Maniraja

Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination – March 2021
I Semester

Class : I UG | **Time : 3 Hours**
Major : Mathematics | **Max. Marks: 100**

18BSMC01 Analytical Geometry and Classical Algebra

Part A | **10 x 1 = 10**
Choose the Correct Answer

1. If a circle bisects the circumference of another circle, their radical axis must pass through the _______ of second circle.
   a. centre b. chord c. intersection point d. any point

2. A system of circles to a system in which every pair of circles has the same radical axis is called a _________ system.
   a. Orthogonal b. Limiting Point c. *Coaxal* d. Perpendicular

3. The polar of the point with respect to the parabola $y^2 = 4ax$ is ______.
   a. $yy_1 = a(x + x_1)$ b. $yy_1 = 2a(x + x_1)$ c. $yy_1 = (x + x_1)$ d. $yy_1 = \frac{a}{2}(x + x_1)$

4. Chords of a parabola are drawn through a fixed point. Then the locus of the middle points is another _______.
   a. Circle b. Rectangle c. Hyperbola d. Parabola

5. The quotient and remainder of the equation $3x^3 + 8x^2 + 8x + 12$ is divided by $x - 4$are
   a. $3x^2 + 20x + 88$ & 364 b. $x^2 + 20x$ & 5
   c. $x^2 - 20x - 88$ & -5 d. $x - 88$ & -5

6. An equation ______ cannot have more positive roots than there are changes of sign in $f(x)$.
   a. $f(x) < 0$ b. $f(x) > 0$ c. $f(x) = 0$ d. $f(x) \leq 0$

7. The number of integers less than 729 and prime to it ___.
   a. 81 b. 200 c. 729 d. 486

8. The number of all the divisors of 360 _____.
   a. 24 b. 36 c. 120 d. 60

9. Find out the remainder when 2^{1000} is divisible by 17.
   a. -1 b. 25 c. 17 d. 1

10. A number N is divisible by 9 if and only if the sum of its digits is divisible by _____.
    a. 6 b. 3 c. 9 d. 2

Part B 5 x 6 = 30

Answer ALL questions

Each answer should not exceed 400 words or two pages

11.a. Show that the circles $x^2 + y^2 - 6x - 9y + 13 = 0$ and $x^2 + y^2 - 2x - 16y = 0$ touch each other and find the co-ordinates of the point of contact.

(or)

11.b. Prove that the tangents drawn from any point of a fixed circle of a coaxal system to two other fixed circles of the system are in a constant ratio.

12.a. Derive the equation of a chord of the parabola in terms of its middle point (x_1, y_1).

(or)

12.b. Find the locus of the poles of chords of a parabola subtending a right angle at the vertex.

13.a. Find the roots of the equation $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$

(or)

13.b. Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$ then obtain the transformed equation.

14.a. Find the highest power of 3 dividing 1000!

(or)

14.b. Derive the formula for divisors of a given number N.

15.a. If $a \equiv b(mod\ m)$ and $a_1 \equiv b_1(mod\ m)$ and if q, r are integers, then prove that $qa + ra_1 \equiv qb + rb_1(mod\ m)$

(or)

15.b. Show that$13^{2n+1} + 9^{2n+1}$ is divisible by 22.

Part C 5 x 12 = 60

Answer ALL questions

Each answer should not exceed 800 words or fourpages

16.a. i. Find the equation of the two circles passing through the two points (0, a), (0, -a) and touching the straight line $y = mx + c$. If the two circles cut at right angles show that $c^2 = a^2(2 + m^2)$.

ii. Obtain the equation of a circle which passes through the point (1, 2) bisect the circumference of the circle $x^2 + y^2 = 9$ and cuts orthogonally the circle $x^2 + y^2 - 2x + 8y - 7 = 0$.

(or)

16.b. i. Explain the Orthogonal circles of a coaxal system.

ii. Find the radical centre of the three circles $x^2 + y^2 - x + 3y - 3 = 0$, $x^2 + y^2 - 2x + 2y + 2 = 0 \text{and} x^2 + y^2 + 2x + 3y - 9 = 0$.

17.a. Show that the locus of the intersection of tangents to $y^2 = 4ax$ which intercept a constant length d on the directrix is $(y^2 - 4ax)(x + a)^2 = d^2x^2$.

(or)

17.b. The polar of a point P with respect to the parabola $y^2 = 4ax$ meets the curve in Q and R. Show that if P lies on the line $lx + my + n = 0$, then the middle point of QR lies on the parabola $l(y^2 - 4ax) + 2a(lx + my + n) = 0$.

18.a. i. Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.

ii. Diminish the roots of $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ by 2.

(or)

18.b. i. If a, b, c be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the equation whose roots are $bc - a^2, ca - b^2, ab - c^2$.

ii. Determine completely the nature of the roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$.

19.a. Prove that every composite number can be resolved into prime factors and this can be done only in one way.

(or)

19.b. Derive the value of $\phi(N)$. Also show that $\phi(N) = \phi(a).\phi(b)$ if N = ab where a and b are prime to one another.

20.a. i. Let x be congruent with r with respect to the modulus m, then prove that $f(x)$ will be congruent with $f(r)$ with respect to modulus m where $f(x)$ is a polynomial in x.

ii. Show that $x^5 - x$ is divisible by 30.

(or)

20.b. i. If $a \equiv b(mod\ m)$ and $a_1 \equiv b_1(mod\ m)$, then show that $aa_1 \equiv bb_1(mod\ m)$.

ii. If $ax \equiv bx(mod\ m)$ and if h is H.C.F. of x, m then $a \equiv b\ (mod \frac{m}{h})$.

iii. Find a number having the remainders 5, 4, 3, 2 when divided by 6, 5, 4, 3 respectively.

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