is a fuzzy almost γ^* generalized continuous mapping but not a fuzzy semi continuous mapping. Since the fuzzy set G_3^c = ⟨y, $(0.5_u, 0.5_v)$⟩is a fuzzy closed set in Y, but $f^{-1}(G_3^c)$ is not a fuzzy semi closed set in X as int(cl($f^{-1}(G_3^c)$)) = $G_1 \nleq$ $f^{-1}(G_3^c)$.

Theorem 3.2.7:

Every fuzzy pre continuous mapping is a fuzzy almost γ^* generalized continuous mapping but not conversely in general.

Proof:

Let f: (X,τ_1) → (Y, τ_2) be a fuzzy pre continuous mapping. Let V be a fuzzy regular closed set in Y. Since every fuzzy regular closed set is a fuzzy pre closed set in Y, V is a fuzzy pre closed in Y. Then $f^{-1}(V)$ is a fuzzy pre closed set in X. Since every fuzzy pre closed set is afuzzy γ^* generalized closed set, $f^{-1}(V)$ is a fuzzy γ^* generalized closed set in X. Hence f is a fuzzy almost γ^* generalized continuous mapping.

Example 3.2.8:

Let X = {a, b} and Y = {u, v}. Then τ_1 = {$\bar{0}$, $\bar{1}$, G_1,G_2} and τ_2 = {$\bar{0}$, $\bar{1}$, G_3} are fuzzy topologies on X and Y respectively, where G_1 = ⟨x, $(0.6_a, 0.6_b)$⟩ and G_2 = ⟨x, $(0.4_a, 0.5_b)$⟩ and G_3 = ⟨y, $(0.5_u, 0.5_v)$⟩. Then (X,τ_1) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: (X,τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. Then f is a fuzzy almost γ^* generalized continuous mapping but not a fuzzy pre continuous mapping. Since the fuzzy set G_3^c = ⟨y, $(0.5_u, 0.5_v)$⟩is a fuzzy closed set in Y, but $f^{-1}(G_3^c)$ is not a fuzzy pre closed set as cl(int($f^{-1}(G_3^c)$)) = $G_2^c \nleq f^{-1}(G_3^c)$.

Theorem 3.2.9:

Every fuzzy α continuous mapping is a fuzzy almost γ^* generalized continuous mapping but not conversely in general.

Proof:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy α continuous mapping. Let V be a fuzzy regular closed set in Y. Since every fuzzy regular closed set is a fuzzy α closed set in Y, V is a fuzzy α closed set in Y. Then $f^{-1}(V)$ is a fuzzy α closed set in X. Since every fuzzy α closed set is afuzzy γ^* generalized closed set, $f^{-1}(V)$ is a fuzzy γ^* generalized closed set in X. Hence f is a fuzzy almost γ^* generalized continuous mapping.

Example 3.2.10:

Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1, G_2\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_3\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.6_a, 0.6_b)\rangle$ and $G_2 = \langle x, (0.4_a, 0.5_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.5_v)\rangle$. Then (X,τ_1) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ by f(a) = u, f(b) = v. Then f is a fuzzy almost γ^* generalized continuous mapping but not a fuzzy α continuous mapping. Since the fuzzy set $G_3^c = \langle y, (0.5_u, 0.5_v)\rangle$is a fuzzy closed set in Y, but $f^{-1}(G_3^c)$ is not a fuzzy α closed set in X as $cl(int(cl(f^{-1}(G_3^c)))) = G_2^c \nsubseteq f^{-1}(G_3^c)$.

Theorem 3.2.11:

Every fuzzy γ continuous mapping is a fuzzy almost γ^* generalized continuous mapping but not conversely in general.

Proof:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ continuous mapping. Let V be a fuzzy regular closed set in Y. Since every fuzzy regular closed set is a fuzzy γ closed set in Y, V is a fuzzy γ closed set in Y. Then $f^{-1}(V)$ is a fuzzy γ closed set in X. Since every fuzzy γ closed set is a fuzzy γ^* generalized closed set, $f^{-1}(V)$ is a fuzzy γ^* generalized closed set in X. Hence f is a fuzzy almost γ^* generalized continuous mapping.

Example 3.2.12:

Let X = {a, b} and Y = {u, v}. Then τ_1 = {$\bar{0}$, $\bar{1}$, G_1,G_2} and τ_2 = {$\bar{0}$, $\bar{1}$, G_3} are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.6_a, 0.4_b)\rangle$ and $G_2 = \langle x, (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.3_v)\rangle$. Then (X,$\tau_1$) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: (X,τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. Then f is a fuzzy almost γ* generalized continuous mapping but not a fuzzy γ continuous mapping. Since the fuzzy set $G_3^c = \langle y, (0.7_u, 0.7_v)\rangle$is a fuzzy closed set in Y, but $f^{-1}(G_3^c)$ is not a fuzzy γ closed set in X as $cl(int(f^{-1}(G_3^c))) \wedge int(cl(f^{-1}(G_3^c)))$ = $\bar{1} \nleq f^{-1}(G_3^c)$.

Theorem 3.2.13:

Every fuzzy generalized continuous mapping is a fuzzy almost γ* generalized continuous mapping but not conversely in general.

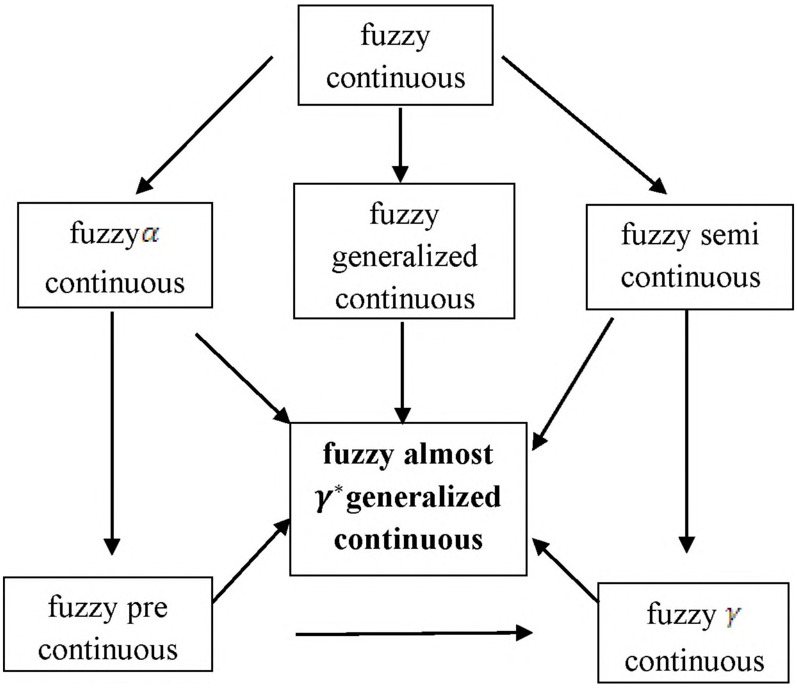
Proof:

Let f: (X,τ_1) → (Y, τ_2) be a fuzzy generalized continuous mapping. Let V be a fuzzy regular closed set in Y. Since every fuzzy regular closed set is a fuzzy generalized closed set in Y, V is a fuzzy generalized closed set in Y. Then $f^{-1}(V)$ is a fuzzy generalized closed set in X. Since every fuzzy generalized closed set is afuzzy γ* generalized closed set, $f^{-1}(V)$ is a fuzzy γ* generalized closed set in X. Hence f is a fuzzy almost γ* generalized continuous mapping.

Example 3.2.14:

Let X = {a, b} and Y = {u, v}. Then τ_1 = {$\bar{0}$, $\bar{1}$, G_1, G_2} and τ_2 = {$\bar{0}$, $\bar{1}$, G_3}are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.6_a, 0.6_b)\rangle$ and $G_2 = \langle x, (0.4_a, 0.3_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.5_v)\rangle$. Then (X,$\tau_1$) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: (X,τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. Then f is a fuzzy almost γ* generalized continuous mapping but not a fuzzy generalized continuous mapping. Since the fuzzy set $G_3^c = \langle y, (0.5_u, 0.5_v)\rangle$is a fuzzy closed set in Y, but $f^{-1}(G_3^c)$ is not a fuzzy generalized closed set in X as $cl(f^{-1}(G_3^c)) = G_2^c \nleq G_1$.

The relation between various types of fuzzy continuity is given in the following diagram.



Theorem 3.2.15:

A mapping f: (X, τ_1) → (Y, τ_2) is a fuzzy almost γ^* generalized continuous mapping if and only if the inverse image of each fuzzy regular open set in Y is afuzzy γ^* generalized open set in X.

Proof: Necessity:

Let A be a fuzzy regular open set in Y. Then A^c is a fuzzy regular closed set in Y. Since f is a fuzzy almost γ^* generalized continuous mapping, $f^{-1}(A^c)$ is a fuzzy γ^* generalized closed set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a fuzzy γ^* generalized open setin X.

Sufficiency:

Let A be a fuzzy regular closed set in Y. Then A^c is a fuzzy regular open set in Y. By hypothesis $f^{-1}(A^c)$ is a fuzzy γ^* generalized open set in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a fuzzy γ^* generalized closed set in X. Hence f is a fuzzy almost γ^* generalized continuous mapping.

Theorem 3.2.16:

Let f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping. If $f^{-1}(int(B)) \leq int(f^{-1}(B))$ for every fuzzy set B in Y, then f is a fuzzy almost γ^* generalized continuous mapping.

Proof:

Let $B \leq Y$ be a fuzzy regular open set. By hypothesis, $f^{-1}(int(B)) \leq int(f^{-1}(B))$. Since B is a fuzzy regular open set, it is a fuzzy open set in Y. Therefore int(B) = B. Hence $f^{-1}(B) = f^{-1}(int(B)) \leq int(f^{-1}(B)) \leq f^{-1}(B)$. This implies $f^{-1}(B)$ is a fuzzy open set in X and hence $f^{-1}(B)$ is a fuzzy γ^* generalized open set in X. Thus f is a fuzzy almost γ^* generalized continuous mapping.

Theorem 3.2.17:

Let f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping. If $\gamma cl(f^{-1}(B)) \leq f^{-1}(\gamma cl(B))$ for every fuzzy set B in Y, then f is a fuzzy almost γ^* generalized continuous mapping.

Proof:

Let $B \leq Y$ be a fuzzy regular closed set. By hypothesis, $\gamma cl(f^{-1}(B)) \leq f^{-1}(\gamma cl(B))$. Since B is a fuzzy regular closed set, it is a fuzzy γ closed set in Y. Therefore $\gamma cl(B) = B$. Hence $f^{-1}(B) = f^{-1}(\gamma cl(B)) \geq \gamma cl(f^{-1}(B)) \geq f^{-1}(B)$. This implies $f^{-1}(B)$ is a fuzzy γ closed set in X and hence $f^{-1}(B)$ is a fuzzy γ^* generalized closed set in X. Thus f is a fuzzy almost γ^* generalized continuous mapping.

Definition 3.2.18:

Let A be a fuzzy set in a fuzzy topological space in (X, τ). Then the γ^* generalized interior and γ^* generalized closure of A are defined as

$$\gamma^*gint(A) = \vee \{G \,/\, G \text{ is a } F\gamma^*GOS \text{ in X and } G \leq A\}$$

$$\gamma^*gcl(A) = \wedge \{K \,/\, K \text{ is a } F\gamma^*GCS \text{ in X and } A \leq K\}$$

It is to be noted that for any fuzzy set A in (X, τ), we have $\gamma^*gcl(A^c) = (\gamma^*gint(A))^c$ and $\gamma^*gint(A^c) = (\gamma^*gcl(A))^c$.

Theorem 3.2.19:

Let f: $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a mapping. If f is a fuzzy almost γ^* generalized continuous mapping, then $\gamma^*gcl(f^{-1}(A)) \leq f^{-1}(cl(A))$ for every fuzzy γ open set in Y.

Proof:

Let A be a fuzzy γ open set in Y. Then $A \leq cl(int(A)) \vee int(cl(A))$. This implies $cl(A) \leq cl(cl(int(A))) \vee cl(int(A))) = cl(int(A)) \vee cl(int(cl(A))) = cl(int(cl(A))) \leq cl(cl(A)) = cl(A)$. Therefore $cl(A) = cl(int(cl(A)))$. Hence $cl(A) \in FRC(Y)$. By hypothesis $f^{-1}(cl(A))$ is a fuzzy γ^* generalized closed set in X. Then $\gamma^*gcl(f^{-1}(cl(A))) = f^{-1}(cl(A))$. Now $\gamma^*gcl(f^{-1}(A)) \leq \gamma^*gcl(f^{-1}(cl(A))) = f^{-1}(cl(A))$. That is $\gamma^*gcl(f^{-1}(A)) \leq f^{-1}(cl(A))$.

Theorem 3.2.20:

A function f: $X \rightarrow Y$ is a fuzzy almost γ^* generalized continuous mapping if and only if $f^{-1}(A) \leq \gamma^*gint(f^{-1}(int(cl(A))))$ for a fuzzy pre open set A of Y.

Proof:

Necessity:

Let A be a fuzzy pre open set in Y. Then $A \leq int(cl(A))$ and $int(cl(A))$ is a fuzzy regular open set in Y. Since f is a fuzzy almost γ^*generalized continuous mapping, $f^{-1}(int(cl(A)))$ is a fuzzy γ^*generalized open set in X and hence we obtain that $f^{-1}(A) \leq f^{-1}(int(cl(A))) \leq \gamma^*gint(f^{-1}(int(cl(A))))$.

Sufficiency:

Let A be a fuzzy regular open set in Y. Then A is a fuzzy pre open set in Y. By hypothesis, $f^{-1}(A) \leq \gamma^*gint(f^{-1}(int(cl(A)))) = \gamma^*gint(f^{-1}(A)) \leq f^{-1}(A)$ is a fuzzy γ^* generalized open set in X and hence f is a fuzzy almost γ^* generalized continuous mapping.

Theorem 3.2.21:

Let $\mu_{\tilde{p}}(x)$ be a fuzzy point in X. A mapping f: $X \rightarrow Y$ is a fuzzy almost γ^* generalized continuous mapping if for every fuzzy open set A in Y with $\mu_{\tilde{p}}(x) \in A$, there exists a fuzzy open set B in X with $\mu_{\tilde{p}}(x) \in B$ such that $f^{-1}(A)$ is fuzzy dense in B.

Proof:

Let A be a fuzzy regular open set in Y. Then A is a fuzzy open set in Y. Let $\mu_{\tilde{p}}(x) \in B$, then there exists a fuzzy open set in B in X such that $\mu_{\tilde{p}}(x) \in B$ and $cl(f^{-1}(A)) = B$, by hypothesis. Therefore $cl(f^{-1}(A))$ is also a fuzzy open set in X and $int(cl(f^{-1}(A))) = cl(f^{-1}(A))$. Now $f^{-1}(A) \leq cl(f^{-1}(A)) = int(cl(f^{-1}(A))) \leq int(cl(f^{-1}(A))) \vee cl(int(f^{-1}(A)))$. This implies $f^{-1}(A)$ is a fuzzy γ open set in X and hence fuzzy γ^* generalized closed set in X. Thus f is a fuzzy almost γ^* generalized continuous mapping.

Theorem 3.2.22:

Let f: X → Y be a mapping where X is a fuzzy $\gamma^*T_{1/2}$ space. Then the following are equivalent:

(i) f is a fuzzy almost γ^* generalized continuous mapping

(ii) $\gamma cl(f^{-1}(A)) \leq f^{-1}(cl(A))$ for every fuzzy semi open set A in Y

(iii) $f^{-1}(A) \leq \gamma int(f^{-1}(int(cl(A))))$ for every fuzzy pre open set A in Y

Proof:

(i) ⟹ (ii) Let A be a fuzzy semi open set in Y. Then $A \leq cl(int(A))$. Now $cl(A) \leq cl(cl(int(A))) \leq cl(int(cl(A))) \leq cl(A)$. Therefore $cl(A) = cl(int(cl(A)))$. This implies cl(A) is a fuzzy regular closed set in Y. By hypothesis, $f^{-1}(cl(A))$ is a fuzzy γ^* generalized closed set in X. Since X is a fuzzy $\gamma^*T_{1/2}$ space, $f^{-1}(cl(A))$ is a fuzzy γ closed set in X. Therefore $\gamma cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$. Now $\gamma cl(f^{-1}(A)) \leq \gamma cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$. Thus $\gamma cl(f^{-1}(A)) \leq f^{-1}(cl(A))$.

(ii) ⟹ (i) Let A be a fuzzy regular closed set in Y. Then $A = cl(int(A))$. Therefore A is a fuzzy semi open set in Y. By hypothesis $\gamma cl(f^{-1}(A)) \leq f^{-1}(cl(A)) = f^{-1}(A) \leq \gamma cl(f^{-1}(A))$. Thus $f^{-1}(A)$ is a fuzzy γ closed set and hence $f^{-1}(A)$ is a fuzzy γ^* generalized closed set in X. Thus f is a fuzzy almost γ^* generalizedcontinuous mapping.

(i) $\Longrightarrow$ (iii) Let A be a fuzzy pre open set in Y. Then $A \leq int(cl(A))$. Since $int(cl(A))$ is a fuzzy regular open set in Y, by hypothesis, $f^{-1}(int(cl(A))$ is a fuzzy γ^* generalized open set in X. Since X is a fuzzy $\gamma^*T_{1/2}$ space, $f^{-1}(int(cl(A)))$ is a fuzzy γ open set in X. Therefore $f^{-1}(A) \leq f^{-1}(int(cl(A))) = \gamma int(f^{-1}(int(cl(A))))$.

(iii) $\Longrightarrow$ (i) Let A be a fuzzy regular open set in Y. Then A is a fuzzy pre open set in X. By hypothesis, $f^{-1}(A) \leq \gamma int(f^{-1}(int(cl(A)))) = \gamma int(f^{-1}(A)) \leq f^{-1}(A)$, this implies $f^{-1}(A)$ is a fuzzy γ open set in X and hence is a fuzzy γ^* generalized open set in X. Thus f is a fuzzy almost γ^*generalized continuous mapping.

CHAPTER 4

Fuzzy γ^* generalized irresolute mappings

In this section we have introduced fuzzy γ^* generalized irresolute mappings and studied some of their properties.

Definition 4.1:

A mapping f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ is called a fuzzy γ^* generalized irresolute mapping if $f^{-1}(V)$ is a fuzzy γ^* generalized closed set in (X, τ_1) for every fuzzy γ^* generalized closed set V of (Y, τ_2).

Example 4.2:

Let X = {a, b} and Y = {u, v}. Then $\tau_1 = \{\bar{0}, \bar{1}, G_1\}$ and $\tau_2 = \{\bar{0}, \bar{1}, G_2\}$ are fuzzy topologies on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b)\rangle$ and $G_2 = \langle y, (0.4_u, 0.5_v)\rangle$. Then (X,τ_1) and (Y, τ_2) are fuzzy topology spaces. Define a mapping f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ by f(a) = u, f(b) = v.Then f is a fuzzy γ^* generalized irresolute mapping.

Theorem 4.3:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ^* generalized irresolute mapping, then f is a fuzzy γ^* generalized continuous mapping but not conversely in general.

Proof:

Let f be a fuzzy γ^* generalized irresolute mapping. Let V be a fuzzy closed set in Y. Then V is a fuzzy γ^* generalized closed set and by hypothesis $f^{-1}(V)$ is a fuzzy γ^* generalized closed set in X. Hence f is a fuzzy γ^* generalized continuous mapping.

Example 4.4:

Let X = {a, b} and Y = {u, v}. Then τ_1 = {$\bar{0}$, $\bar{1}$, G_1, G_2 } and τ_2 = {$\bar{0}$, $\bar{1}$, G_3} are fuzzy topologies on X and Y respectively, where G_1 = ⟨x, $(0.6_a, 0.4_b)$⟩,G_2 = ⟨x, $(0.8_a, 0.8_b)$⟩ and G_3 = ⟨y, $(0.3_u, 0.3_v)$⟩. Then (X,τ_1) and (Y, τ_2) are fuzzy topological spaces. Define a mapping f: (X,τ_1) → (Y, τ_2) by f(a) = u, f(b) = v. Then f is a fuzzy γ* generalized continuous mapping. Now the fuzzy set A = ⟨y, $(0.7_u, 0.7_v)$⟩ is a fuzzy γ* generalized closed set in Y, but $f^{-1}(A)$ isnot a fuzzy γ* generalized closed set in (X, τ_1) as $f^{-1}(A)$ = ⟨x, $(0.7_a, 0.7_b)$⟩ ≤ G_2 whereas cl(int(f^{-1}(A)) ∧ int(cl(f^{-1}(A)) = $\bar{1}$ ≰ G_2.Therefore f is not a fuzzy γ* generalized irresolute mapping.

Theorem 4.5:

A mapping f: (X,τ_1) → (Y, τ_2) is called a fuzzy γ* generalized irresolute mapping if and only if the inverse image of eachfuzzy γ* generalized open set in Y is a fuzzy γ* generalized open set in X.

Proof:

The proof is obvious from the Definition 3.2.1, since $f^{-1}(A^c) = (f^{-1}(A))^c$.

Theorem 4.6:

Let f: (X,τ_1) → (Y, τ_2) be a fuzzy γ* generalized irresolute mapping, then

(i) f is a fuzzy γ irresolute mapping if X is a fuzzy γ* $T_{1/2}$ space

(ii) f is a fuzzy pre irresolute mapping if X is a fuzzy $\gamma *_p$ $T_{1/2}$ space

Proof:

(i) Let V be a fuzzy γ closed set in Y. Then V is a fuzzy γ* generalized closed set in Y. Therefore $f^{-1}(V)$ is a fuzzy γ* generalized closed set in X, by hypothesis. Since X is a fuzzy γ* $T_{1/2}$ space, $f^{-1}(V)$ is a fuzzy γ closed set in X. Hence f is a fuzzy γ irresolute mapping.

(ii) Let V be a fuzzy pre closed set in Y. Then V is a fuzzy γ* generalized closed set in Y. Therefore $f^{-1}(V)$ is a fuzzy γ* generalized closed set in X, by hypothesis. Since X is a fuzzy $\gamma^*{}_pT_{1/2}$ space, $f^{-1}(V)$ is a fuzzy pre closed set in X. Hence f is a fuzzy pre irresolute mapping.

Theorem 4.7:

Composition of two fuzzy γ^* generalized irresolute mapping is a fuzzy γ^* generalized irresolute mapping.

Proof:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ and g: $(Y,\tau_2) \rightarrow (Z, \tau_3)$ be fuzzy γ^* generalized irresolute mappings. Let V be a fuzzy γ^* generalized closed set in Z. Then $g^{-1}(V)$ is a fuzzy γ^* generalized closed set in Y. Since f is a fuzzy γ^* generalized irresolute, $f^{-1}(g^{-1}(V))$ is a fuzzy γ^* generalized closed set in X, by hypothesis. Hence $g \circ f$ is a fuzzy γ^* generalized irresolute mapping.

Theorem 4.8:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ^* generalized irresolute mapping and g: $(Y,\tau_2) \rightarrow (Z, \tau_3)$ be a fuzzy γ^* generalized continuous mapping, then $g \circ f$: $(X,\tau_1) \rightarrow (Z, \tau_3)$ is a fuzzy γ^* generalized continuous mapping.

Proof:

Let V be a fuzzy closed set in Z. Then $g^{-1}(V)$ is a fuzzy γ^* generalized closed set in Y. Since f is a fuzzy γ^* generalized irresolute mapping, $f^{-1}(g^{-1}(V))$ is a fuzzy γ^* generalized closed set in X. Hence $g \circ f$ is a fuzzy γ^* generalized continuous mapping.

Theorem 4.9:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be fuzzy γ^* generalized irresolute mapping and g: $(Y,\tau_2) \rightarrow (Z, \tau_3)$ be a fuzzy continuous mapping, then $g \circ f$: $(X,\tau_1) \rightarrow (Z, \tau_3)$ is a fuzzy γ^* generalized continuous mapping.

Proof:

Let V be a fuzzy closed set in Z. Then $g^{-1}(V)$ is a fuzzy closed set in Y. Since every fuzzy closed set is afuzzy γ^* generalized closed set, $g^{-1}(V)$ is a fuzzy

γ^*generalized closed set in Y. Therefore $f^{-1}(g^{-1}(V))$ is a fuzzy γ^* generalized closed set in X, by hypothesis. Hence $g \circ f$ is a fuzzy γ^* generalized continuous mapping.

Theorem 4.10:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ^* generalized continuous mapping and g: $(Y,\tau_2) \rightarrow (Z, \tau_3)$ be a fuzzy γ^* generalized irresolute mapping, then $g \circ f$: $(X,\tau_1) \rightarrow (Z, \tau_3)$ is a fuzzy γ^* generalized irresolute mapping if Y is a fuzzy $\gamma^*{}_cT_{1/2}$ space.

Proof:

Let V be a fuzzy γ^* generalized closed set in Z. Then $g^{-1}(V)$ is a fuzzy γ^* generalized closed set in Z, by hypothesis. Since Y is a fuzzy $\gamma^*{}_cT_{1/2}$ space $g^{-1}(V)$ is a fuzzy closed set in Y. Therefore, $f^{-1}(g^{-1}(V))$ is a fuzzy γ^* generalized closed set in X, by hypothesis. Hence $g \circ f$ is a fuzzy γ^* generalized irresolute mapping.

Theorem 4.11:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a mapping. Then the following conditions are equivalent if X and Y are fuzzy $\gamma^*T_{1/2}$ spaces:

(i) f is a fuzzy γ^* generalized irresolute mapping

(ii) $f^{-1}(B)$ is a fuzzy γ^* generalized open set in X for each fuzzy γ^* generalized open set B in Y

(iii) $f^{-1}(\gamma int(B)) \leq \gamma int(f^{-1}(B))$ for each fuzzy set B of Y

(iv) $\gamma cl(f^{-1}(B)) \leq f^{-1}(\gamma cl(B))$ for each fuzzy set B of Y

Proof:

(i) $\Leftrightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (iii) Let B be any fuzzy set in Y and $\gamma int(B) \leq B$. Also $f^{-1}(\gamma int(B)) \leq f^{-1}(B)$. Since $\gamma int(B)$ is a fuzzy γ open set in Y, it is a fuzzy γ^* generalized open set in Y. Therefore $f^{-1}(\gamma int(B))$ is a fuzzy γ^* generalized open set in X, by hypothesis. Since X is a fuzzy $\gamma^*T_{1/2}$ space, $f^{-1}(\gamma int(B))$ is a fuzzy γ open set in X. Hence $f^{-1}(\gamma int(B)) = \gamma int(f^{-1}(\gamma int(B))) \leq \gamma int(f^{-1}(B))$.

(iii) $\Rightarrow$ (iv) is obvious by taking complement in (iii).

(iv) ⇒ (i) Let B be any fuzzy γ^* generalized closed set in Y. Since Y is a fuzzy $\gamma^*T_{1/2}$ space, B is a fuzzy γ closed set in Y and γcl(B) = B. Hence $f^{-1}(B) = f^{-1}(\gamma cl(B)) \geq \gamma cl(f^{-1}(B)) \geq f^{-1}(B)$. Therefore $\gamma cl(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is a fuzzy γ closed set and hence it is a fuzzy γ^* generalized closed set in X. Thus f is a fuzzy γ^* generalized irresolute mapping.

Theorem 4.12:

Let f: $(X,\tau_1) \rightarrow (Y, \tau_2)$ be a fuzzy γ^* generalized irresolute mapping. Then $f^{-1}(B) \leq \gamma int(f^{-1}(cl(int(B)) \vee int(cl(B))))$ for every fuzzy γ^* generalized open set in Y, if X and Y are fuzzy $\gamma^*T_{1/2}$ spaces.

Proof:

Let B be a fuzzy γ^* generalized open set in Y. Then by hypothesis $f^{-1}(B)$ is a fuzzy γ^* generalized open set in X. Since X is a fuzzy $\gamma^*T_{1/2}$ space, $f^{-1}(B)$ is a fuzzy γ open set in X. Therefore $\gamma int(f^{-1}(B)) = f^{-1}(B)$. Since Y is a fuzzy $\gamma^*T_{1/2}$ space, B is a fuzzy γ open set in Y and $B \leq (cl(int(B)) \vee int(cl(B)))$. Now, $f^{-1}(B) = \gamma int(f^{-1}(B))$, implies $f^{-1}(B) \leq \gamma int(f^{-1}(cl(int(B)) \vee int(cl(B)))$.

Summary and conclusion

In **Chapter 1,** the preliminary definitions are discussed.

In **Chapter 2,** fuzzy γ^* generalized closed sets and fuzzy γ^*generalized open sets are introduced and their basic properties are investigated.

In **Chapter 3,** fuzzy γ^* generalized continuous mappings and fuzzy almost γ^* generalized mappings are introduced and their characterizations theorems are obtained.

In **Chapter 4,** fuzzy γ^*generalized irresolute mappings are introduced and their basic properties are obtained.

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